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SYSTEMATIC OBSERVATION OF VERBAL INTERACTION AS A METHOD OF COMPARING MATHEMATICS LESSONS.

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VARIOUS PATTERNS OF VERBAL INTERACTION WERE INVESTIGATED BETWEEN PUPILS AND TEACHERS IN CLASSROOMS WHERE FOUR TYPES OF MATHEMATICS LESSONS WERE EMPLOYED. TO DETERMINE IF SIGNIFICANT DIFFERENCES EXISTED IN THE AMOUNT OF VERBAL INTERACTION IN THESE 4 APPROACHES, 12 SENIOR HIGH SCHOOL AND FRESHMAN COLLEGE MATHEMATICS CLASSES WERE USED TO TEST THE AUTHORS' DEVELOPED SCHEME OF SYSTEMATIC OBSERVATION CALLED THE WRIGHT-PROCTOR OBSERVATIONAL INSTRUMENT. DISTINCTIVE PATTERNS IN THE AREAS OF CONTENT, PROCESS, AND ATTITUDE WERE ESTABLISHED FOR THE 4 TYPES OF CLASSES INVESTIGATED. THE RESULTS SUPPORTED THE EFFICACY OF THE INSTRUMENT. THE OPPORTUNITIES FOR OBSERVATION MADE POSSIBLE AN EXTENSIVE REVISION OF THE OBSERVER'S MANUAL WHICH APPEARS AS AN APPENDIX TO THIS REPORT. (GD)

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SYSTEMATIC OBSERVATION OF VERBAL INTERACTION AS A METHOD OF COMPARING MATHEMATICS LESSONS

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PREFACE

Considerable variety in mathematics classrooms of this community, St. Louis, Missouri, may be attributed to current concern that the teaching of mathematics should be more systematic at all levels of instruction, and that, particularly at early levels, the student should take part in the development as well as in the application of important principles. Some of the apparent differences may be considered a direct result of adoption of an experimental program, such as that of the University of Illinois Committee on School Mathematics; some are effected by individual teacher development of new material or adaptation of old; even more may be expected as gradual modification of College Entrance Examination Board papers occurs and as school systems find the need for replacing worn out texts.

Immediate comparison of experimental and non-experimental programs in mathematics may be made by direct observation of classroom discussions. An instrument specifically designed for study of mathematical discourse has been developed and successively modified by Wright and Proctor and was at hand for such comparison.

The proposal consequently was made and support was granted by the Cooperative Research Program, U. S. Office of Education and by Washington University, St. Louis, for intensive study of verbal interaction between teacher, pupils and subject matter in a small sample of four markedly different types of mathematics classrooms selected at the extremes of each continua of mathematical rigor and of student participation. These classroom types were characterized: high rigor, high participation; low rigor, high participation; low rigor, low participation; and high rigor, low participation. Using the Wright-Proctor instrument, this study was directed to establishing the similarities and the differences in the content, the process and the attitude of verbal interaction in these broadly different classroom types.

A rationale of the study is detailed in Chapter I. Description and comparison of the four extreme classroom types occurs in Chapter II. Assessment of the technical aspects of observer reliability, observer training and sample stability follow in Chapter III, Appendix B and Chapter IV. Exploratory work in comparison of problem solving behaviors of the individual by himself and in classroom discussion as initially proposed was rounded to a more complete design in consideration of the behaviors met in all possible classroom subgroups during problem solving: entire classroom; small group of pupils; small group of pupils with teacher; individual pupil and individual pupil with teacher. This is described in Chapter V. A brief resume is then made of this entire study and consequent steps that would be desirable in later research.

All pertinent materials for the analyses basic to Chapters II--V are presented in Appendix A.

Finally, all these aspects contributed to improvements in the use of the Wright-Proctor Instrument and to production of materials suitable for training purposes. A refined observers' manual, Appendix B, therefore could be prepared with the intent of self training away from this laboratory for an observer adequately prepared in mathematics for the use of this instrument. As a supplement to the manual, a carefully edited tape--corresponding to typescripts appearing in the manual--was prepared to facilitate an intermediate step from typescripts to the live situation in laboratory or field observation.

The extensive observation of classrooms necessary for this project was made possible and indeed a very rewarding experience by principals and teachers in the suburban St. Louis area. The opportunities for visiting in these school systems have not been limited to the period covered by this report. In particular the exploratory comparison of problem solving in the classroom and its several subgroups was carried out in association with Mrs. Alice Kimmel of University City High School, and with the generous cooperation of one of her geometry classes in after-school time.

The arduous task of working through our preliminary drafts of the final report and of the observers' manual was facilitated by Mrs. Mildred Scott, secretary; the final draft was typed by Mrs. E. R. Dunn. Further secretarial assistance throughout the entire project was given by Mrs. Charlotte Clark and Mrs. Irene McGough of the Mathematics staff.

Despite the extensive use of computers considerable hand calculation remained. Much of this was carried out by Mr. George Sidney, a senior student at the University.

It was particularly pleasing that during the course of the study Virginia Proctor completed a Master's Thesis dealing with refinement of an earlier form of the Wright-Proctor Instrument.

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St. Louis, Missouri
June, 1961

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GLOSSARY OF SYMBOLS

*Content:

Symbols used in Report		Symbols used in Manual	
C 1	Structure	C 1	Structure
C 2	Deductive	C 2	Techniques
C 3	Mathematical	C 3	Deductive
C 4	Other	C 4	Inductive
C 5	Techniques	C 5	Statements
C 6	Inductive	C 6	Mathematical
		C 7	Other

Process:

P 1	Analyzing
P 2	Synthesizing
P 3	Specializing
P 4	Generalizing
P 5	Relevant

Attitude:

A 1	Curiosity
A 2	Independence
A 3	Receptivity

Neutral: N

V	Verbal
T	Teacher
P	Pupils
B	Boy
G	Girl
C	Classroom
O	Observer

RP	High rigor, high participation
rP	Low rigor, high participation
rp	Low rigor, low participation
Rp	High rigor, low participation

---

* Symbols used for the Content categories differ in report and in manual because of addition of a new category and more logical ordering

CHAPTER I

RATIONALE

Central in the current renewed interest in the teaching of mathematics is a concern with emphasis of mathematical rigor and student involvement in all phases of system development. Direct observation of classroom discussion provides perhaps the most valid means of seeking immediate answers to questions of success of experimental programs. The Wright-Proctor Instrument designed for direct study of mathematical discourse is seen within a broad model of essential aspects of language. As yet only tentative consideration of the place of this model in a general learning situation has been possible. This has been stimulated by existing models of the teaching-learning situation viewed merely as a problem in interaction between teacher and pupil.

MATHEMATICS CLASSROOMS--1961

The remarkable rennaissance of concern with the teaching of mathematics during the past decade is producing in the mathematics classroom a teaching-learning situation of considerable variability. Two controversial ideas are at the center of differences to be found: the rigorous building of the mathematical system under study at a level suitable for the students and the participation of the students in the building of the mathematical system as well as in its application.

While each idea has been argued from one extreme to the other, increasingly, programs directed to achieving one or both of these aims have been gaining widespread approval. The brilliant program of the University of Illinois Committee on School Mathematics, the Commission on Mathematics of the College Entrance Examination Board, the Madison Project, the Ball State Project, the Catherine Stern and the Cuisenaire materials, the School Mathematics Study Group, the Institutes for teachers sponsored by the National Science Foundation, the Committee on the Undergraduate Program for teacher training--all these, and many others have developed in a climate for reconsideration and indeed for daring in the teaching of mathematics.

What effects will these experimental programs have on the teaching? on the learning? How will experimental classrooms differ from non-experimental classrooms and how will these differences relate to pupil change? What measures may be used in determining some of the answers to these and related questions?

MEASUREMENT OF CLASSROOM VARIABILITY

Systematic study of the classroom on the whole has emphasized indirect assessment of the classroom variables by measurement involving paper and pencil tests given in the pre and post lesson periods. Most promising of these is a consideration of pupil change during the experimental period. Despite the general level of refinement of this type of measure, and the additional improvements in scaling likely to develop from Guttman's concern with ordering of task complexity in correspondence with the real numbers (1959) or in experimental design in the use of residual scores as discussed by DuBois (1959), there remains the question of validity of such indirect assessment in evaluation of experimental programs introducing additional topics or emphasizing more rigorous or more creative thinking. In use of achievement measures based on the work studied it would have to be assumed that topics and acceptable reasoning of the control groups which provide a practical common ground of testing materials constitute a valid sample for measurement of knowledge of the experimental groups. It might be feasible to turn away from testing achievement in the specific subject matter to measures of problem solving, critical thinking or other abilities such as those analyzed by Guilford (1956) which, it might be inferred, would be affected by changes in mathematics instruction. A better indirect study of classroom effects would be in long term comparison of success in later academic work of experimental and control groups. If our interest is more immediate than could be served by this long term study and directed particularly to classroom variability, then the value of the refined indirect measure of somewhat questionable validity should be compared with that of the less precise direct measure possessing high validity obtained by systematic observation in the classroom itself.

Considerable support for the use of carefully developed schemes for systematic observation has been accumulating over the past several decades from the pioneer work of Thomas (1933) and of Olson (1931) to the richer tools and techniques of Bales (1950), Cartwright and Zander (1953), and Guetzkow (1950) in social interaction generally and of Anderson (1939), Withall (1948), Ryans and Wandt (1952) in the classroom.

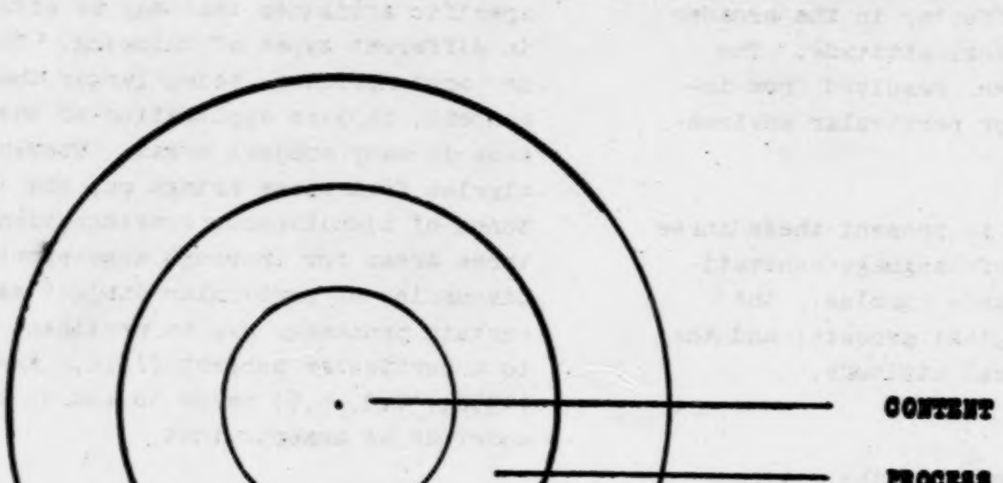
While current investigations continue to develop new systems or to modify and refine earlier ones within broad social psychology frameworks (Trow, 1960; Flanders, 1960; Newell, Lewis and Withall, 1960), a variety of operational definitions of the teaching-learning situation are implicit in instruments now reaching publication level (Hughes and associates, 1959; Kowatrakul, 1959). These operational definitions necessarily fall into a multiple-criterion prediction in their approach to description, modification and prediction of teaching-learning phenomena. Of particular note in the attempt to study the classroom from observable behavior with a minimum of theoretical preconceptions is the work of B.O. Smith (1960), Mary Jane Aschner (1960), and associates, in directing their study of linguistic behavior of teacher and pupil in a classroom situation to a fundamental aspect of language, itself: the process or logical framework.

Also of high interest is Suchman's (1961) concern with the low incidence of pupil-initiated questions in most classroom discussion. This concern has taken form, not in systematic field observation of existing discussion, but in an active laboratory program for developing in children a method of inquiry for determining answers about intriguing natural phenomena from an informed individual (the investigator).

MEASUREMENT OF MATHEMATICS CLASSROOM VARIABILITY
by
WRIGHT-PROCTOR OBSERVATIONAL INSTRUMENT

Our own experience has been that an operational definition may gradually make explicit a broad theoretical framework, which may then direct further resolution of the initial operational definition. Beginning in 1956 from the multiple-criterion operational approach to classification of language used in mathematical discourse, our instrument developed through several levels of refinement of category definition and interpretation (Wright, 1957, 1959, 1960; Proctor, 1960). Modifications resulted from use of the instrument in the field interspersed with reconsideration of the categories in terms of a minimum number necessary yet sufficient to classify the language used by the teacher and the pupils in their thinking about mathematical subject matter. Because the key aspect of the classroom is the mastery of particular subject matter, it was with the belief that the important yet peripheral aspects of the classroom forming the background of many of the studies cited above would not be neglected but would be placed in proper perspective that we developed our categories and their definitions from the species of mathematical language in the classroom. Thus as in the case of Smith et al., because we were studying the teaching-learning situation as evidenced by language we concentrated on the essential

TRIPLE VIEWPOINT OF THE WRIGHT-PROCTOR INSTRUMENT



MATHEMATICAL CONTENT		PSYCHOLOGICAL PROCESS		SOCIOLOGICAL ATTITUDE
Structure	Fundamentals	Analyzing	Syllogistic	Curiosity
Techniques		Synthesizing		Independence
Deductive	Relations	Specializing	Classificatory	Receptivity
Inductive		Generalizing		
Mathematical	Applications	Relevant		
Other				

Figure 3. The three viewpoints and the defining categories of the Wright-Proctor Observational Instrument. Further definition and technical aspects are detailed in Observers' Manual, Appendix B.

aspects of language; however, we did not limit ourselves simply to the logical process.

It became apparent that language involving mathematical argument was based on content which had to be carried in the broad vehicle of psychological process and together these were effected in the broader framework of sociological attitude. The cognitive aspects, then, resulted from impetus of the general or particular environment of the speaker.

It was convenient to present these three aspects of the study of language schematically by three concentric circles: the middle area, psychological process; and the outer area, sociological attitude. (Figure 3)

This schema implies that the environment has induced certain processes in the consideration of particular subject matter. This schema also is in marked agreement with Piaget's (1950) analysis of logical thinking where he points out the affective and cognitive aspects of thinking, his cognitive aspect here divided into process and content. This division of the cognitive aspect is in keeping with Cassirer's (1953, p.9, 345) consideration of thought as "thing-concepts" and "relation-concepts" with science presenting the things as distinct and certain, and psychology presenting the relations as a temporal process:

> "Psychology in this sense gives the approach to problems, which must seek their (complete) solution in logic and in their application to science."

The categories of process were selected at a functional level in consideration of subject matter rather than at the finest division of formal logic (Smith et al.), that is, syllogistic and classificatory levels and relevant (but non-directed to immediate problem). It is also noted that the categories of the Process frame developed under the above considerations are in keeping with the broad division of the problem solving process reached empirically by Duncker (1945, p.45).

The order of these concentric circles also may be commented on. The outer area, attitude, being larger than the process, permits inclusion of discursive thinking and other types of thought and the many specific attitudes that may be affective in different types of thinking. The center area, process, being larger than the content, implies application of the process in many subject areas. Viewing the circles from above brings out the importance of simultaneous consideration of the three areas for thorough assessment of the discussion of particular subject matter, certain processes may be pertinent only to a particular subject field. Kantor (1950), V.I, p.6) seems to sum up this model as he asserts that

> "every particular interbehavior (of organisms and stimulus objects) is set in a special reference frame. As a result any specific type of logic, whether formal, empirical symbolic, or methodological is not only conditioned by the type of material dealt with, but also by general conditions under which the work is done."

While the language of the classroom--including its mathematical refinements and its approximations by gesture, hieroglyphics, or drawings--may be studied from these three reference areas, the place of this verbal behavior as a sampling of the thinking actually occurring in the speaker should be questioned. Langer (1948, p.65, 70,73,79) points out that

> "language has a form which requires us to string out our ideas even though their objects rest one within the other"

and that this implies the limitation on language that only thoughts that can be arranged in such an order can be spoken at all. It seemed important at this stage to avoid the assumption than language is the only means of articulating thought and that everything which is not speakable thought is feeling. Such assumptions are surely inadmissible in the light of the mind's organized storing of the meaningless impressions of relatively simple sense-experience or in view of the complex symbolism of the musician and the painter. Langer suggests the term "presentational thinking "for thoughts which cannot readily be ordered temporally and therefore cannot be expressed in language. However, while thought itself must include more than discursive thinking, mastery of mathematical subject matter implies an understanding of the specific mathematical ideas which finally may be demonstrable by the discursive symbolism of language.

From the general nature of these three frames of reference and their categories, it is interesting to consider that by modification and redefinition of the exact categories to meet the peculiarities of another subject area, comparison of categories common to two or more subject matter areas would be possible.

PLACE OF THIS LANGUAGE MODEL IN THE GENERAL LEARNING PROBLEM

While the language of mathematical argument encountered in the classroom may thus be viewed in the broader context of a model of thinking, the place of this within a learning situation has not yet become clear to us.

Two models of a teaching-learning situation viewed as a problem of interaction have been stimulating to us in consideration of the broader problem: that of DeCharms (1961) and that of B.O. Smith (1960).

DeCharms suggests for the pedagogic situation the use of a "contingency" model from studies of social interaction of Jones and Thibaut. The distinguishing criterion "contingency" indicates the dependency of the behavior of each person in a dyad on that of the other.

> "Three possible forms of interaction in a dyad may be considered: non-contingent, asymmetrically contingent and symmetrically contingent. In a non-contingent interaction, the degenerate case, each person would follow a rigid predetermined course of behavior unaffected by the other. In an asymmetrically contingent interaction one of the two persons follows a rigid predetermined procedure while the second responds in behaviors depending on those of the first. In the third type, symmetric contingency, the behavior of each person is determined to some extent by the behavior of the other."

It is this final type that Smith considers in the two action steps (R_t, R_p) in his detailing of reciprocal behaviors in a teaching cycle:

$$\| P_t \rightarrow D_t \rightarrow R_t \mid P_p \rightarrow D_p \rightarrow R_p \|$$

P_t-teacher's perception of pupil's behavior
D_t-teacher's diagnosis of pupil's state
R_t-action taken by teacher
P_p-pupil's perception of teacher's behavior
D_p-pupil's diagnosis of teacher's state
R_p-pupil's reaction to teacher's actions

Each cycle

> "consists of a teacher-pupil interaction. Within this teaching cycle are two subunits divided by the single vertical line. The subunit $P_t \rightarrow D_t \rightarrow R_t$ is what we refer to as an act of teaching; the subunit $P_p \rightarrow D_p \rightarrow R_p$ is what we call the act of taking instruction. These are reciprocating acts and when performed under proper conditions they issue in achievement."

In the practical situation of the classroom with interaction occurring among one teacher and many pupils the teacher's contingent actions may result from immediate effect of one pupil but more often will be "statistically contingent" upon the combined effect of the several pupils of this classroom and upon past experience. This is seen in even more extreme form in the apparently asymmetric situations of carefully programmed materials in formal lectures or in teaching machines.

Despite the insight and obvious fruitfulness of these models within their carefully limited sphere of interaction of teacher and pupils, as models of an aspect of the learning situation each seems curiously limited to affective aspects: essential, yes, but surely to be placed in more appropriate perspective with the central protagonists in learning--the learner and the subject matter.

Now learning may take place in a classroom under constant interaction of teacher and pupils as affective stimuli; it may occur in school or home assignments after a broad initial stimulus by the teacher; it may occur at home as a direct result of a well stocked library; indeed it may occur in a great variety of situations under a great variety of stimuli. By whatever method effected--and the classroom interaction of teacher and pupils may be a major cause--it is the interaction of the learner and the subject matter that is essential for learning.

I would suggest then, that interbehavior of learner and subject matter is the key condition prerequisite to learning and should be implicit in models even of aspects of the general learning problem.

That interbehavior of learner and of subject matter may be a symmetrical interaction has been evidenced by many examples in our observation of the classroom, and more broadly in the development of mathematics. Several instances are cited.

(1) The solution of a problem in Euclidean geometry required proof that two lines were parallel, and that, in addition, a particular relationship of size held. Two teams of students were observed working on this problem. Each selected the first aspect. The first team noted that application of the theorem "A line joining the midpoints of two sides of a triangle will be parallel to the third side and equal to one-half of it" would establish this first aspect. One student quoted the complete statement; immediately a second student noted that the first part was sufficient to establish the point under study, and that the second part was just what was needed for the second aspect of the problem. Immediate solution resulted. The second team also noted that this theorem could be used for the first aspect. However, only the part necessary for establishing this point was stated. Thus no opportunity for the additional aspect of theory to act on the learner occurred as in team one. Indeed the second team failed to consider the same theorem a second time and did not reach a solution.

(2) Historically mathematical systems and their use in description of certain aspects of natural phenomena were not distinct. Recognition of an isomorphism rather than an identity freed the subject matter from a limitation imposed by the learner and permitted modification of initial axioms and development of a variety of new mathematical systems. Some systems found immediate application; others are as yet unrelated to physical models.

(3) One of the happiest experiences of teaching is observing the recognition by the student of significant relationships or of other methods of solution or of families of problems related to a specific one under study. While, except at sophisticated levels completely new subject matter will not emerge, as far as the student is concerned, he is indeed involved in mathematical invention.

Perhaps Shahn (1960, p.57) presents the case of the effect of developing subject matter on the person working with or creating it most vividly in his "Biography of a Painting".

"From the moment at which a painter begins to strike figures

> of color upon a surface, he must become acutely sensitive to the feel, the textures, the light, the relationships which arise before him. At one point he will mold the material according to an intention. At another he may yield intention--perhaps his whole concept--to emerging forms, to new implications within the painted surface....One must say that painting is both creative and responsive. It is an intimately communicative affair between the painter and his painting, a conversation back and forth, the painting telling the painter even as it receives its shape and form."

The application of the idea of contingency to teacher-pupil interaction by DeCharms and the teaching cycle suggested by B.O. Smith then, have proved of considerable stimulus in recent months in our tentative attempts to clarify the relationship of our own language model with existing fruitful models of learning in the classroom. In particular the considerations cited above have pointed up our own conviction that a broad model of the general learning situation should emphasize interbehavior between subject matter and learner and permit placing of models of various aspects of the general problem--in these cases stimulation by teacher-pupil interaction--in appropriate perspective.

We now turn to the comparison of the four types of classrooms characterized as high rigor, high participation; low rigor, high participation; low rigor, low participation, and high rigor, low participation using the direct method of systematic observation of the mathematics classroom in the terms of the Wright-Proctor Observational Instrument.

CHAPTER II

DESCRIPTION AND COMPARISON OF CLASSROOMS DIFFERING IN EMPHASIS OF MATHEMATICAL RIGOR AND OF STUDENT PARTICIPATION

Four types of conditions are achieved by selection of mathematics classrooms at the extreme of each continua of mathematical rigor and of student participation. The consequent measures of content, of process and of attitude dependent on these conditions, derived from the Wright-Proctor Instrument are expressed as scores in each frame singly but also across the three frames in combination. Both the individual frame and the triple frame scores confirmed the division into types made by the independent measures. An index of initiative based on the categories of attitude also confirms this division, thus gaining support for its own validity.

The individual frame scores are most valuable in indicating the level of emphasis of the various categories, the similarities, and the differences most marked in the four types. Patterns and comparisons are illustrated by bar graphs. A number of hypotheses relating the dependent and the independent variables are establihsed. For instance, i. Deductive varies directly as rigor. ii. Deductive + inductive (relations) varies jointly and directly as rigor and participation.

The triple frame scores--interpreted from the first ten ranks--are most valuable in placing some body on the differences between types. For instance, while the individual frame score might indicate an increase in synthesizing from student behavior in type rP to RP, the triple score will indicate that this synthesizing is directed in the former to solving specific mathematical problems: mathematical/synthesizing/independence; but in the latter to deductive establishment of relations: deductive/synthesizing/independence.

Within each type while examination of individual frame scores shows firm establishment of certain single categories, the remaining categories may be quite widely varied. If our study had been limited only to these widely varying categories the classrooms even within each type might have been placed on a continuum perhaps labelled good to weak; yet from our broad judgment and the known reputations of the teachers this would have been quite inappropriate. This suggests a fruitful line of inquiry with reference to the question of wide variation in behaviors observed in "good" teachers in certain studies limited to only one or two dimensions of the classroom. In our study, a complete view of the classroom could be gained by consideration of the three frames of single category scores all at once; of some combinations of these single categories (such as relations above) to give a broader view; or of the more complex and comprehensive triple frame scores. By whatever means a complete view of all measured classroom variables was taken, certain apparent discrepancies in earlier limited assessment were resolved, and marked variation in a particular dimension could be accepted as reasonable and indeed as quite desirable.

Finally it was apparent that in the high rigor, high participation type where teachers are concerned theoretically with both independent variables, in practice this concern is taking form as would be desired in emphasis of structure (fundamentals and relations) and in successful pupil assistance in construction of the systems. It is apparent that both an increase of participation, rigor constant, and an increase of rigor, participation constant, results in greater emphasis of the structure of the mathematics without lack of attention to technical skills.

DESCRIPTION AND COMPARISON

INTRODUCTION

Four markedly different types of mathematics classrooms selected at the extremes of each continua of mathematical rigor and of student participation were chosen for description and comparison of classroom discussion in experimental and non-experimental programs. The discussion types were characterized: (1) high rigor, high participation (RP); (2) low rigor, high participation (rP); (3) low rigor, low participation (rp); (4) high rigor, los partitipation (Rp). Guided by the triple viewpoint and the defining categories of the Wright-Proctor Observational Instrument the consequent similarities and differences in the mathematical content, the psychological process and the sociological attitude of the language of the classroom were sought.

METHOD

Design

The design is outlined in Table 10.

Subjects The subjects were twelve classrooms drawn from a total of twenty visited in senior high school and in first year university on the basis of measures of the independent and of the controlled variables to provide a balanced design--3 classrooms of each of the 4 extreme types. The experimental conditions, then were not induced, but were achieved by selection of the twelve classrooms from a larger initial population. The RP was found in classrooms using the University of Illinois materials or where the teacher was experimenting with the ideas of rigor and participation himself; the rp and the rP were most characteristic of the present high school; the Rp with one exception was found only in university lectures.

Independent variables were mathematical rigor and student participation. Mathematical rigor--was merely dichotomized hi-lo and determined by careful examination of text materials, course outlines, assignments, teacher conversations about the new ideas, followed by confirming descriptions of the classrooms. Student participation--also was merely dichotomized hi-lo from two considerations: an assessment of quantity (proportion of student response to entire response) and of quality (student responses appearing in the entire range of aspects of the topic under discussion)--these determined by simple checking of frequency of student responses and by broad descriptions of class organization. With each of these variables separated into scores of hi and lo, there were four scores describing variation of the independent variables: RP, rP, rp, Rp.

Dependent variables The dependent variables were drawn directly from the Wright-Proctor Instrument (Appendix B) designed for the study of mathematics discussions from three viewpoints: mathematical content, psychological process and sociological attitude, viewed either separately or simultaneously, each delineated by several categories:

Mathematical content--

fundamentals:	structure, techniques
relations:	deductive, inductive
applications:	mathematical, other including historical emphasis

Psychological process--
  analyzing; synthesizing; specializing; generalizing; relevant information
Sociological attitude--
  curiosity; independence ; receptivity

These complete the aspects of discursive thinking outlined by this instrument.

The two types of scores obtained from this instrument are derived from a 10-day sample of classroom discussion: category scores in individual frames, and scores in triple frame combinations of the categories. In each case these scores are "standardized" by simple proportion to a base 1,000, the number of behaviors anticipated in the 10-day hour long classifications.

Intervening variable The observer determining classrooms at the extremes of mathematical rigor and student participation, and particularly using the more complex Wright-Proctor Instrument, was an important intervening variable. It was felt that effects due to variable judgment would be minimal in

Table 10 Design for description and comparison of the four classroom types.

D E S I G N

INDEPENDENT VARIABLES

1. **Mathematical rigor**
2. **Student participation**

SUBJECTS

12 classes:

Teacher
Pupils
Subject matter

DEPENDENT VARIABLES

1. **Mathematical content**
2. **Psychological process**
3. **Sociological attitude**

INTERVENING VARIABLE

Observer

CONTROLLED VARIABLES

1. **Teacher characteristics**
2. **Pupil group characteristics**
3. **Subject matter characteristics**

UNCONTROLLED VARIABLES

1. **Time of day**
2. **School**
3. **Lesson organization**
4. **Topic length**

the former case because either of the broad independent variables could be readily dichotomized by an observer much less highly trained than was necessary for judgment of the dependent variables. In the case of the dependent variables continual check of the reliability between the two experienced observers making the classifications was made (Chapter III). This was highly satisfactory. Secondly, to minimize the effect of an individual observer, wherever joint classification had been made mean scores were obtained and used as the "best" scores for that period of observation. In addition a check of the adequacy of the sample (Chapter IV) providing truly "representative" scores of a classroom supported the ten-day classification period. Some further support to the adequacy of observer judgment was given by the level of agreement reached during a ten-day training program of five novice observers (Appendix B). It must be noted, however, that while general satisfaction was felt about limiting effects of observer judgment in this immediate study, a full scale comparison of agreement among many observers trained independently of this laboratory would be desirable for adequate estimation of the effect of this intervening variable.

Controlled variables To permit clear-cut judgment of the effects of high and low mathematical rigor and student participation, many characteristics of the teacher, the pupils and the subject matter were limited in variability. It was decided that pertinent teacher characteristics to control were: experience, mathematical preparation, professional interest, teaching loads, other school responsibilities, and any unusual interest in teaching the class in this experiment. Two considerations might be noted--first, minimum values for the teacher characteristics assured sound preparation but did not eliminate considerable variation above these ground values. Second, to avoid undue bias because of teacher enthusiasm in the experimental mathematics programs, considerable attention was directed to arousing and maintaining teacher interest in this study: follow-up discussion of results at departmental meetings, speaking at mathematics clubs, forwarding of copies of papers read, reading paper at local meeting of National Council Teachers of Mathematics, providing particular liaison with the University. Pertinent pupils characteristics were controlled: age, mathematical ability and preparation, college expectancy, class size, broad indication of intelligence, and economic status. One distinction may be noted here in the classrooms of type high rigor, low participation. These were found almost entirely in the first year university where a lecture format was closely adhered to. Both age and class size were above the usual level encountered in the other types. It was felt that because of the pre-determined lecture format any effects in a given class of the higher levels in age and class size would be minimized. On the other hand it was noted that in the university setting the initial decision to use essentially a straightforward lecture was dictated by expected classroom size...a smaller class might have naturally permitted more student participation. This latter view was supported by the slightly higher level of participation in the low participant groups of the high school. Finally, algebra was chosen as the broad area of subject matter because of the limited attention to its careful logical development in most school classes in contrast to the more general attempt to present geometry as a coherent structure. In addition the topics under observation did not appear logically to require any fixed degree of student participation nor of mathematical rigor--as might be expected, for instance, at the beginning of the school year or during a review period at the end of a term.

Uncontrolled variables Several conditions that might affect the three main aspects of the classroom were noted but were not controlled. Their effect would not be directed to any one of our three key aspects of the classroom, but might influence an entire discussion: time of class; school; organization of lesson including amount of supervised study; topic length. Any variation in the results that might be explained by these uncontrolled conditions was to be noted. For instance, study habits and climate of attention might be markedly affected by broad school policies; or classrooms with considerable supervised study might require observation over a longer period of time to provide an adequate sample of behaviors.

Individual frame scores

Materials Following the method outlined for the Wright-Proctor Instrument, verbal discussion of teacher and pupils in each of the twelve classrooms of the study was classified by one or both of the two experienced observers during ten periods, yielding samples of approximately one thousand behaviors. This classification was expressed as frequencies of behaviors in each category of Content, of Process and of Attitude, and in Neutral.

Analysis An example of the classification of discussion in classroom I by Observer I was recorded in Table 13. It may be noted that frequencies were expressed in teacher and pupils components and that record of negative as well as of positive behaviors in each category of Content was made by use of superscripts. This latter practice was followed in Process and in Attitude also, but so few negative behaviors occurred in these frames that it was decided to absorb them subsequently in a total frequency for each category. In most later work all negatives were absorbed. In periods where joint classification had occurred the "best" scores were determined from the arithmetic mean of the two observers' scores in each category.

The frequencies for these individual periods then were totalled to provide a raw score sample of behavior for each of the twelve classrooms expressed in teacher and pupils components.

Representative teacher and pupils components for each class were calculated by standardizing the raw score frequencies by simple proportion to a base of 1,000 behaviors. For instance, Teacher I raw score sample frequency in Structure was 109^2 in a total of 868 classroom behaviors.

Teacher I representative score in Structure:

$$\frac{109}{868} \times 1{,}000 = 126; \quad \frac{2}{868} \times 1{,}000 = 2$$

Thus: 126^2

The teacher and pupils components were combined by addition to provide classroom representative scores.

Preliminary to studying differences category by category, it was desirable to establish statistically the same distinctions in type of classrooms in measures of the dependent variables that had been made by the two independent variables.

A limited test of the appropriateness of the division into the four types was made by attention only to the Attitude categories. Where student involvement occurred in the development of the mathematical system as well as in its application, it was apparent that a high degree of initiative would be demonstrated by the students. An Index of Initiative based on a simple weighting of teacher and pupil categories taking into account both frequency and quality of behaviors should separate the classrooms into the same two types as the student participation variable: RP and rP; rp and Rp. Such an index was feasible because the three categories of this frame--receptivity, independence and curiosity--may be considered as increasingly indicative of this broad attitude of initiative characterizing the entire frame. Proctor (1960, p.87) suggested a weighting which seemed to take fair account both of frequency and quality of behaviors and further considered pupil behaviors as somewhat more valuable than teachers:

	Teacher	Pupils
Curiosity	3	4
Independence	2	3
Receptivity	1	2
Neutral	0	0

For classroom I where teacher representative scores were 25, 540, 153, 21 and pupils representative scores were 8, 230, 21, 1 both

Table 13 Frequency of behaviors in each category of Content, of Process and of Attitude, and in Neutral for Teacher I and Pupils I as recorded by Observer I for the ten periods of classroom visits. Superscripts indicate frequency of negative behavior in individual categories.

DAYS CATEGORY	1	2	3	4	5	6	7	8	9	10
				TEACHER I						
Content										
Structure	10	11	6	3	14	0	9	22	15	18
Techniques	13	7	5	6	6	9	8	6	4	6
Deductive	20	3	18	21	11	10	15	27	38^1	12
Inductive	0	12	8	12	6	2	15	3	1	5
Mathematical	21	22^1	23	18^1	17	7	18	8^2	17	18
Other	0	0	0	3	10	11	7	2	0	0
Process										
Analyzing	7	8	10	7	11	1	10	14	15	4
Synthesizing	11	15	18	11	4	6	6	28	25	9
Specializing	17	15	18	26	31	6	36	18	19	34
Generalizing	12	3	7	8	3	5	10	0	9	3
Relevant	17	15	7	12	15	21	11	10	8	9
Attitude										
Curiosity	1	0	0	4	5	7	3	4	0	0
Independence	52	48	47	43	52	25	55	54	58	50
Receptivity	11	8	13	17	7	7	15	12	18	9
Neutral	1	9	0	1	0	-	1	2	-	4
TOTAL TEACHER	65	65	60	65	64	39	74	72	76	63
				PUPILS I						
Content										
Structure	6	0	1	0	1^1	0	2^1	8^3	3^1	3^1
Techniques	1	1	1	1	0	1	1	0	2	4
Deductive	2^1	0^1	4^1	16^1	4	2^1	5^1	8	7^1	5^1
Inductive	0	7	0	0	1	2	3	0	0	1
Mathematical	14^2	15^2	18	8	13^1	4^1	5	1	1^1	14
Other	0	0	0	0	1	0	0	0	0	0
Process										
Analyzing	1	1	1	1	0	0	1	2	4	1
Synthesizing	15	14	17	11	7	5	4	9	5	7
Specializing	4^1	9	5	9	13	3	8	7	4	16
Generalizing	3	0	0	2	2	1	4	1	2	1
Relevant	2	2	2	3	0	2	1	1	1	4
Attitude										
Curiosity	1	0	0	3	2	0	1	0	0	0
Independence	23	24	23	22	20	10	17	19	16	25
Receptivity	2	2	2	1	0	1	0	1	0	4
Neutral	-	-	-	1	0	-	-	-	-	-
TOTAL PUPILS	26	26	25	27	22	11	18	20	16	29
TOTAL T & P	91	91	85	92	86	50	92	92	92	92

in the order listed, the Index of Initiative was calculated:

$$\frac{(3.25+2.540+1.153+0.21+4.8+3.230+2.21+0.1)}{1,000}$$

$= 2.072$

It should be noted here that the Index was proposed initially as a measure for the independent variable of student participation. This was early discarded for two reasons. First, it was realized that while this simple weighting seemed quite reasonable logically, it should be validated empirically; and second, that while high student involvement in the development of the mathematics as well as in its application would result in a high Index, an equally high Index could result from considerable pupil participation in just one aspect of the lesson e.g. application of principles.

The Indexes were illustrated graphically and the differences between those derived from classrooms designated high participation and low participation were tested by means of analysis of variance within and between the types RP and rP; rp and Rp.

For a more conclusive test of distinction of types of classrooms consideration of category frequencies in each frame was necessary. Hoping to be able to use the X^2 technique in this analysis by comparing the group of categories of each frame of reference, it was first necessary to confirm a tentative conclusion reached in an earlier more limited study that the categories within each frame were statistically independent. It may be noted that the Instrument ensures that within a frame each category is exclusive, but this of itself does not ensure independence. Further, the question might be raised in the frame Attitude whether or not the apparent qualitative logical relationship among the categories (increasing degree of initiative from receptivity through curiosity) should be considered even if statistical independence is demonstrated. It was assumed at present that statistical independence would be sufficient to fulfill the necessary condition for use of chi-square. Accordingly, the sixteen total sample sets of scores in each category of the frames of Content, or Process and of Attitude were used in plotting scattergrams between pairs of categories within each frame: i.e. C_iC_j, P_iP_j, A_iA_j with C(i,j from 1 to 6) P(i,j from 1 to 5), and A(i,j from 1 to 3). The independence of the categories within each frame was confirmed and hope was raised that the analysis of broad classroom types might be carried out across complete frames rather than by assessment of individual categories, a tedious aspect of earlier analyses. This was confounded, however, for using a rough ranking check of classroom scores not only were the 3 classrooms of each set alike, but the four sets of 3 were alike; while moving to the more stringent chi-square, not only were the four sets different but also the 3 classrooms within each set were different. Thus while wide differences in certain categories were masked by ranking, they were sufficiently large to dominate the chi-square and hide any similarity of remaining categories in a frame. We returned regretfully to the study of individual categories. Formally our plan was to use analysis of variance in individual categories:

(1) to determine if the twelve classrooms formed one population,
or
if each group of three classrooms identified as RP, rP, rp, and Rp might be considered to form single populations differing from each other,
and
(2) to determine how these groups of three classrooms differed among themselves, e.g. Rp different from rP, (1)permitting.

In considering (1) the variance within and between types as distinguished by the independent variable measures was analyzed. An example of this analysis of variance for the Classroom Structure scores follows:

RP	rP	rp	Rp
169	161	116	120
120	46	50	106
253	88	53	85

Estimate of population variance:

from means of columns:	6480
from within columns:	2400

$$F = \frac{6,480}{2,400} = 2.70$$

In this example the two estimates of variance were not significantly different, so that the four types were not distinguished in this particular category. Decision about distinction of the four types would be made on the collected results of this analysis of variance applied to all categories. It should be noted further that the differentiation of classroom types was limited to study of classroom scores. If the types could be distinguished in these totals of classroom (t+p) scores, even greater distinction could be inferred for the components, from the variability of the ratio of teacher behaviors to pupils behaviors alone. This was assessed by analysis of variance in pupils totals.

If the distinction of types suggested above was indeed made, detailed comparisons category by category (2) could be made using the X^2, 2 x 2 comparison of two independent samples. To this end, the three classrooms of each type were noted: RP--I, XI, XIII; rP--IX,XV,XVII; rp--VIII, XII, XVI; and Rp--III, V, VI. Aggregate representative scores in each category were obtained for each type and expressed as classroom scores and as teacher and pupils components, all negatives absorbed. The table set up for comparison P_1 in (Rp) and (RP) groups would be as follows:

Classroom score

Category	(Rp)	(RP)	Totals
P_1	A	B	A + B
Total Verbal - P_1	C	D	C + D
Totals	A + C	B + D	N

$$X^2 = \frac{N(|AD - BC| - N/2)^2}{(A + B)(C + D)(A + C)(B + D)} \qquad df = 1$$

(Siegel, 1956, p. 107)

Considering the four broad types of lessons tending towards the extremes of high and low rigor, high and low participation a number of differences between categories might be anticipated logically.

Hypotheses of the relationships of individual frame dependent measures with the two independent variables were as follows:

CONTENT:

Structure varies jointly and directly as rigor and participation.

Technique varies jointly but inversely as rigor and participation.

Deductive varies directly as rigor.

Inductive varies directly as participation.

Mathematical varies inversely as rigor.

Other is independent of rigor and of participation.

PROCESS:

Analyzing varies jointly and directly as rigor and participation.

Synthesizing varies inversely as participation.

Specializing varies directly as participation and inversely as rigor.

Generalizing varies inversely as participation.

Relevant varies jointly but inversely as rigor and participation.

ATTITUDE:

Curiosity varies directly as rigor.

Independence varies directly as participation.

Receptivity varies inversely as participation.

NEUTRAL: is independent of rigor and participation.

Operational definitions of these hypotheses are indicated in Table 17. Because the initial plan to use the Index of Initiative as the measure for the second independent variable had been discarded the original proposal did not suggest specific hypotheses for comparison in the Attitude frame. However, such hypotheses were made here and included in Table 17.

Although expressed as if a one-tailed rejection region should be considered, these hypotheses are viewed as guides, and if rejected, it would be desirable to know if the difference is in the opposite direction whether or not it is significant. Thus a two-tailed rejection region. Therefore the null and alternative hypotheses will actually be phrased as:

Null hypothesis
H_0 The proportion of behavior falling into the particular category was the same in the two groups of classes.
H_1 The proportion of behavior falling into the particular category differed in the two groups of classes.

Statistical test
Because the data are in discrete categories, the X^2 test was appropriate.

Significance level
$p = 0.05$. May also record actual probability.

Sampling distribution
X^2 from above is distributed approximately as chi square with df = 1. The probability associated with the occurrence under H_0 of values as large as an observed value of X^2 may be found, Fisher and Yates (1957, table IV).

Region of rejection
The region of rejection consists of all values of X^2 which are so large that the probability associated with their occurrence under H_0 is equal to or less than the chosen 0.05.

For example: comparison of classroom scores, analyzing, in type Rp, RP:

Category	(Rp)	(RP)	Totals
P_1	363	447	810
Total Verbal $- P_1$	2637	2553	5190
Totals	3000	3000	6000

$X^2 = 9.83$ significant at $p = 0.01$

Thus type high rigor, low participation has significantly less analyzing in total classroom scores than type high rigor, high participation. This would support the hypothesis stated above.

While these calculations were based on aggregate scores, tabulation of separate category frequencies for each type of classroom was made in "standardized" form (based on 1,000 behaviors) for more ready comparisons with the familiar levels of the representative classroom scores. Thus with type RP:

Representative Scores in Structure:	Class I	Class XI	Class XIII
Classroom	161^8	115^5	244^9
Teacher	126^2	101	183
Pupils	35^6	14^5	61^9

resulting in:
Aggregate Scores (Standardized)

Classroom	181	
Teacher	138	n.b. negative
Pupils	43	values absorbed.

These were illustrated by means of bar graphs.

Table 17 Original logical hypotheses of expected differences in classroom scores between individual categories of Content, Process and Attitude among the four types of classrooms taken in pairs: high rigor, high participation (RP), low rigor, high participation (rP), low rigor, low participation (rp), and high rigor, low participation (Rp).

ATEGORY	Hypotheses	Rp & rp	RP & rP	Rp & RP	rp & rP	Rp & rP	rp & RP
ontent							
Structure	$S \propto rp$	>	>	<	<	-	<
Technique	$T \propto 1/rp$	<	<	-[1]	>	<[2]	>
Deductive	$D \propto r$	>	>	-	-	>	<
Inductive	$I \propto P$	-	-	<	<	<	>[3]
Mathematical	$M \propto 1/r$	<	<	-	-	<	>
Other	$O = \text{constant}$	-	-	-	-	-	-
ocess							
Analyzing	$A \propto rp$	>	>	<	<	-	<
Synthesizing	$Sy \propto 1/p$	-	-	>	>	>	>
Specializing	$Sp \propto p/r$	<	<	-	-	<	>
Generalizing	$G \propto 1/p$	-	-	>	>	>	>
Relevant	$R \propto 1/rp$	<	<	>	>	<[4]	>
itude							
Curiosity	$C \propto r$	>	>	-	-	>	<
Independence	$I \propto p$	-	-	<	<	<	<
Receptivity	$R \propto 1/p$	-	-	>	>	>	>
utral	$N = \text{constant}$	-	-	-	-	-	-

: "varies as"

2, 3, 4 Several specific differences did not correspond to the broad hypotheses, e. g. should read Rp = rP. However in practice it was felt that participation would have the eater effect on this category. Similarly, (1) should read >, (3) <, and (4) =.

DESCRIPTION AND COMPARISON

The question also was raised of differences between types in teacher and in pupils scores. The same analysis would have been confounded, however, because of the wide differences in teacher and pupils total frequencies across the four classroom types. Accordingly, the mean teacher and pupils scores of each type were used to determine the proportions of teacher behaviors falling into each category and the proportion of pupils behaviors falling into each category. For example, type RP:

Teacher Structure: 138
Total Teacher: 710
Proportion:

$$\frac{\text{Teacher Structure}}{\text{Total Teacher}} = \frac{138}{710} = 0.194$$

Pupils Structure: 43
Total Pupils: 290
Proportion:

$$\frac{\text{Pupils Structure}}{\text{Total Pupils}} = \frac{43}{290} = 0.149$$

The individual category proportions of teacher and of pupils were then compared type by type exactly as the classroom proportions had been examined by the 2 x 2 chi-square tables. Comparison of the differences in the $\frac{T_i}{T_v}$ and $\frac{P_i}{P_v}$ scores was made with the set of hypotheses, Table 17, based on classroom scores.

Similar comparisons of classroom, teacher and pupils scores: $\frac{C_i}{C_v}$, $\frac{T_i}{T_v}$, $\frac{P_i}{P_v}$ in combinations of categories was also made:

In Content:
- (1) Structure + Deductive + Inductive
- (2) Deductive + Inductive
- (3) Mathematical + Other
- (4) Inductive + Mathematical

In Process:
- (1) Analyzing + Synthesizing
- (2) Specializing + Generalizing

Each combination of categories was linked in some fashion:

Structure + Deductive + Inductive → Theory
Deductive + Inductive → Relations
Mathematical + Other → Applications
Inductive + Mathematical → Practice in specific problems
Analyzing + Synthesizing → Syllogistic reasoning
Specializing + Generalizing → Classificator reasoning

These comparisons were also illustrated in bar graphs. Analysis of variance of these classroom scores in these combinations was completed to determine if distinction by types occurred.

Finally, analysis of variance of pupils total scores was carried out to confirm further distinction of types that would occur in use of teacher and pupils components.

Triple frame scores

Materials Following the method outlined for the Wright-Proctor Instrument, verbal discussion of teacher and pupils in each of the twelve classrooms was classified by one or both of two experienced observers during ten periods yielding samples of approximately one thousand behaviors. For this aspect of the study, the classification was expressed as frequencies of behaviors in all possible triple combinations of the categories across the three frames of Content, or Process and of Attitude. Neutral scores were available also.

Analysis An example of the classification of discussion of Teacher I by Observers I and II was recorded in Table 19. It may be noted that certain periods--in this case 1,3, 4,5,7,8,10 --were observed jointly and from these a "best" matrix for the common days was obtained. This was combined with the matrix of the remaining periods--collected in this case by Observer I--to provide the raw score sample from the ten days of classification for Teacher I. It may also be noted that certain of the triples contained negative content behaviors. These were indicated by superscripts. While negative behaviors in Process and in Attitude had been recorded,

Table 19 Cross classification of Teacher I behaviors in Content, Process and Attitude categories: work matrices and final matrix. Negative in Process and Attitude disregarded; in Content where noted by superscripts. Symbolically: $C_iP_jA_k$

i--1 to 6
j--1 to 5
k--1 to 3

	P_1A_1	P_2A_1	P_3A_1	P_4A_1	P_5A_1	P_1A_2	P_2A_2	P_3A_2	P_4A_2	P_5A_2	P_1A_3	P_2A_3	P_3A_3	P_4A_3	P_5A_3
#1							Observer I -- Periods 1,3,4,5,7,8,10								
C_1	1		2			22	4	25^1	3	3	5		12	5	
2			1		1	21	33	27	16	6	1	5	8	4	1
3			1			7	36^1	59^1	1	2	2	1^1	7	2	5^1
4					10				1	5		1	3	2	
5					1			2	1	31					15
6						4	6	27	8				4		
#2							Observer II -- Periods 1,3,4,5,7,8,10								
C_1	2	1	1	1		20	6	26^1	4	2	1	3	13	5	1
2	1		1			13	35	22^1	8		1	5	10	3	1
3						3	36	54	2			8	21		2
4			2		6			3		1			3	2	
5								3		41			1		28
6						7	4	20	9				3	4	
Mean 1 & 2: #3							Joint Observers I & II								
C_1	1		1			21	5	25^1	3	2	3	1	12	5	
2			1			17	34	24	12	3	1	5	9	3	1
3						5	36	56	1	1	1	4	14	1	3
4			1		8			1	2	3			3		
5								2		36					21
6						5	5	23	8				3	2	
#4							Observer I -- Additional Periods 2,6,9								
C_1						11	1	6	2	1	1	2		2	
2						6	15^1	9	6	1	1	2	4	3	4
3						2	17^1	12	1	10		3	1		
4					7	1		1		1			1		
5										12			1		7
6						2	4	5	3						1
#3 + 4							Summary final matrix								
C_1	1		1			32	6	31	5	3	4	3	12	7	
2			1			23	49	36	18	4	2	7	13	6	5
3						7	53	68	2	11	1	7	15	1	3
4			1		15			2	2	4			4		
5								2		48			1		28
6						7	9	28	11				3	2	1

they occurred so infrequently that it was decided to absorb them immediately. All negatives recorded, then, indicate merely negative content. In most later work all negatives were absorbed.

Similar raw score samples were derived for Pupils I and for the remaining fifteen classrooms.

Representative teacher and pupils components for each class were calculated by standardizing the raw score frequencies from the ten-day sample by simple proportion to a base of 1,000 behaviors. For instance, Teacher I raw score sample frequency in the triple

Structure was 32 with classroom
Analyzing total of 868
Independence

Thus Teacher I representative score in

Structure was $\frac{32}{868} \times 1{,}000 = 37$
Analyzing
Independence

The teacher and pupils components were combined by addition to provide classroom representative scores. As a general practice a frequency $\geqslant 5$ for representative scores was required for record to be made.

As in the case of the individual scores, it was desirable first to establish statistically at an appropriate significance level, $p = 0.05$, the difference in dependent scores of the four types selected by the independent measures.

Formally our approach was

(1) to determine if the twelve classrooms formed one population

or

to determine if each group of three classrooms identified as RP, rP, rp, and Rp might be considered to form single populations differing from each other,

and

(2) to determine how these groups of three classrooms differed among themselves, e.g. RP different from rP, (1) permitting.

As in the case of the individual frame scores, the study was first limited to classroom scores. If the types could be distinguished in these totals of teacher and pupils components even greater distinction could be inferred for the components from the variability of the ratio of teacher behaviors to pupil behaviors alone.

An initial step in this comparison was the ranking of representative triple frame scores for each classroom. Examination of Table 19 will emphasize the large number of triple combinations that were untouched in the verbal behaviors of Teacher I, and which might be expected throughout the twelve classromms. If the cells left empty in the several classrooms were different, a major difficulty in comparing incomplete rankings was to be anticipated. In the rankings within each group of three such difficulty did not occur. Instead clustering in triple combinations of each type of classroom was marked. To facilitate comparison, it was decided to include all triples in each classroom ranking where the mean of the three classrooms $\geqslant 5$. Because of almost complete identity of empty cells determination of the similarity of emphasis of the set of triples within the three classrooms of each type was considered by means of the Kendall Coefficient of Concordance (Siegel, 1956, p.229). The similarity of emphasis would be reflected by the degree of variance among the three sums of ranks; the larger the variance, the greater the similarity. The Kendall Coefficient of Concordance, W, may be computed:

$$W = \frac{s}{1/12\, k^2(N^3-N)-k\sum_{T} T} \qquad \text{(Siegel, 1956 p. 234)}$$

whereas: sum of squanes of observed deviations from the mean of R_j (the sums of ranks), i.e.

$$s = \sum\left(R_j - \frac{R_j}{N}\right)^2$$

k: number of sets of rankings, i.e. the number of classrooms in each type.

N: number of triples being ranked

$1/12\ k^2(N^3-N)$: maximum possible sum of the squared deviations, the sum which would occur with perfect agreement among the k rankings.

T: $\frac{\sum(t^3 - t)}{12}$ with t the number of observations in a group tied for a given rank: directs summation over all groups of ties within any one of the k rankings.

In the example of rankings of the three classrooms within type RP, Table 22,

s: 22,952 N: 34

k: 3 T: $\frac{186}{12}$ W = 0.78

Under an assumption of independence of the three sets of rankings H_0 the probability of occurrence of a value as large as the observed W was determined by finding the X^2 associated with the observed W, noting that for N larger than 7, k(N - 1)W is distributed approximately as chi-square, df = N - 1. In the particular example of the type RP classrooms, $X^2 = 3(34 - 1)(0.78) = 77.2$ with p = 0.001. Thus the null hypothesis of independence of the three sets would be rejected and it may be inferred that the emphasis of the set of triples listed for this type, RP, was similar in each classroom, and a mean ranking could be used as the best indication of emphasis for that type. If the W for each of the other groups of classrooms permitted inference of similarity within each classroom type, comparison was planned across the four mean rankings. It was apparent that sets of incomplete ranks would occur. While an exact treatment was proposed by Benard and Van Elteren (1953), the elegant theoretical treatment masked the massive computation task with the some 36 ranks of this particular comparison, certainly out of keeping with the rapid calculations usually an advantage of ranking methods. It was felt, therefore, that application of the above W formula would be made to the four sets of mean rankings determined in several ways:

(1) Use of all triples with mean across the four sets $\geqslant 5$. Where cells in the individual rankings were empty, they would be tied for the final rank regardless of the multiplicity of the tie encountered.

(2) Because of the great bias towards similarity that might be encountered in (1) some of the empty cells were eliminated by limiting the selection of triples to those with mean $\geqslant 10$. Any remaining empty cell in any ranking would then be considered as the final rank as in (1).

(3) Only triples occurring in each of the four mean rankings were used. If the rankings of these could be shown to be different, some cautious inference of difference among all the triples could be made.

It may be noted that the W calculated in case (1) would undoubtedly be much inflated, in case (2) somewhat inflated and in case (3) somewhat depressed. Taken together a practical indication of the approximate significance level could be obtained.

The results of the analysis of variance of pupils total scores carried out under the "individual frame score" section were noted to confirm further distinction of types that would occur with use of teacher and pupil components.

The mean frequencies of each teacher, pupils and classroom type were then obtained and the first ten ranks where frequencies $\geqslant 5$ listed.

Table 22 Calculation of Kendall Coefficient of Rank Correlation, W, among the rankings of triple frame scores for the three classrooms of type RP.

Numerical Symbols of Triple Scores	Frequencies by Classrooms			Rankings by Classrooms			ΣR	$\lvert R_j - \bar{R}_j \rvert$	$(R - R_j)^2$
	I	XI	XIII	I	XI	XIII			
112	44	42	38	7	6	7'	20'	32	1024
212	31	21	27	10'	15	13	38'	14	196
321	9	13	38	22	21'	7'	51	1'	2"
612	9	13	23	22	21'	16	59'	7	49
113	5	17	16	26'	17	20	63'	11	121
213	3	12	3	29	23	34	86	33'	1122"
313	2	5	13	31	30'	21'	83	30'	930"
122	10	-	7	19'	34	31'	85	32'	1056"
222	90	77	29	3	3	12	18	34'	1190"
322	128	116	71	1	1	2	4	48'	2352"
622	16	30	19	16	11	18	45	7'	56"
223	8	19	9	24'	16	27'	68	15'	240"
323	12	27	26	18	13	14	45	7'	56"
623	-	9	8	33'	26	29'	89	36'	1332"
431	2	5	10	31	30'	26	87'	35	1225
631	-	8	12	33'	28	24	85'	33	1089
132	57	22	76	5	14	1	20	32'	1056"
232	50	32	40	6	9'	6	21'	31	961
332	100	38	57	2	7	3	12	40'	1640"
632	42	72	44	8	4	5	17	35'	1260"
133	17	16	49	15	18'	4	37'	15	225
233	17	29	20	14	12	17	43	9'	90"
333	20	43	37	12	5	9'	26'	26	676
433	4	1	12	28	33	24	85	32'	1056"
533	2	16	6	31	18'	33	82'	30	900
633	5	35	37	26'	8	9'	44	8'	72"
142	10	9	25	19'	26	15	60'	7	49
242	31	11	13	10'	24	21'	56	3'	12"
642	15	3	7	17	32	31'	80'	28	784
143	9	9	17	22	26	19	67	14'	210"
243	8	14	9	24'	20	27'	72	19'	361
451	17	6	8	14	29	29'	72'	20	400
552	63	32	12	4	9'	24	37'	15	225
553	38	92	31	9	2	11	22	30'	930"
									22,951.75

$$W = \frac{s}{(1/12)K^2(N^3-N)-K\sum_T T} \qquad W = \frac{12\,(22{,}952)}{3^2(34^3-34)-3(186)}$$

$$= 0.78$$

$\chi^2 = K(N-1)W$

$S = 22{,}952$

$K = 3$

$N = 34$

$\sum_T T = \frac{186}{12}$

$\chi^2 = 77.2$ df = 33

$p = \ll 0.001$

' indicates 1/2

" indicates 1/4

RESULTS

Design

Measures of the variables controlled in the teacher, the pupils and the subject matter were recorded, Tables 24,25,26, for the twelve classrooms selected for the study and for the remaining four classrooms seriously considered. Measures of the uncontrolled variable were summarized in Table 27. Four other classrooms were discarded on preliminary visits because of marked differences: very high student ability, very low college expentancy, unusually high emphasis of supervised study and in the final case completion of the rp section of the design before a fourth classroom of this type was visited.

The four classrooms seriously considered were rejected for this design because:

Classroom II: same teacher as classroom I.

Classroom X: while of Rp type the pupil participation was much higher than in III, V, VI, a function it was believed of small classroom size in the high school setting.

Classroom XIV: lower ability of pupils

Classroom XVIII: same teacher as classroom XVII.

Of the 12 selected, 3 Rp were in university freshmen algebra where the instructors' emphasis of rigorous approach caused considerable discussion among the faculty. Of the 3 RP, one in UICSM; the second teacher was a participant in several N.S.F. summer Institutes; the third teacher was very active in modernizing curriculum in all levels of school district;--all 3 were heads of department. Of the 3 rP--2 very interested in discussing the value of developing a rigorous program in their classes if feasible since they are modifying classes and materials towards this; the 3rd emphasized the need to provide much participation for his class in traditional-type applications as a means of developing steady work habits. He was enrolled for the following summer in a N.S.F. Institute and hoped to be prepared to turn toward rigor. Finally of the 3 rP: one was in the same department as the first two discussed above, in rP and held the same views; a second followed the traditional text very closely, even to its errors; the 3rd presented much of the work systematically but without reference to an overall structure. The twelve classrooms finally selected to complete the 2 x 2 design were tabulated.

	p	P
R	III V VI	I XI XIII
r	VIII XII XVI	IX XV XVII

Individual frame scores

In addition to the example of individual frame raw score classification for classroom I, Table 13 similar tables for the remaining fifteen classrooms occur in Appendix A, Tables:146-174. Summary of these raw scores for the 10-day sample for the twelve classrooms appears in Table 28. Tabulation of raw scores for the other four classrooms were included in Appendix A, as noted above.

The representative teacher, pupils and classroom scores of the twelve subjects derived from the raw score tables were recorded in Appendix A, Tables 215 and 216.

Values of the indexes of initiative determined for the twelve classrooms selected appeared in Table 29 and Figure 29. Analysis of variance applied to these indices indicated distinction between the six classrooms specified as high participant, RP and rP, and the remaining six classrooms specified as low participant, rp and Rp, by the measures of the independent variable: $F = 234.3$ with $n_1 = 1$, $n_2 = 10$;, $p \ll 0.001$.

The results of the analysis of variance of each category were detailed in Table 30. Included were the F ratios of the individual category variances within and

Table 24 Measures of controlled teacher characteristics.

					Number			
T.	Sch. or Univ.	Exp. (yr)	Math. Prep. (sem.hr)	Related Courses (sem.hr)	Math. Cl.	P.	Other Cl.	Remarks
				TWELVE SUBJECTS				
I	S	5 - 10	40 - 50	18	5	100	-	Department head; UICSM instructor in summer institute
III	U	5 - 10	Ph.D.in Math.		2	175	-	Assoc.Prof.Mathematics
V	U	5 - 10	(M.A. in Math. (Ph.D.in Ed.		2	160	-	Asst.Prof.Mathematics
VI	U	>10	Ph.D.in Math.		2	75	-	Professor Mathematics
VIII	S	5	30	?	4	90	-	Part-time textbook ed.
IX	S	>10	40 - 50	8	4	92	-	Dept.head; Dir.summer sessio
XI	S	>10	40 - 50	14	5	127	-	Participant in N.S.F. Summer session.
XII	S	>10	40 - 50	-	4	120	-	Experience in elementary college courses and in high school.
XIII	S	>10	55	10	2	56	-	Dept.head; Assist.principal
XV	S	5	40 - 50	3	4	80	-	Participant in N.S.F.Summer Institute: 3 hrs.
XVI	S	>10	40 - 50	27	5	110	-	Department chairman
XVII	S	>10	24	6	4	91	-	Additional preparation and experience in science.
				FOUR OTHER CLASSROOMS				
II	S	5 - 10	40 - 50	18	5	100	-	Same as I
X	S	5 - 10	65	--	5	110	1	Curriculum committee, Math. Contest Chairman, Mo.M.A.A.
XIV	S	5 -10	40 - 50	--	4	130	1	Also teaches chemistry
XVIII	S	>10	24	6	4	91	-	Same as XVII

T.: Teacher
Sch.: School
Univ.: University
Exp.: Experience

Sem.: Semester
Cl.: Classes
P.: Pupils

Table 25 Measures of controlled pupils characteristics

Class	Class size			Age		#Yrs. Math.	Aptitude							Coll. Expec.	Economic Status
							S. A. T.				General I.Q.				
	B	G	Tot.	Mean	s.d.	gr.8	Verbal	s.d.	Math.	s.d.	Mean	s.d.	Type		
TWELVE SUBJECTS															
I	13	3	16	16.7	0.3	3	557	89	628	93	--	--	--	High	High*
III	236	26	262	19.7	0.9	5	517	86	594	79	--	--	--	Here	High*
V	236	26	262	19.7	0.9	5	517	86	594	79	--	--	--		High*
VI	52	8	60	19.1	0.9	5	508	71	556	73	--	--	--	Here	High*
VIII	7	9	16	16.6	0.4	3	546	76	540	75	120	6.6	Otis	High	High*
IX	12	10	22	16.7	0.3	3	637	85	642	67	126	7.8	Otis	High	High*
XI	21	0	21	17.8	0.5	4	513	93	662	49	121	9.2	Mean	Good	High
XII	15	7	22	17.7	0.3	4	572	90	637	64	115	9.2	Otis	Good	Medium
XIII	22	2	24	16.7	0.3	4	mean V+M:		690	59	125	6,6	Comp.	High	High
XV	13	4	17	17.1	0.5	3	--	--	--	--	122	13.1	--	Good	Medium
XVI	15	5	20	17.9	0.3	4	560	75	640	80	125	13.2	--	High	high
XVXI	17	5	22	16.8	0.4	3	601	34	618	80	122	6.9	Otis	High	High*
FOUR OTHER CLASSROOMS															
II	5	6	11			3	567	82	552	70	--	--	--	High	High*
X	13	4	17	17.6	0.3	4	511	75	574	60	118	7.3	Calif.	High	High
XIV	14	12	26	17.0	0.5	3	mean V+M:		474	51	118	5.4	Mean	Low	High
VIII	15	5	20	17.0	0.3	3	613	118	630	98	124	7.3	Otis	High	High*

III, V - Measures derived for combined classes as common examinations taken.

* private

Table 26 Algebra topics: subject matter characteristics in the 16 classrooms.

TWELVE SUBJECTS

I UICSM. Exponential, logarithmic functions; arithmetic, geometric series; A.M.,G.M.,H.M.

III & V Allendoerfer & Oakley-Adapted. Polynomials and polynomial functions: fund, th^m, factor th^m, no. roots of complex polynomial; synthetic division; rational roots rational polynomial eq^n; real root, real polynomial eq^n; exponential, inverse functions.

VI Allendoerfer & Oakley-Adapted. Def^n 6 trigonometric functions in terms x, y, r; rectangular and polar coordinates; law of signs--applications; law of cosines --application; def^n wrt real numbers instead angles; graphing $y = A \sin (Bx+C)$; half-angle formulae.

VIII Welchons-Krickenberger. Graphing polynomial 2nd degree; introduction i--defined, graphed; def^n complex no.; quadratic eq^n--applications, general formula, reduction to.

IX Welchons-Krickenberger. Logarithms-triangle solution and in simple, compound interest; exponential functions; trigonometric functions--sine, cosine, properties, use of tables; law of sines, cosines treated as problems; approximate numbers used correctly; basic identities.

XI Allendoerfer & Oakley-Adapted. Group, field characteristics; review problems in logic; Boolean algebra application; functions--defined, domain, range, inverse inequalities; graphing; A.M.,G.M.,H.M.; operations for polynomial, rational, algebraic functions.

XII Middlemiss. Straight line--slope; triangle--altitude, area; symmetry; sums of series by mathematical induction; characteristics right circular cone and sections; translation axes.

XIII Allendoerfer & Oakley; Middlemiss--Adapted. Emphasis precision; relation, function; simultaneous linear polynomial eq^n; inequalities; graphing; sequence, progression A.M.,G.M.; questions on National Mathematics Contest; Exponential, logarithmic, trigonometric functions.

XV Hart, Schult, Swain--Second Year Algebra. Application trigonometric functions in triangle solution; application logarithms; arithmetic series--A.M., sum.

XVI Welchons-Krickenberger. A.P.,G.P.,A.M.,G.M., solution algebraic eq^n; general formula quadratic; determinants 2 x 2; 3 x 3 stated as technique; properties polynomial eq^n, stated; introductory remarks on technical aspects calculus.

XVII Welchons-Krickenberger. General formula quadratic; equivalent equations; student presentation conic sections; translation axes.

FOUR OTHER CLASSROOMS

II Same as I.

X Middlemiss--Adapted. Relation complex with reals; i; roots; general quadratic formula; complex roots; ..graph-functions, inequalities in rectangular coords; basic th^m inequalities.

XIV Rider-College Algebra. Manipulation non-rationals; i; solution algebraic eq^n but no reference equivalent equations; technical method taking square root.

XVIII Welchons-Krickenberger--Adapted. Conic sections presented by student, teacher; A.P. sum, means G.P., sum, means; binomial theorem.

Table 27 Measures of uncontrolled variables

CLASS	Time of Day	School	Topic length approx. (days)	Proportion supervised Study or omitted	
		TWELVE SUBJECT CLASSROOMS		p	n
I	Mon. Thurs.: 12:54-1:40 Tues.Wed.Fri.: 11:00-11:46	A	10	0.046	908
III	Mon. Fri.: 10:00-11:00	B	8	0.002	979
V	Mon. Fri.: 2:00-3:00	B	8	0.003	1095
VI	Mon.Wed.Fri.: 10:00-11:00	B	8	0.006	1004
VIII	Mon.: 11:14-12:02 Tues. Thurs.: 10:55-11:42	C	10	0.082	891
IX	Mon.: 9:25-10:12 Wed.Thurs.Fri.: 8:45-9:32	C	10	0.063	901
XI	Mon. Fri.: 10:30-11:30	D	10	0.039	1088
XII	Mon. Fri.: 9:30-10:00	F	10	0.417	990
XV	Mon. Fri.: 10:30-11:30	G	12	0.250	1018
XVI	Mon.Wed.Fri.: 8:15-9:10	H	10	0.188	925
XVII	Mon.: 10:15-11:02 Tues.Wed.Fri.: 9:35-10:22	C	10	0.030	939
		FOUR OTHER CLASSROOMS			
II	Mon. Thurs.: 1:44-2:30 Tues.Wed.Fri.: 12:54-1:40	A	10	0.006	881
X	Mon. Fri.: 2:30-3:30	I	10	0.084	925
XIV	Mon. Fri.: 10:30-11:30	F	12	0.234	1026
XVIII	Mon.: 10:15-11:02 Tues.Wed.Fri.: 9:35-10:22	C	10	0.013	817

Table 28 Raw scores from 10-day sample in each of the twelve classrooms of the four types: high rigor, high participation (RP); low rigor, high participation (rP); low rigor, low participation (rp); high rigor, low participation (Rp). Teacher and Pupils components derived from the sum of 10 periods of classification using mean scores of Observers 1 and 2 where observation is joint. All scores to nearest whole number 1,000. Superscripts negatives.

TYPE	RP			rP			rp			Rp		
CLASS	I	XI	XIII	IX	XV	XVII	VIII	XII	XVI	III	V	VI
CATEGORY												
Content						TEACHERS						
Structure	109^2	106	197	108^1	22	60	79^5	20^1	152^9	115	113	82^1
Techniques	82	148	55	119	107	90	170	82	153	30	25	165
Deductive	162^2	111^1	104	123	24	161	128^2	85^1	36^3	277^1	315^1	334
Inductive	61	164	160^1	59	46	53	60	119^1	87	211	383	120
Mathematical	171^3	140	159	220	117^1	222	238^1	194	187^1	251^1	186	185
Other	31	29	58	24	28	8	16^2	9	9	25	18^1	52
Process												
Analyzing	80	112	149	54	24	61	33	37	26	143	120	94
Synthesizing	138	100	106	125	57	123	129	138	126	288	375	243
Specializing	221	297	336	306	149	225	278	215	215	247	327	331
Generalizing	59	51	85	73	13	71	78	35	30	180	185	87
Relevant	125	139	58	96	102	114	182	87	103	53	35	184
Attitude												
Curiosity	22	34	51	21	6	22	12	8	1	42	59	63
Independence	468	318	340	430	137	398	312	35	98	72	32	84
Receptivity	133	347	343	203	202	174	376	469	401	797	951	792
Neutral	19	34	10	5	32	29	13	9	10	18	10	32
Content						PUPILS						
Structure	30^6	14^6	65^{10}	17^{10}	11^2	15^5	10^2	7^1	9^7	2	1^2	1^1
Techniques	51^9	113^6	15^1	183	30	8^2	6	5	29^1	3^1	3	4
Deductive	90^8	130^3	61^1	35^7	16^4	85^{10}	19^6	7^5	3^1	5^3	10^6	7^2
Inductive	1	6	41^1	4	13^1	7	6^1	10	2	2	3	3
Mathematical	12^1	7	108^5	88^8	295^5	139^{14}	53^6	14^1	169^7	31^1	15	9
Other	17	25^2	20^6	2^1	12^2	1^1	1	2^1	2	0	0	0
Process												
Analyzing	15	32	68	11	24	9	13	7	13	5	5	2
Synthesizing	96	197	94	60	241	132	27	1	137	0	2	4
Specializing	79	72	133	86	81	106	55	35	62	38	25	16
Generalizing	19	4	27	14	9	22	4	4	2	1	4	1
Relevant	16	7	12	14	31	18	5	5	25	4	4	4
Attitude												
Curiosity	7	5	11	4	8	7	4	9	1	6	6	1
Independence	200	275	281	119	353	206	63	30	214	10	12	16
Receptivity	18	32	42	62	25	74	37	13	24	32	22	10
Neutral	1	1	0	0	1	1	0	2	2	0	0	0
Total:												
Teacher & Pupil	868	1046	1078	844	764	911	817	575	751	977	1092	998

Table 29 Index of initiative for each of twelve classrooms

Class	I	XI	XIII	IX	XV	XVII
Index	2.07	1.91	1.99	1.92	2.14	2.01
Class	VIII	XII	XVI	III	V	VI
Index	1.61	1.24	1.72	1.21	1.19	1.22

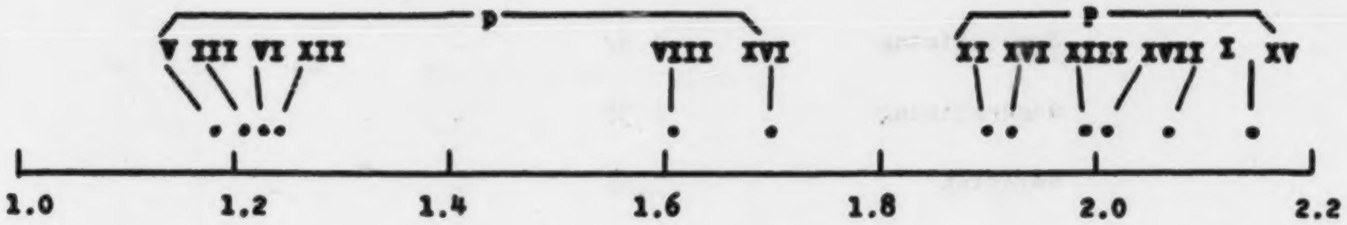


Table 30 F ratios of the individual category variances within and between groups for classroom scores, with corresponding degrees of freedom and differences significant at $p = 0.01$, $= 0.05$ and 0.10: *, ($n_1 = 3$, $n_2 = 8$).

	F	Significance at p		
CATEGORY		0.01	0.05	0.10
Content				
Structure	2.70	-	-	-
Techniques	3.07	-	-	*
Deductive	4.00	-	*	*
Inductive	2.29	-	-	-
Mathematical	7.27	*	*	*
Other	1.35	-	-	-
Process				
Analyzing	6.23	-	*	*
Synthesizing	2.38^{-1}	-	-	-
Specializing	1.32	-	-	-
Generalizing	2.58	-	-	-
Relevant	1.26	-	-	-
Attitude				
Curiosity	7.90	*	*	*
Independence	18.47	*	*	*
Receptivity	19.57	*	*	*
Neutral	2.90^{-1}	-	-	-

between groups and the significant differences between the four groups of classroom scores, at probability levels 0.01, 0.05, 0.10. At the chosen probability level, $p = 0.05$, the following categories could be distinguished into the four types RP, rP, rp and Rp:

Content --relations-deductive
applications-mathematical
Process --analyzing

Attitude --curiosity
independence
receptivity

Considering then the ratio of teacher and pupils frequencies, the analysis of variance of the pupils contribution to the total representative score of 1,000 across the twelve classrooms resulted in an F ratio of 5.92, $n_1 = 3$, $n_2 = 8$ significant at $p = 0.025$. Altogether, the four types selected by the measures of the independent variables were clearly distinguished by an overall view of the individual frame measures of the dependent variables, $p = 0.05$.

The detailed comparison category by category of pairs of types of classrooms could then be made interpreting with some caution significant differences in categories not cited above as distinctive. Accordingly, following comparison of the aggregate representative scores summary of the differences significant at $p = 0.05$ were recorded in Table 32. It is noted that Table 32 records the actual results of differences hypothesized in Table 17. Where the actual result did not confirm the hypothesis this was emphasized by an asterisk. The standardized aggregate values of each type of classroom, teacher and pupils scores were illustrated in descriptive and comparative bar graphs, Figures 34-39. Comparison of Tables 17 and 32 and reference to Table 33 permits selection of hypotheses unconfirmed and the direction of the disagreement.

Confirmation or rejection of hypotheses may be detailed as follows: (Categories firmly separated by analysis of variance starred).

CONTENT:

Structure varies jointly and directly as rigor and participation.

Confirmed - The one exception rp and RP in expected direction but not significant.

Technique varies jointly but inversely as rigor and participation.

Confirmed - with one exception where $Rp < RP$

*Deductive varies directly as rigor.

Confirmed - In Rp and RP; rp and rP where rigor was constant and participation varied equality was expected. No effect related to participation was noted, however, for $Rp > RP$ and $rp < rP$.

Inductive varies directly as participation.

Rejected - However the following would be confirmed "Inductive varies directly as rigor and inversely as participation".

*Mathematical varies inversely as rigor

Confirmed - with one exception $Rp < RP$ though related tendency observed in $rp < rP$, $p = 0.02$.

Other is independent of rigor and of participation.

Rejected - "Other varies directly and jointly as rigor and participation" would be confirmed.

PROCESS:

*Analyzing varies jointly and directly as rigor and participation.

Confirmed - although while $rp < rP$ this difference was not significant. Further $Rp \neq rP$ so that the effect of rigor and participation was not

Table 32 Direction of differences significant at p = 0.001 for the four classroom types RP, rP, rp and Rp. Classroom scores for differences significant merely at p = 0.05, 0.02, 0.01 these levels indicated. Where differences differ from hypotheses marked with asterisk.

TYPE CATEGORY	Rp & rp	RP & rP	Rp & RP	rp & rP	Rp & rP	rp & RP	
Content							
Structure	>	>	<	- *	-	<	
Techniques	<	<	< *	>	<	>	
Deductive	>	>	> *	< *	>	<	
Inductive	< *	> *	> *	> *	> *	- *	
Mathematical	<	<	< *	0.02 <	<	>	
Other	0.01 >	> *	< *	0.01 <	-	< *	
Process							
Analyzing	>	>	0.01<	- *	> *	<	
Synthesizing	> *	<	>	0.01*<	- *	- *	
Specializing	<	- *	< *	0.05 >	<	0.05 >	
Generalizing	> *	-	>	- *	>	- *	
Relevant	<	<	< *	>	<	>	
Attitude							
Curiosity	>	>	0.01 >	0.05<	>	<	
Independence	< *	-	<	<	<	<	
Receptivity	> *	-	>	>	>	>	
Neutral	-	-	-	0.01>	-	-	
No. disagreements	5/15	4/15	7/15	6/15	3/15	4/15	29/90

Table 33 Standardized aggregate individual frame score for each type of classroom: high rigor, high participation (RP); low rigor, high participation (rP); low rigor, low participation (rp); high rigor, low participation (Rp) in each category of Content, or Process and of Attitude. Classroom scores. Teacher and Pupils components. (C, T, P) Correct to nearest 1,000

Type	RP			rP			rp			Rp		
Classes	(I, XI, XIII)			(IX, XV, XVII)			(VIII, XII, XVI)			(III, V, VI)		
Component	C	T	P	C	T	P	C	T	P	C	T	P
CATEGORY												
Content												
Structure	181	138	43	98	74	24	73	57	16	104	102	2
Techniques	108	96	12	152	127	25	204	185	19	76	73	3
Deductive	211	130	81	178	118	60	137	121	16	314	303	11
Inductive	153	125	28	83	63	10	146	132	14	232	229	3
Mathematical	275	160	115	439	220	219	404	293	111	223	204	19
Other	50	40	10	32	24	8	19	16	3	31	31	0
Process												
Analyzing	149	112	37	72	54	18	62	46	16	121	117	4
Synthesizing	246	117	129	297	119	178	261	189	72	296	294	2
Specializing	378	284	94	376	268	108	404	334	70	321	295	26
Generalizing	82	65	17	78	61	17	70	65	5	149	147	2
Relevant	123	111	12	150	124	26	186	170	16	94	90	4
Attitude												
Curiosity	42	35	7	27	19	8	18	10	8	58	53	5
Independence	638	387	251	651	375	276	329	191	138	74	62	12
Receptivity	298	267	31	294	232	62	636	603	33	848	827	21
Neutral	22	21	1	28	27	1	17	15	2	20	20	0
Totals	1,000	710	290	1,000	653	347	1,000	819	181	1,000	962	38

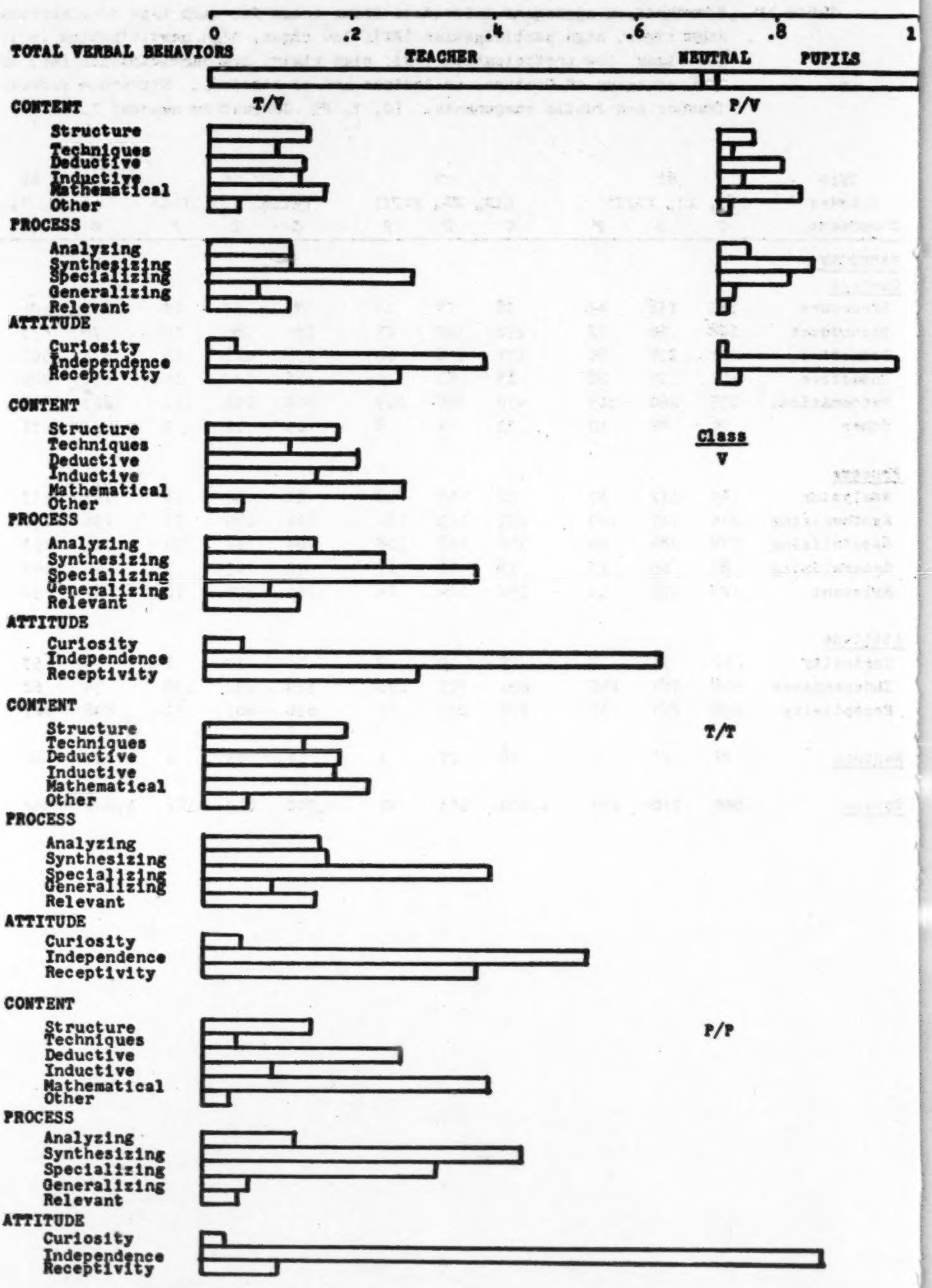


Figure 34 Classroom type RP. Individual frame scores in Content, Process and Attitude for classroom (T&P)/V, teacher T/V, pupil P/V; and teacher proportions T/T, pupil proportions P/P.

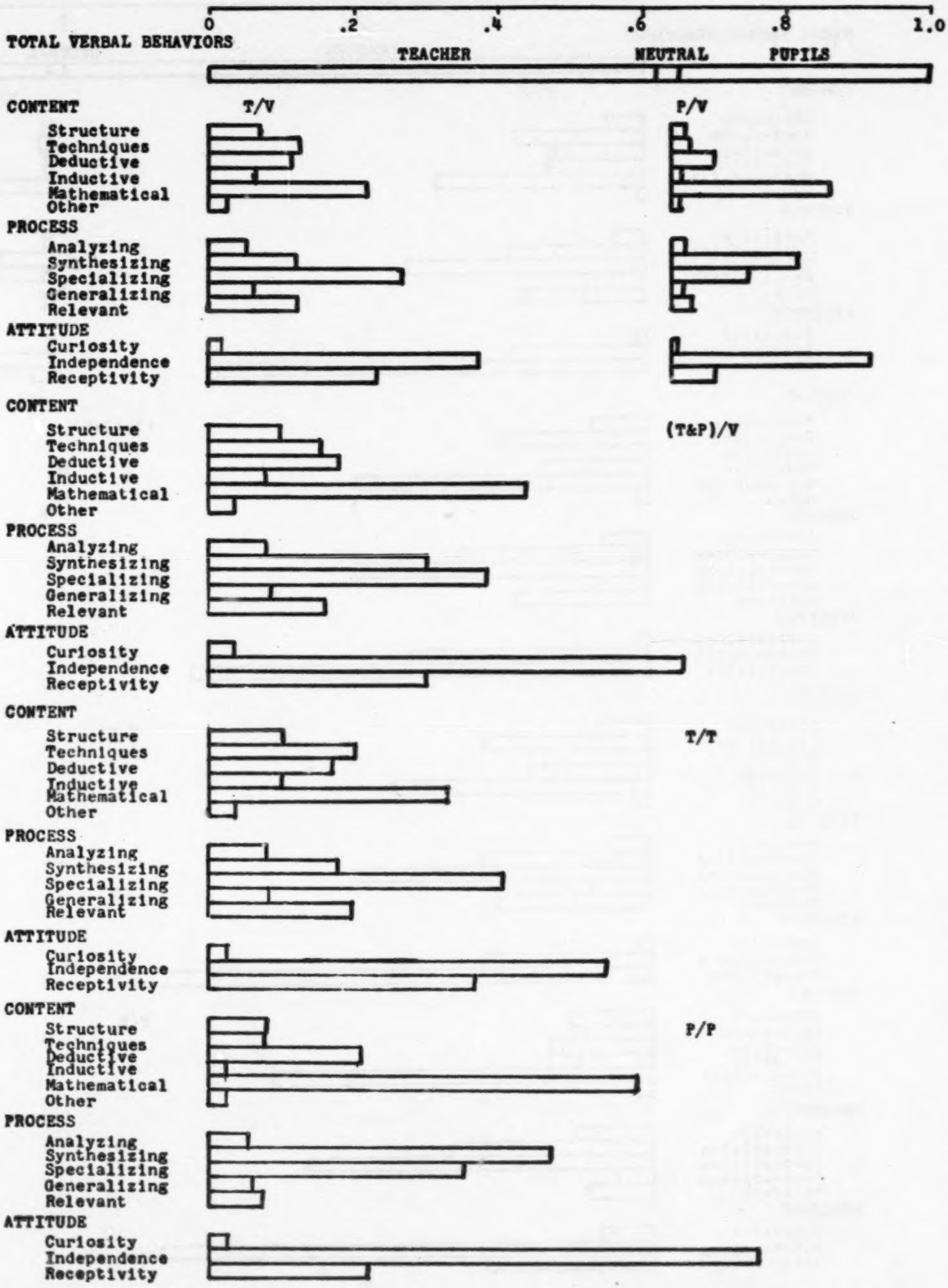


Figure 35 Classroom type r^P. Individual frame scores in Content, Process and Attitude for classroom (T&P)/V, teacher T/V, pupil P/V; and teacher proportions T/T, pupil proportions P/P.

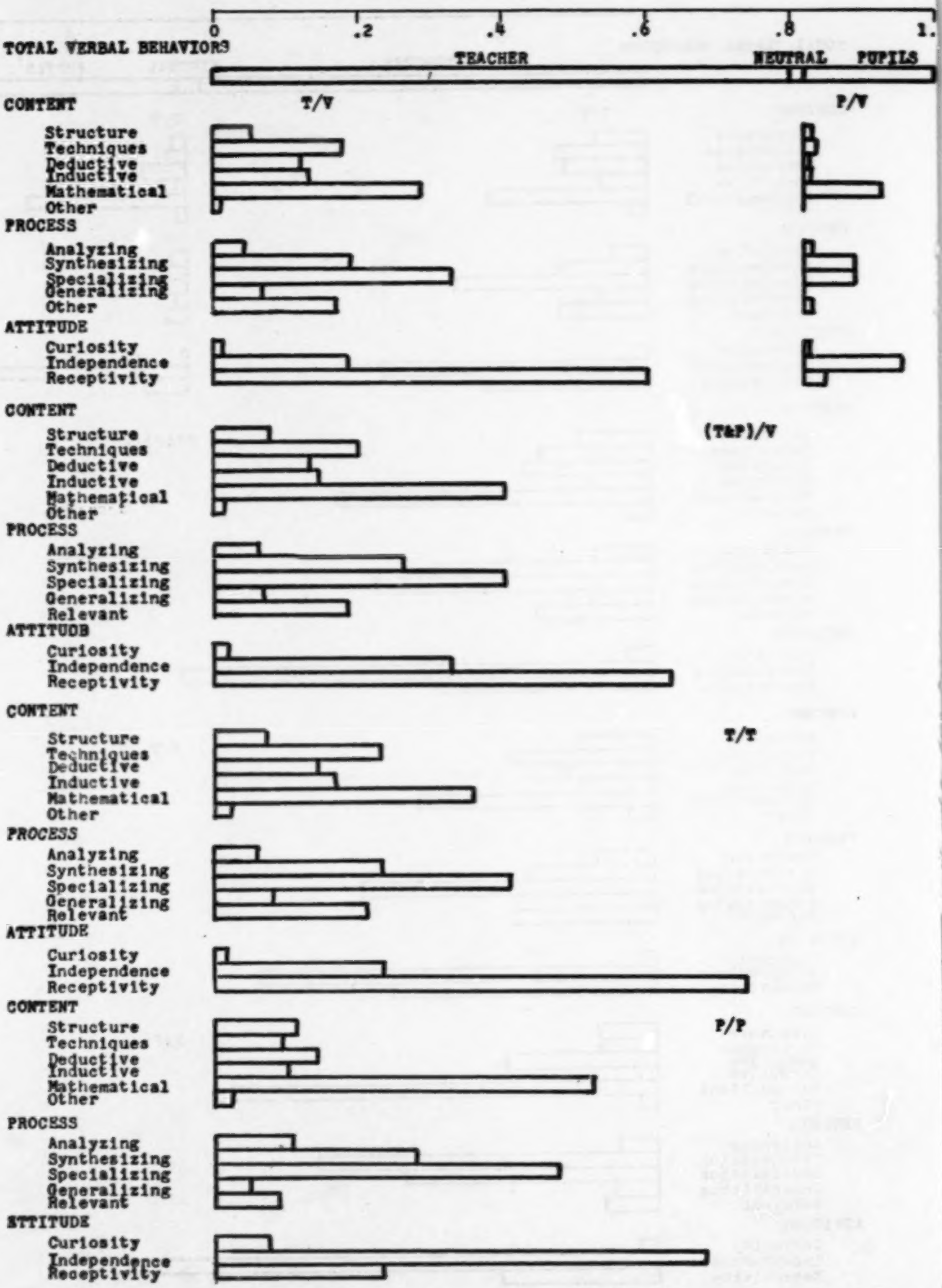


Figure 36 Classroom type rp. Individual frame scores in Content, Process and Attitude for classroom (T&P)/V, teacher T/V, pupil P/V; and teacher proportions T/T, pupil proportions P/P.

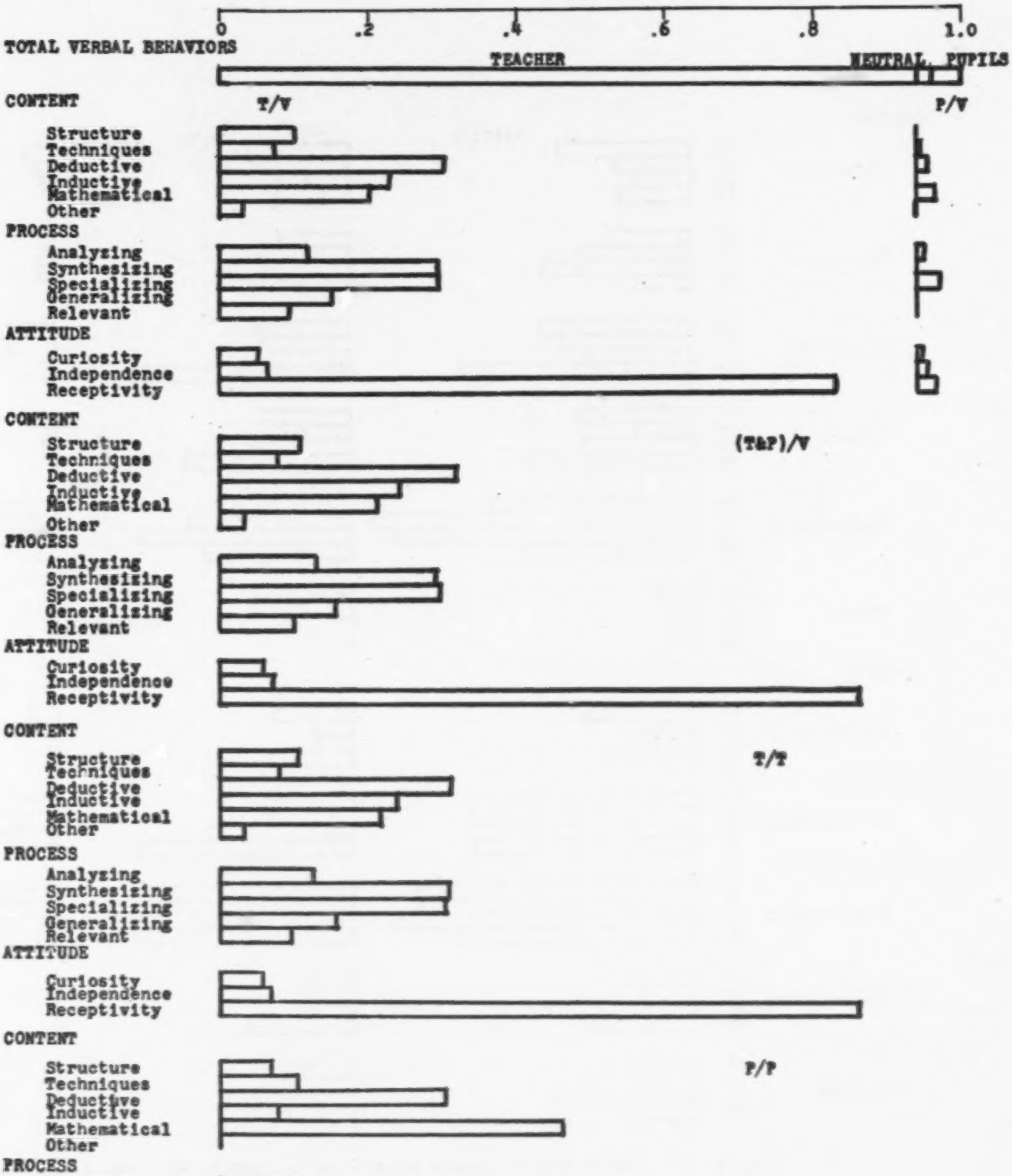


Figure 37 Classroom type Rp. Individual frame scores in Content, Process and Attitude for classroom (T&P)/V, teacher T/V, pupil P/V; and teacher proportions T/T, pupil proportions P/P.

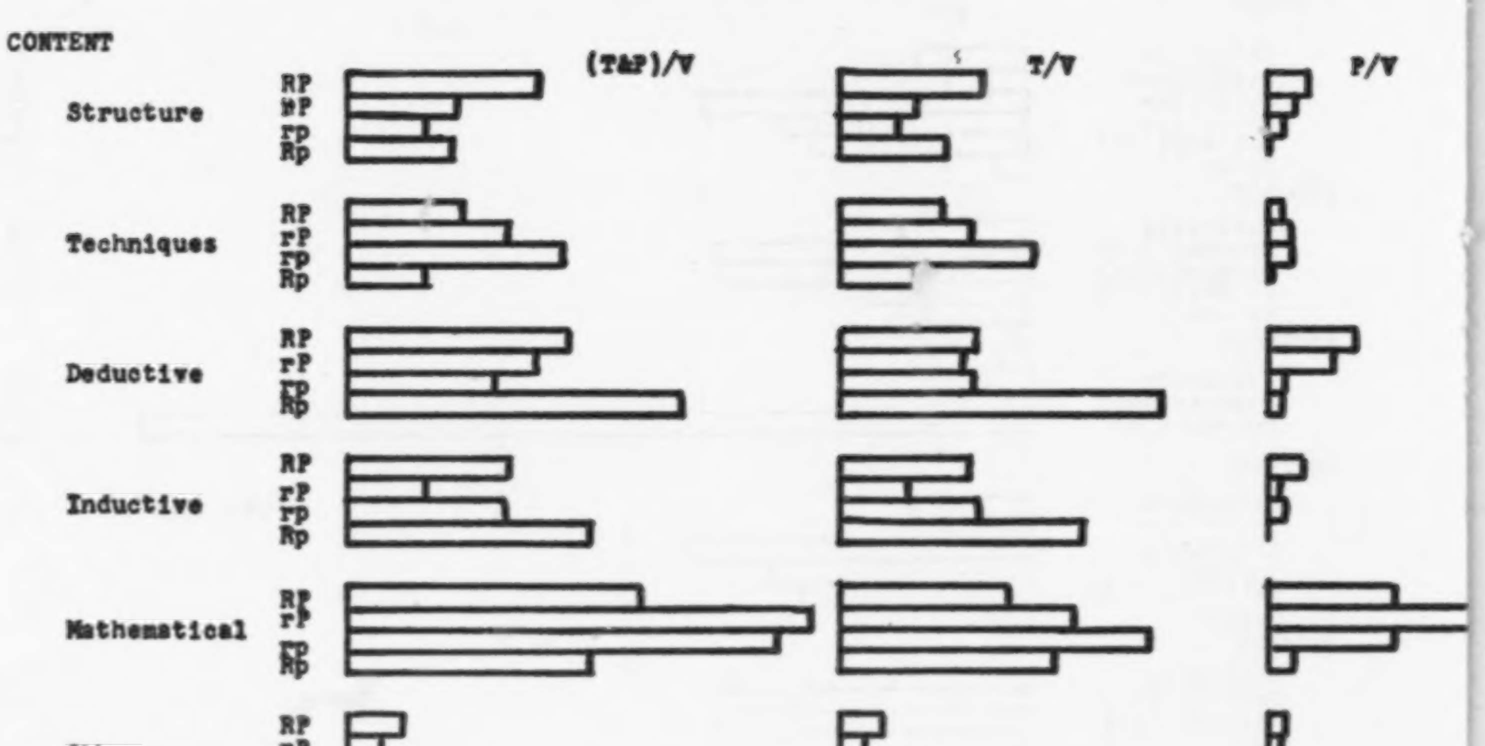


Figure 38 Comparison of single categories in individual frame scores of the four classroom types, RP, rP, rp, Rp. In the Content and Process frames only. Scale as before.

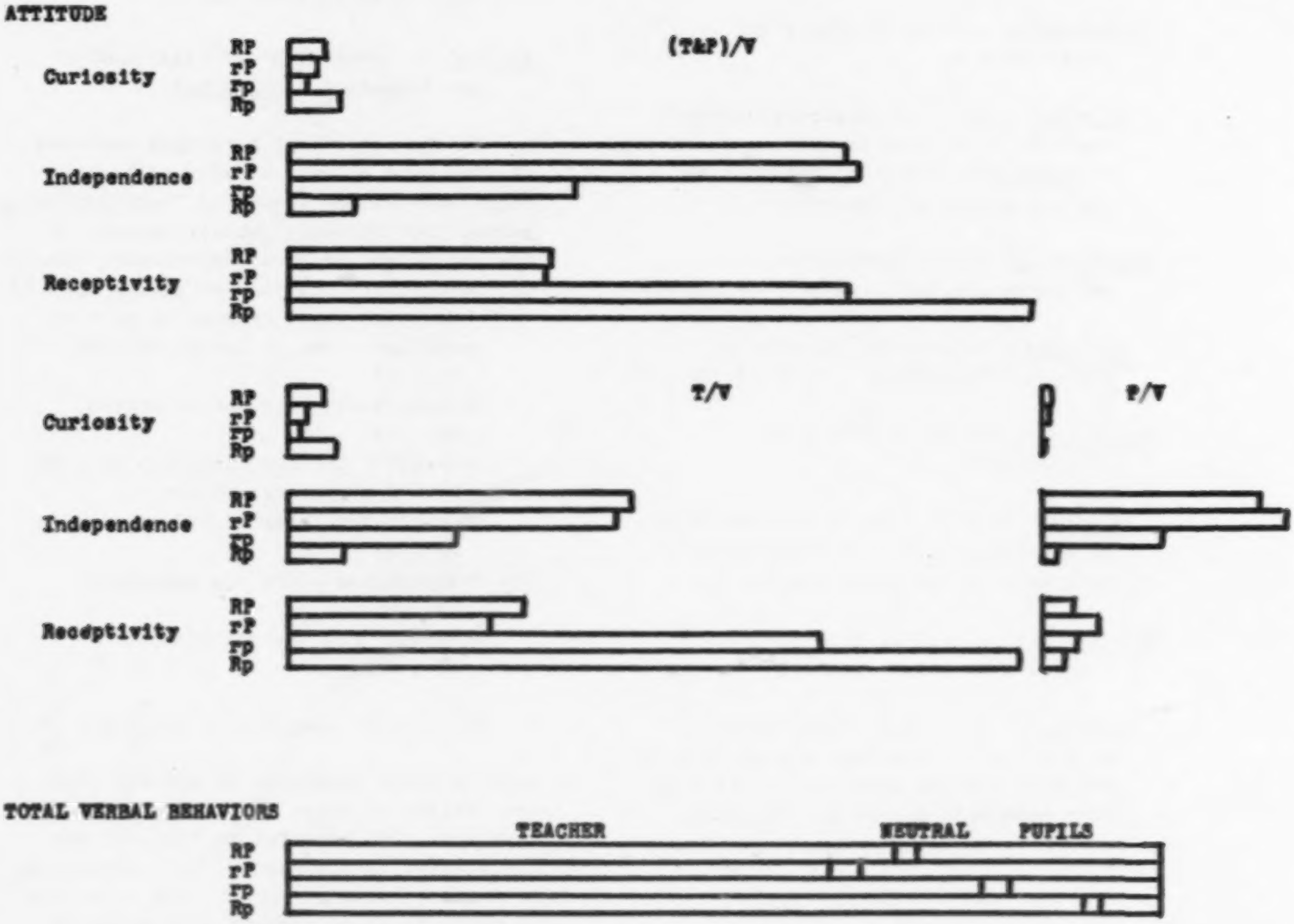


Figure 39 Comparison of single categories in individual frame scores of the four classroom types, RP, rP, rp, Rp. In the Attitude frame only. Comparison of division of total verbal behaviors.

such as to exactly equalize each other - rather, the effect of rigor was apparently greater.

Synthesizing varies inversely as participation.

Rejected - while synthesizing differs somewhat from type to type no clear-cut relationship either to rigor or to participation was apparent.

Specializing varies directly as participation and inversely as rigor.

Confirmed - in four of the six instances. Exceptions - $RP = rP$; $Rp < RP$.

Generalizing varies inversely as participation.

Rejected Only in type Rp did the level of generalizing differ significantly from each of the other types.

Relevant varies jointly but inversely as rigor and as participation.

Confirmed - one exception in $Rp < RP$. Rp might well have been stated $Rp = rP$ but some feeling that rigor likely to have more effect than participation.

ATTITUDE:

**Curiosity* varies directly as rigor.

Confirmed - the differences in Rp and RP, rp and rP just significant as $p = 0.01$ and $p = 0.05$, respectively, were not of the same order as in the four other cases.

**Independence* varies directly as participation.

Confirmed - Only exception $Rp < rp$. The frequency of participation in rp was indeed greater than in Rp.

**Receptivity* varies inversely as participation.

Confirmed - Only exception $Rp > rp$. Actually the frequency of participation in rp was indeed greater than in Rp as noted above.

Neutral is independent of rigor and of participation. *Confirmed*.

It was noted that a recognition that (1) the type Rp was actually of higher rigor that RP and (2) Rp of lower participation than rp would resolve several of the exceptions to these hypotheses, namely:

(1) Techniques - the one exception $Rp < RP$ would at least resolve to $Rp = RP$
Deductive - one of two exceptions $Rp > RP$
Mathematical - the one exception $Rp < RP$
Relevant - the one exception $Rp < RP$ with R of greater effect.
Specializing - one of two exceptions $Rp < RP$
(2) Independence - the one exception $Rp < rp$
Receptivity - the one exception $Rp > rp$.

The related comparisons of $\frac{T_i}{T_v}$ and $\frac{P_i}{P_v}$ were recorded in Tables 41 and 42. The mean values of these proportions type by type were also recorded in Table 43 and illustrated in Figures 44, 45. Comparison of Table 43 with 41 and 42, and reference to Table 17 again permits indication of relationship of observed differences in teacher and in pupils proportions with those hypothesized for classroom scores in Table 17.

From the comparisons of $\frac{T_i}{T_v}$, it was clear that the hypotheses stated above for classroom scores were confirmed or rejected in essentially the same fashion. Specifically, *confirmation* was more complete in Content - mathematical and in Process--analyzing and synthesizing; *rejection* was more definite in Content - Other. Rejection of hypothesis for Neutral was also noted.

Table 41 Direction of differences significant at p = 0.001 for the four classroom types RP, rP, rp, Rp: Teacher proportions. For differences significant merely at p = 0.05, 0.02, 0.01 these levels indicated. Where differences differ from classroom hypotheses marked with asterisk.

TYPE CATEGORY	Rp & rp	RP & rP	Rp & RP	rp & rP	Rp & rP	rp & RP	
Content							
Structure	>	>	<	<	-	<	
Techniques	<	<	< *	>	<	>	
Deductive	>	- *	> *	< *	>	<	
Inductive	> *	> *	> *	> *	> *	- *	
Mathematical	<	<	-	-	<	>	
Other	> *	> *	< *	< *	-	< *	
Process							
Analyzing	>	>	<	<	> *	<	
Synthesizing	> *	-	>	>	>	>	
Specializing	<	- *	< *	-	<	- *	
Generalizing	> *	-	>	- *	>	- *	
Relevant	<	<	< *	- *	<	>	
Attitude							
Curiosity	>	>	-	< *	>	<	
Independence	< *	-	<	<	<	<	
Receptivity	> *	-	>	>	>	>	
Neutral	-	< *	< *	< *	<	-	
No. disagreements	6/15	5/15	7/15	7/15	2/15	4/15	31/90

Table 42 Direction of differences significant at p = 0.001 for the four classroom types RP, rP, rp, Rp: Pupils proportions. For differences significant merely at p = 0.05, 0.02, 0.01, these levels indicated. Where differences differ from classroom hypotheses marked with asterisk.

TYPE CATEGORY	Rp & rp	RP & rP	Rp & RP	rp & rP	Rp & rP	rp & RP	
Content							
Structure	< *	>	<	> *	0.01<	<	
Techniques	- *	<	> *	0.02 >	> *	>	
Deductive	>	>	-	< *	>	<	
Inductive	0.01 <	-	0.01<	> *	> *	- *	
Mathematical	<	<	> *	< *	<	>	
Other	< *	> *	> *	-	< *	< *	
Process							
Analyzing	- *	>	0.01<	> *	> *	<	
Synthesizing	< *	-	< *	< *	< *	<*	
Specializing	> *	- *	< *	> *	> *	>	
Generalizing	0.02 >	-	- *	>	- *	< *	
Relevant	> *	<	>	- *	> *	>	
Attitude							
Curiosity	>	- *	> *	> *	>	> *	
Independence	< *	> *	<	<	<	<	
Receptivity	> *	< *	>	- *	>	>	
Neutral	< *	-	0.02<	> *	< *	> *	
No disagreements	10/15	5/15	7/15	11/15	9/15	6/15	48/90

Table 43 Proportions of representative teacher individual frame scores of the four classroom types in each category to total teacher verbal; proportions of representative pupil individual frame scores of the four classroom types in each category to total pupil verbal. For each category of Content, Process and Attitude and in Neutral. All decimals omitted. Negatives absorbed in each category score.

TYPE	RP	rP	rp	Rp
CATEGORY		TEACHERS		
Content				
Structure	194	106	68	105
Techniques	135	203	232	75
Deductive	183	170	141	315
Inductive	178	99	162	238
Mathematical	225	334	359	213
Other	55	41	20	33
Process				
Analyzing	159	81	56	122
Synthesizing	165	179	231	305
Specializing	400	407	408	306
Generalizing	92	86	79	153
Relevant	154	200	208	93
Attitude				
Curiosity	50	28	12	55
Independence	540	551	232	65
Receptivity	380	374	738	859
Neutral	30	47	18	21
Total Teacher	1,000	1,000	1,000	1,000
		PUPILS		
Content				
Structure	149	83	110	64
Techniques	43	75	92	102
Deductive	277	203	141	300
Inductive	96	27	99	75
Mathematical	399	590	525	459
Other	34	20	18	0
Process				
Analyzing	124	51	103	101
Synthesizing	446	468	282	66
Specializing	326	347	478	670
Generalizing	59	59	40	53
Relevant	43	73	82	110
Attitude				
Curiosity	27	23	70	104
Independence	868	756	683	367
Receptivity	103	219	232	529
Neutral	2	2	15	0
Total Pupils	1,000	1,000	1,000	1,000

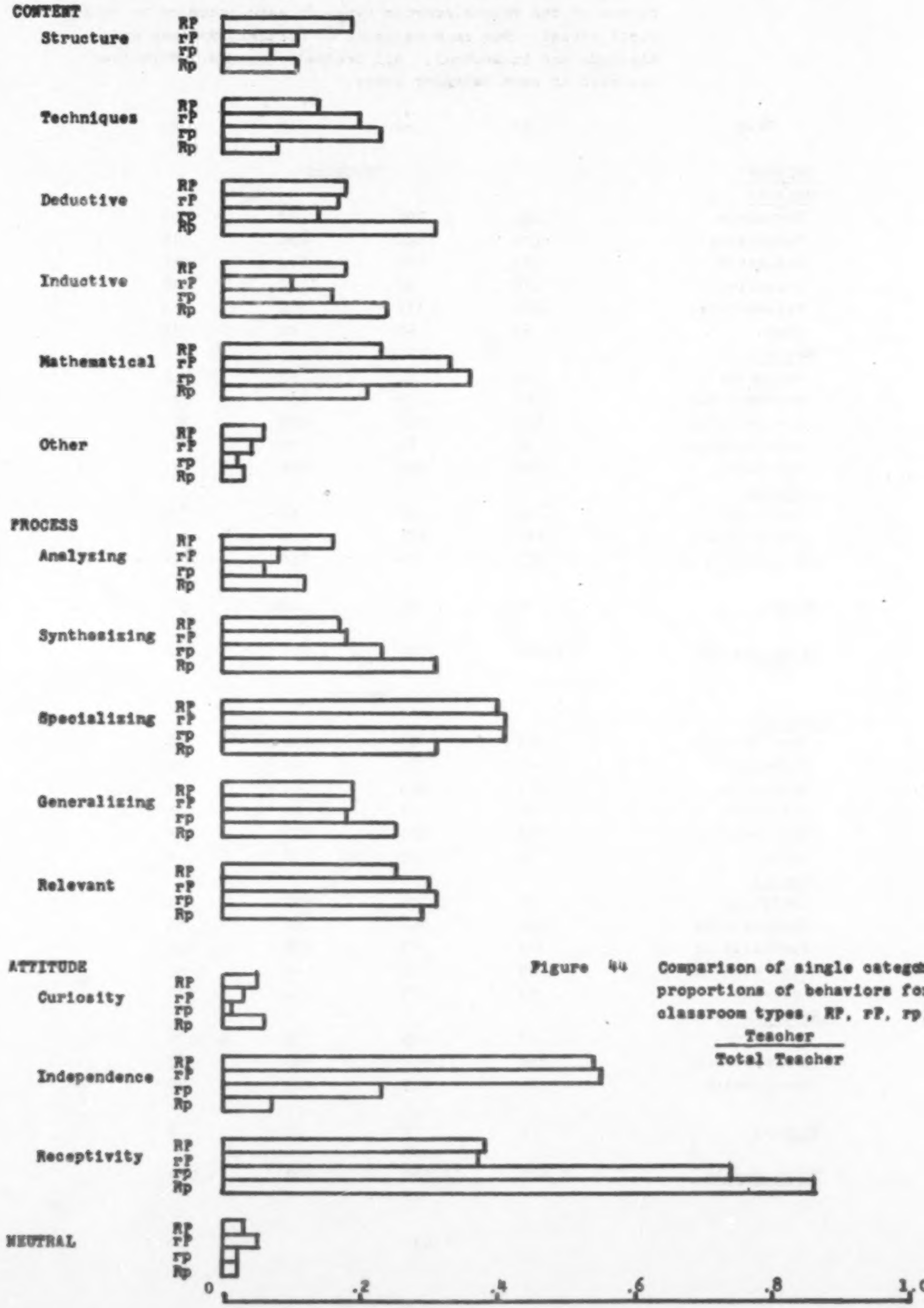


Figure 44 Comparison of single categ
proportions of behaviors fo
classroom types, RP, rP, rp

$\frac{\text{Teacher}}{\text{Total Teacher}}$

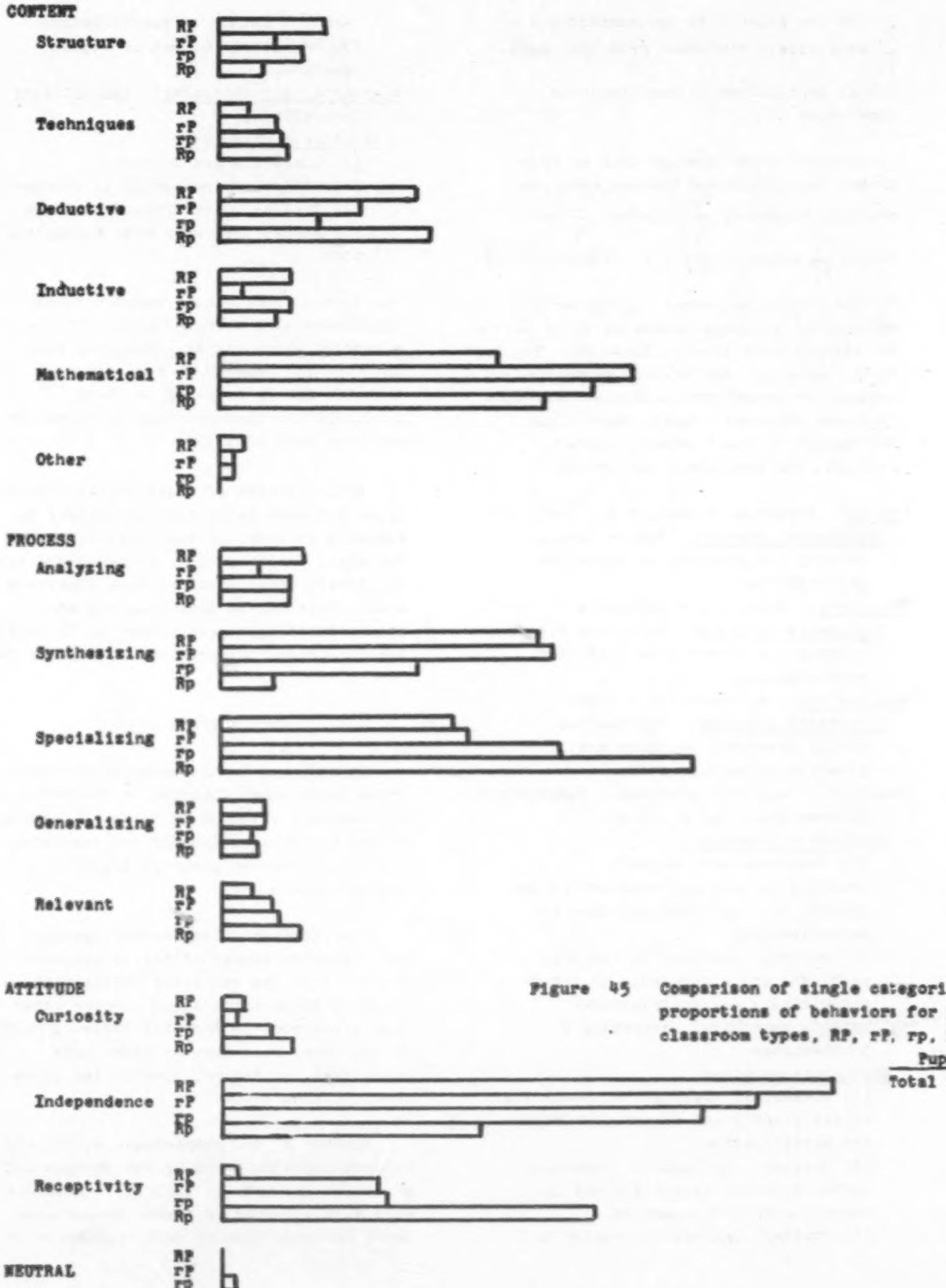


Figure 45 Comparison of single categories: proportions of behaviors for fou
classroom types, RP, rP, rp, Rp
Pupil
Total Pup

On the other hand the comparisons of $\frac{P_i}{P_v}$ were widely different from the operational definitions of the classroom hypotheses.

Several broad combinations of categories in Content and Process were compared in classroom $\frac{C_i}{C_v}$, teacher $\frac{T_i}{T_v}$ and pupils $\frac{P_i}{P_v}$ scores. Table 47, Figures 48,49.

For the classroom scores, as before an analysis of variance indicated the F ratios and significance levels, Table 50. The results, Table 51, can be best summarized by proposal of hypotheses to account for distinctions observed. Again where clear distinction of the 4 types occurred, $p = 0.05$, the hypothesis was starred.

*Theory: Structure + Deductive + Inductive
  Hypothesis proposed: Theory varies jointly and directly as rigor and participation.
*Relations: Deductive + Inductive
  Hypothesis proposed: Relations varies jointly and directly as rigor and participation.
*Application: Mathematical + Other
  Hypothesis proposed: Application varies inversely as rigor and directly as participation.
*Practice in specific problems: Inductive + Mathematical; but $p = 0.05$.
  Hypothesis proposed:
    (1) Classroom and teacher: Practice in specific problems varies jointly but inversely as rigor and participation.
    (2) Pupils: Practice in specific problems varies inversely as rigor and directly as participation.
*Syllogistic reasoning: Analyzing + Synthesizing
  Hypothesis proposed:
    (1) Classroom: Syllogistic reasoning varies jointly and directly as rigor and participation.
    (2) Teacher: Syllogistic reasoning varies directly with rigor and inversely with participation.
    (3) Pupils: Syllogistic reasoning varies directly as participation. (No consistent effect of rigor apparent.)
Classificatory reasoning: Specializing + Generalizing
  Hypothesis proposed:
    (1) Classroom and teacher: Classificatory reasoning is constant.
    (2) Pupils: Classificatory reasoning varies inversely with participation.

The several comparisons support these hypotheses to a marked degree. If, as suggested above, it is considered that the type Rp actually is more rigorous than RP, and Rp actually is lower participation than rp, then confirmation was even more complete.

Establishment of clear distinction of types and some basis for the variety in emphasis in terms of the level of rigor and pupil participation, prepared the way for direct description of each classroom type. This may be made from the mean standardized aggregate scores in Table 33 and the bar graph illustrations of Figures 34-37.

Triple frame scores

In addition to the example of triple frame score classification of Teacher I by Observers I and II in Table 19, similar tables for Pupil I and for the remaining fifteen classrooms occur in Appendix A Tables: 175 to 202.

The representative teacher, pupils and classroom scores of the 12 subjects derived from the raw score tables were recorded in Appendix A (not for the other four classrooms) in Appendix Tables 203-214. In addition, mean type matrixes were calculated for teacher, pupils and classroom. Tables 52-54.

Summary of the comparisons within and between types selected by the independent variables was made in Table 55. Recorded were W, X^2, number of ranks, probability level for rejection of null hypothesis of

Table 47 Frequencies of selected category combinations of individual frame scores for the four classroom types. Classroom, teacher and pupils scores.

TYPE			RP	rP	rp	Rp
CATEGORY COMBINATIONS		SCORES				
Content:	Theory	C	545	349	356	650
		T	555	375	371	657
		P	522	313	350	440
	Relations	C	364	251	283	546
		T	361	269	303	552
		P	373	230	240	376
	Applications	C	325	471	423	254
		T	280	375	379	246
		P	433	610	543	459
	Specific Problems	C	428	512	550	455
		T	403	433	521	451
		P	495	617	524	535
Process:	Syllogistic	C	395	369	323	417
		T	324	260	287	427
		P	569	519	385	167
	Classificatory	C	460	454	474	470
		T	492	493	487	459
		P	385	406	518	723

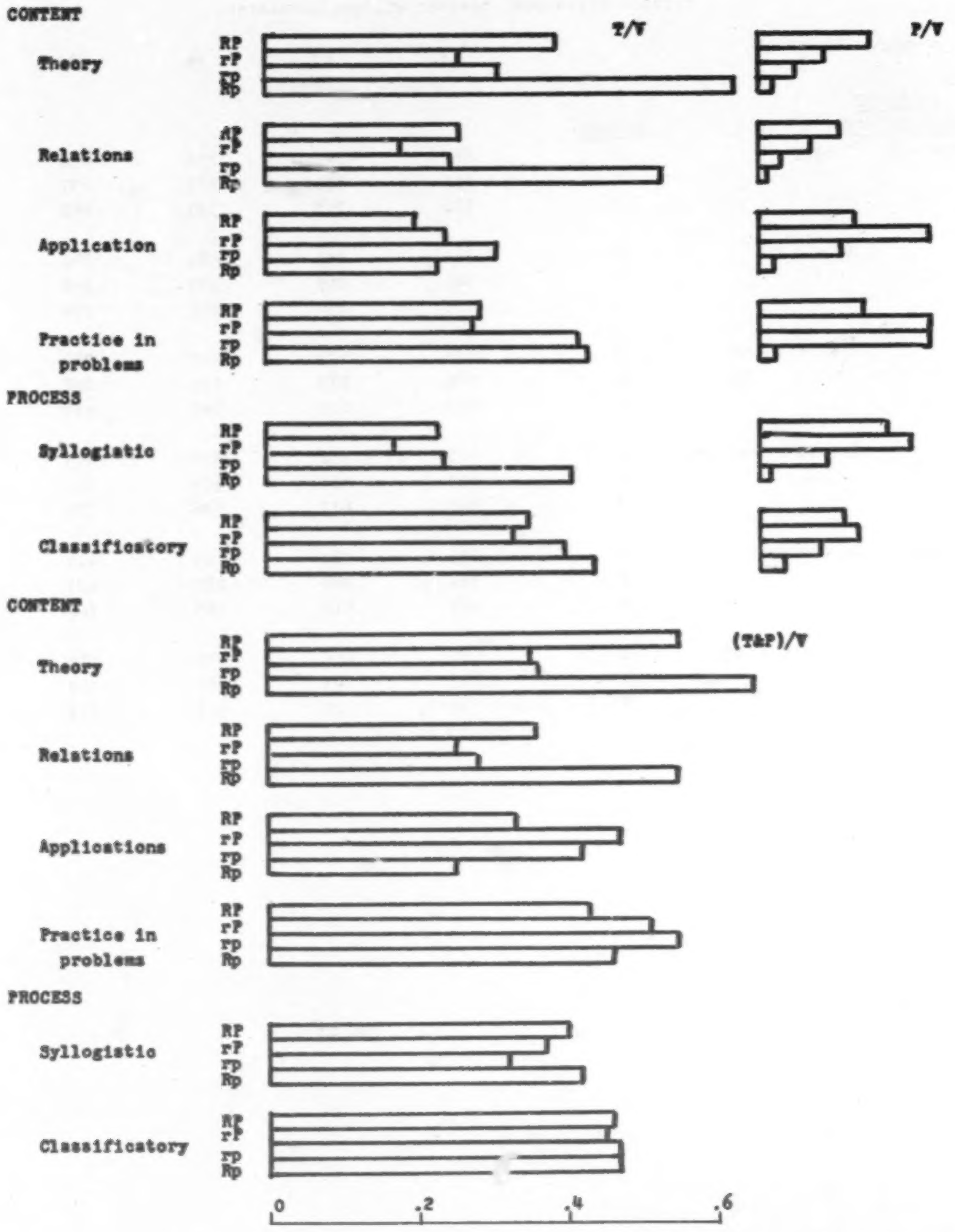


Figure 48 Comparison of combinations of categories in individual frame scores in Content and Process for the four classroom types, RP, rP, rp, Rp.

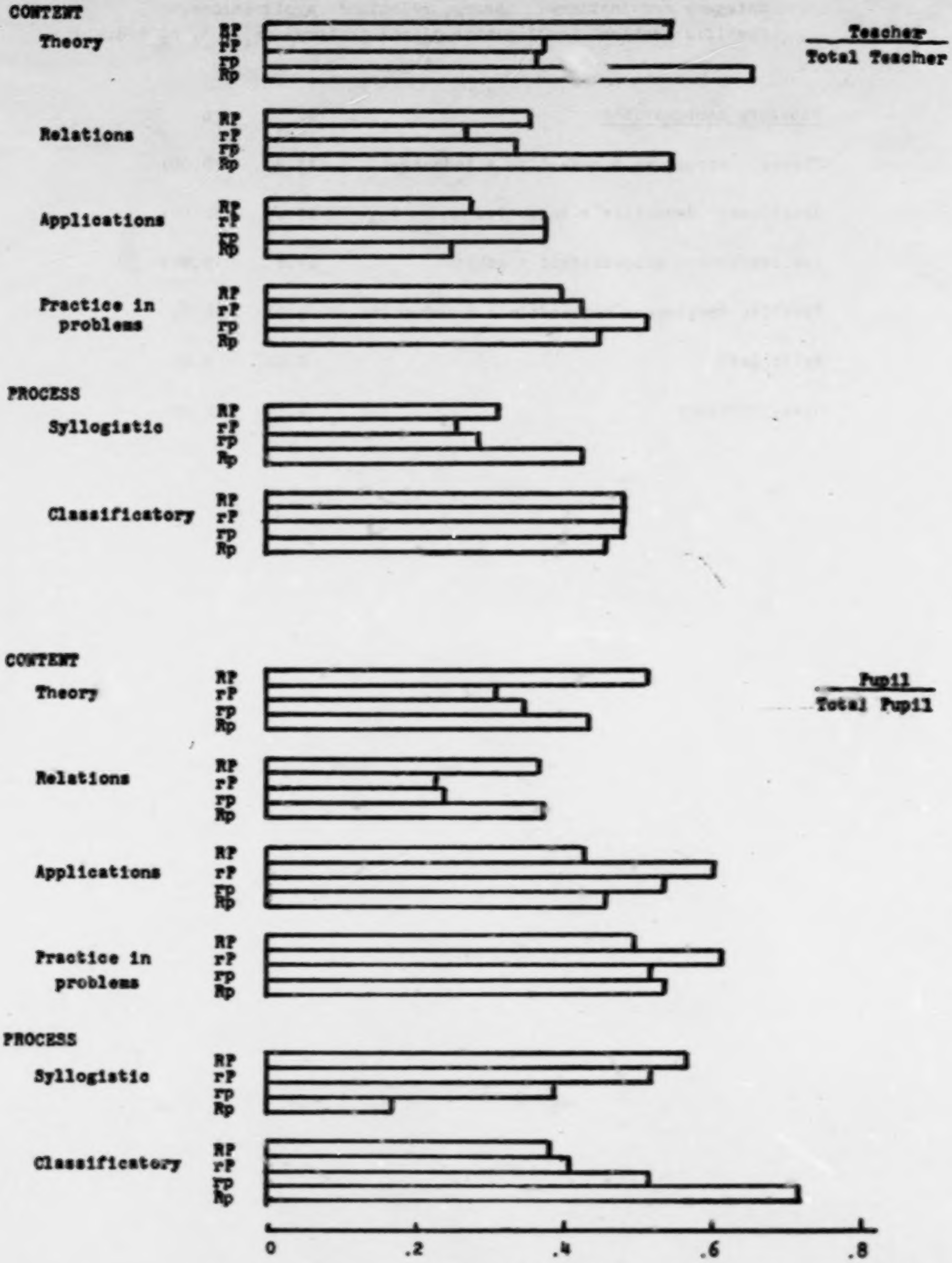


Figure 49 Combinations of categories compared: proportions of behaviors for four classroom types, RP, rP, rp, Rp: $\frac{\text{Teacher}}{\text{Total Teacher}}$ and $\frac{\text{Pupil}}{\text{Total Pupil}}$.

Table 50 F ratios and corresponding probability levels of combinations of individual category differences for classroom scores. Category combinations: theory, relations, applications, specific problems, syllogistic, classificatory. $n_1 = 3$; $n_2 = 8$.

Category combinations:	F	p
Theory: structure + deductive + inductive	17.2	0.001
Relations: deductive + inductive	19.6	0.001
Applications: mathematical + other	17.8	0.001
Specific Problems: mathematical + inductive	4.03	0.05
Syllogistic	4.02	0.05
Classificatory	0.14	0.20

Table 51 Direction of differences significant at p = 0.01 for category combinations for the four classroom types RP, rP, rp, Rp; Classroom scores, teachers proportions, pupils proportions - $\frac{C_i}{C_v}$, $\frac{T_i}{T_v}$, $\frac{P_i}{P_v}$. For differences significant merely at p = 0.05, 0.02, 0.01 these levels indicated.

TYPE CATEGORY Combination	Rp & rp	RP & rP	Rp & RP	rp & rP	Rp & rP	rp & RP
Content: Theory						
C	>	>	>	-	>	<
T	>	>	>	-	>	<
P	>	>	0.01<	-	>	<
Content: Relations						
C	>	>	>	-	>	<
T	>	>	>	-	>	0.01<
P	>	>	-	-	>	<
Content: Applications						
C	<	<	-	0.02<	<	>
T	<	<	-	-	<	>
P	<	<	-	0.01<	<	>
Content: Specific problems						
C	<	<	-	>	0.02<	>
T	0.01<	-	0.05 >	>	-	>
P	-	<	-	<	<	-
Process: Syllogistic						
C	>	-	-	0.05 <	0.05>	<
T	>	>	>	-	>	-
P	<	0.05>	<	<	<	<
Process: Classificatory						
C	-	-	0.05>	-	-	0.01>
T	-	-	-	-	-	-
P	>	-	>	>	>	>

Theory: Structure + Deductive + Inductive
Relations: Deductive + Inductive
Applications: Mathematical + Other
Specific problems: Inductive + Mathematical
Syllogistic: Analyzing + Synthesizing
Classificatory: Specializing + Generalizing

Table 52 Mean triple frame cross classification in Content (C), Process (P), and Attitude (A), for all types of classrooms. Classroom scores. Negatives disregarded. Symbolically: $C_iP_jA_k$. i--1 to 6; j--1 to 5; k--1 to 3. Scores 5 only.

	P_1A_1	P_2A_1	P_3A_1	P_4A_1	P_5A_1	P_1A_2	P_2A_2	P_3A_2	P_4A_2	P_5A_2	P_1A_3	P_2A_3	P_3A_3	P_4A_3	P_5A_3
								MEAN CLASSROOM TYPE RP							
C_1						41	6	52	15		12		31	12	
2						26	65	41	18		6	12	22	10	
3						20	105	65		5	6	22	33		
4			5		10	5		5					6		
5								5		36			8		53
6			7			15	22	53	8			6	26	7	
								MEAN CLASSROOM TYPE rP							
C_1						14	8	33	7				16	7	
2						18	24	52	21			9	16	14	
3						16	199	109		11		35	51		9
4								8					10		
5								20		49			14		59
6						6	6	24	11			6	7	5	
								MEAN CLASSROOM TYPE rp							
C_1								15			7		19	8	
2								22	5		10	15	36	26	
3							100	63		6	6	95	105		8
4													6		
5								6		32			22		123
6						5		20			8	28	56	12	
								MEAN CLASSROOM TYPE Rp							
C_1	6			5							31		24	18	
2								11			37	103	67	74	
3								9			12	96	33		7
4					14										
5										9			9		51
6			6					10			16	82	80	30	

Table 53 Mean triple frame cross classifications in Content (C), Process (P), and Attitude (A), for all types of classrooms. Teacher scores. Negatives disregarded. Symbolically: $C_iP_jA_k$. i--1 to 6; j--1 to 5; k--1 to 3. Scores 5 only.

	P_1A_1	P_2A_1	P_3A_1	P_4A_1	P_5A_1	P_1A_2	P_2A_2	P_3A_2	P_4A_2	P_5A_2	P_1A_3	P_2A_3	P_3A_3	P_4A_3	P_5A_3
								MEAN TEACHER TYPE RP							
C_1						34		32	7		12		25	11	
2						18	26	26	13		6	9	18	9	
3						11	34	49			6	19	29		
4			7		8								5		
5										32			7		48
6			6			12	13	43	7			5	24	8	
								MEAN TEACHER TYPE rP							
C_1						11		24	6				12	6	
2						14	10	35	12			7	11	11	
3						9	49	74		8		25	31		7
4													8		
5								16		42			11		50
6							5	20	8			6	7		
								MEAN TEACHER TYPE rp							
C_1								10			7		18	8	
2								15			10	15	33	27	
3							38	40		5	6	91	96		6
4													6		
5										27			24		116
6								13			7	25	56	10	
								MEAN TEACHER TYPE Rp							
C_1	6			5							31		24	19	
2								7			37	102	65	73	
3								8			12	96	68		6
4					14										
5								5		8			9		48
6			5					12			16	82	79	27	

Table 54 Mean triple frame cross classifications in Content (C), Process (P), and Attitude (A), for all types of classrooms. Pupils Scores. Negatives disregarded. Symbolically: $C_iP_jA_k$. i--1 to 6; j--1 to 5; k--1 to 3. Scores 5 only.

	P_1A_1	P_2A_1	P_3A_1	P_4A_1	P_5A_1	P_1A_2	P_2A_2	P_3A_2	P_4A_2	P_5A_2	P_1A_3	P_2A_3	P_3A_3	P_4A_3	P_5A_3
MEAN PUPILS TYPE RP															
C_1						7		19	7						
2						8	40	17	5						
3						9	71	23							
4															
5															5
6							9	9							
MEAN PUPILS TYPE rP															
C_1								8							
2							14	16	9						
3						5	143	35				9	16		
4															
5										5					9
6															
MEAN PUPILS TYPE rp															
C_1								6							
2								7							
3							63	22					10		
4															
5															8
6								7							
MEAN PUPILS TYPE Rp															
C_1															
2															
3													11		
4															
5															
6															

Table 55 **Comparison by Kendall Coefficient of Rank Correlation, W, of triple frame scores within and between classroom types: RP, rP, rp, Rp. Included are W, X^2. Number of ranks, p.**

	WITHIN				BETWEEN		
	RP	rP	rp	Rp	Case (1)	Case (2)	Case (3)
W	0.78	0.68	0.81	0.82	0.43	0.40	0.40
X^2	77.2	65.3	60.5	59.2	59.9	36.9	22.6
No. ranks	34	33	26	25	36	24	15
p	$\ll 0.001$	$\ll 0.001$	$\ll 0.001$	$\ll 0.001$	0.01	0.040	0.075

independence of ranks. It was noted that within each type RP, rP, rp, Rp the null hypothesis was rejected $p < 0.001$ and similarity of classroom scores within each type was inferred. As discussed in Method, the exact treatment of incomplete ranks was far beyond the demands of the data and several cases providing alternately inflated and depressed W's were determined to provide limits for p. In case 2 where all ranks $\geqslant 10$ in mean value were used the W would be inflated. Here $p = 0.04$. In case 3 where only ranks which had occurred in each mean type were used, W would be depressed. Here $p = 0.075$. At the chosen level $p = 0.05$ and particularly in comparison to the $p < 0.001$ level the null hypothesis could not be rejected and the independence of the types could be inferred. Thus the selection of types by the triple frame scores of the independent variables was confirmed by statistical test of measures of the dependent variables. From the mean matrixes of each type in classroom, teacher and pupils scores, Tables 52 to 54, the first ten ranks of each type or in pupils case frequencies $\geqslant 5$ were listed in Tables 57, 58, 59.

The first 10 ranks of the classroom scores constituted about 70% of the total classroom behaviors. Each triple was listed as a category in content, process and attitude, respectively. Noting results in the order: rp, rP, RP, Rp: Behaviors typical of the rp classroom were technical categories of subject matter--techniques and mathematical. The attitude was largely receptive rather than independent. Not until the 6th rank was any attention to the building or the understanding of the structure of the system given, with the occurrence of relations--inductive, an intuitive approach to the theory by examination of particular examples.

In the rP classroom a remarkable change in attitude was observed, the numbers of categories emphasizing independence jumping from 3/10 to 7/10. While the technical categories still predominated, the weakest "techniques" dropped considerably in frequency. Further, relations--deductive was in 4th place, and fundamentals--structure appeared.

In the change to RP, independence maintained its level, even increasing by one. Mathematical applications were still predominant but dropped in frequency to permit greater variety of behaviors. The deductive development of relations was in second place, the intuitive consideration of relations--4th place, with fundamental structure appearing both in examination of details--specializing, and in directing to original premises--analyzing, one of the strongest possible combinations. The techniques and relevant information categories dropped still further.

The remaining Rp type, occurring in the first year university in a situation demanding a lecture format necessarily emphasized an attitude of receptivity. The theoretical categories of content: deductive, inductive and structure, appear several times in the list. Techniques dropped considerably. In process emphasis was directed to active reasoning toward solution of problem be it deductive relations, mathematical applications or inductive relations - all in the 1st three ranks. Highly evident was deductive/generalizing/receptivity, possible with heavy emphasis of development of the 1st 3 ranks. The emphasis of inductive relations both synthesizing and specializing was noted--much greater in quantity even to RP.

Comparison of the classroom scores with the teacher and pupils components first emphasized the different levels of pupil participation in a variety of the classroom concerns. The 11/1000 behaviors of Rp were merely asking or answering specific questions about mathematical applications. In the smaller high school classrooms type rp, a greater emphasis and variety of the technical aspects occurred: examining and solving specific problems; and small amount of student participation in the theoretical

Table 57 First 10 ranks of triple classifications of classroom behaviors in Content, Process and Attitude in classrooms of high rigor, high participation (RP); low rigor, high participation (rP); low rigor, low participation (rp); and high rigor, low participation (Rp). Numbers indicate frequencies per 1,000.

	RP		rP		rP		Rp
105	Mathematical Synthesizing Independence	199	Mathematical Synthesizing Independence	123	Techniques Relevant Receptivity	103	Deductive Synthesizing Receptivity
65	Deductive Synthesizing Independence	109	Mathematical Specializing Independence	105	Mathematical Specializing Receptivity	96	Mathematical Synthesizing Receptivity
65	Mathematical Specializing Independence	59	Techniques Relevant Receptivity	100	Mathematical Synthesizing Independence	82	Inductive Synthesizing Receptivity
53	Inductive Specializing Independence	52	Deductive Specializing Independence	95	Mathematical Synthesizing Receptivity	80	Inductive Specializing Receptivity
53	Techniques Relevant Receptivity	51	Mathematical Specializing Receptivity	63	Mathematical Specializing Independence	74	Deductive Generalizing Receptivity
52	Structure Specializing Independence	49	Techniques Relevant Independence	56	Inductive Specializing Receptivity	67	Deductive Specializing Receptivity
41	Structure Analyzing Independence	35	Mathematical Synthesizing Receptivity	36	Deductive Specializing Receptivity	51	Techniques Relevant Receptivity
41	Deductive Specializing Independence	33	Structure Specializing Independence	32	Techniques Relevant Independence	37	Deductive Analyzing Recrptivity
36	Techniques Relevant Independence	24	Inductive Specializing Independence	28	Inductive Synthesizing Receptivity	33	Mathematical Specializing Receptivity
33	Mathematical Specializing Receptivity	24	Deductive Synthesizing Independence	26	Inductive Generalizing Receptivity	31	Structure Analyzing Receptivity

Table 58 First 10 ranks of triple classifications of Teacher behaviors in Content, Process and Attitude in classrooms of high rigor, high participation (RP); low rigor, high participation (rP); low rigor, low participation (rp); and high rigor, low participation (Rp). Numbers indicate frequencies per 1,000.

	RP		rP		rp		Rp
49	Mathematical Specializing Independence	74	Mathematical Specializing Independence	116	Techniques Relevant Receptivity	102	Deductive Synthesizing Receptivity
48	Techniques Relevant Receptivity	50	Techniques Relevant Receptivity	96	Mathematical Specializing Receptivity	96	Mathematical Synthesizing Receptivity
43	Inductive Specializing Independence	49	Mathematical Synthesizing Independence	91	Mathematical Synthesizing Receptivity	82	Inductive Synthesizing Receptivity
34	Structure Analyzing Independence	42	Techniques Relevant Independence	56	Inductive Specializing Receptivity	79	Inductive Specializing Receptivity
34	Mathematical Synthesizing Independence	35	Deductive Specializing Independence	40	Mathematical Specializing Independence	73	Deductive Generalizing Receptivity
32	Structure Specializing Independence	31	Mathematical Specializing Receptivity	38	Mathematical Synthesizing Independence	68	Mathematical Specializing Receptivity
32	Techniques Relevant Independence	25	Mathematical Synthesizing Receptivity	33	Deductive Specializing Receptivity	65	Deductive Specializing Receptivity
29	Mathematical Specializing Receptivity	24	Structure Specializing Independence	27	Techniques Relevant Independence	48	Techniques Relevant Receptivity
26	Deductive Synthesizing Independence	20	Inductive Specializing Independence	27	Deductive Generalizing Receptivity	37	Deductive Analyzing Receptivity
26	Deductive Specializing Independence	16	Techniques Specializing Independence	25	Inductive Synthesizing Receptivity	31	Structure Analyzing Receptivity

Table 59 All ranks of triple classifications (frequencies = 5) of pupils behaviors in Content, Process and Attitude in classrooms of high rigor, high participation (RP); low rigor, high particiaption (rP); low rigor, low participation (rp); and high rigor, low participation (Rp). Numbers indicate frequencies per 1,000.

	RP		rP		rp		Rp
71	Mathematical Synthesizing Independence	143	Mathematical Synthesizing Independence	63	Mathematical Synthesizing Independence	11	Mathematical Specializing Receptivity
40	Deductive Synthesizing Independence	35	Mathematical Specializing Independence	22	Mathematical Specializing Independence		
23	Mathematical Specializing Independence	16	Deductive Specializing Independence	10	Mathematical Specializing Receptivity		
19	Structure Specializing Independence	16	Mathematical Specializing Receptivity	8	Techniques Relevant Receptivity		
17	Deductive Specializing Independence	14	Deductive Synthesizing Independence	7	Deductive Specializing Independence		
9	Mathematical Analyzing Independence	9	Mathematical Synthesizing Receptivity	7	Inductive Specializing Independence		
9	Inductive Specializing Independence	9	Deductive Generalizing Independence	6	Structure Specializing Independence		
9	Inductive Synthesizing Independence	9	Techniques Relevant Receptivity				
8	Deductive Analyzing Independence	8	Structure Specializing Independence				
7	Structure Analyzing Independence	5	Techniques Relevant Independence				
7	Structure Generalizing Independence						
5	Deductive Generalizing Independence						
5	Techniques Relevant Receptivity						

aspects of fundamentals--structure; relations--deductive and inductive. Increase in participation in rP shows the same emphasis of technical behaviors as in rp but as early as the third rank and appearing again in the 5th, 7th, 9th ranks come the theoretical categories. Finally, where both mathematical rigor and pupil participation were emphasized, in RP, much greater variety of behaviors appeared. While the categories of mathematical application were still predominant, they were alternated with strong theoretical categories and active problem solving ones in synthesizing and analyzing.

Because of the heavy weight of teacher frequency over pupil frequency even in RP and rP classrooms, the teacher ranks were very similar to those of the classroom. Certain effects of strong pupil participation were noted, however. In rp the high techniques/relevant/receptivity was almost entirely from teachers behaviors. The teacher's emphasis of mathematical specializing in rP and RP occurred at the same time as pupils' emphasis of mathematical synthesizing: teacher dissection of the problem, pupil solution of the problem. In RP the high deductive/synthesizing/independence - second in the classroom was largely of pupils behaviors, 40/65.

Finally, it was interesting to note in these major ranks of the triple scores that in no type of classroom was there marked evidence of concern with applications of mathematics to other fields or in its historical context, despite its emphasis by educational theorists. The detailed matrix of the triple scores or the individual frame scores indicate the precise level of neglect.

DISCUSSION AND INFERENCES

Design

The design generally proved quite satisfactory. While each of the independent variables, rigor and participation, had been dichotomized for this study, it became apparent as observations accumulated that indeed three levels of each could be selected. In terms of rigor it was apparent that there was (1) logical development [illegible] the complete course from initial axioms (2) logical development of topics or parts of topics from rules not of themselves developed as part of a structure from an initial set of axioms. (3) acceptance of rules and application within isolated topics or part topics. In this study, the first level was characterized as high rigor; the second and third levels as low rigor. In the six classrooms of the low rigor group observed the second level was predominant - the third level was rarely maintained where student participation reached a frequency of the order of 20/100.

In participation three levels also could be noted: (1) involvement of pupils frequently in consideration, development and application of the subject matter (2) involvement of pupils frequently in application of the subject matter and (3) involvement of pupils only rarely in any aspect of the discussion. In this study the first level was characterized as high participation; the second and third levels as low participation. In the six classrooms of the low participation group observed, the second level was predominant in the high school - type rp - the third level was apparent in each of the university classes - type Rp.

If desired, classrooms could be distinguished at each of these three levels of rigor and of participation resulting in nine types. In the following 3 x 3 matrix indicating these nine possible types, the twelve classrooms of this study are appropriately placed.

PARTICIPATION

RIGOR		H	M	L
R I	H	I,XI,XIII RP		III,V,VI Rp
G O	M	IX,XV,XVII rP		XII rp
R	L		VIII,XVI rp	

Secondly, comment is important on the distinction of the independent measure of student participation and the categories of attitude of the dependent variable. These two variables certainly had clear similarities. Indeed, as remarked in method, the Attitude frame of the Wright-Proctor Instrument had been considered initially for measuring the independent "student participation" by an "Index of Initiative."

The Index obtained by relatively simple weighting of the three categories in increasing order: receptivity, independence and curiosity and in greater emphasis of pupil response: Teacher--1, 2, 3; Pupils--2, 3, 4 certainly would place classrooms on a continuum of initiative.

The index was not adopted for this purpose because of insufficient validation. It appeared that this was a fortunate circumstance for while high "student participation" as defined, would necessarily result in a high Index, a high Index might occur when student activities were limited to a single aspect of a topic--e.g. mathematical applications.

In addition our results in individual frame scores indicated that while the category independence varied directly and the category receptivity varied inversely with student participation, curiosity was independent of student participation and varied directly with mathematical rigor.

The frame Attitude then does not necessarily indicate the nature of student involvement and in the category curiosity seems quite unrelated to "student participation". Thus the independent variable "student participation" and the dependent variable "attitude" played distinct parts in the design.

One of the uncontrolled variables, in particular, bears comment. While there had been no intention to control for class organization, it was decided to drop one of the initial twenty classrooms considered because of a major incidence of pupil behaviors somewhat related to the supervised study type noted in the Wright-Proctor Instrument, but used throughout the lesson. Classroom discussion was liberally interspersed with pupil work at their seats testing the questions raised, seeking proofs for relations under consideration, applying new relations, all individually rather than the more usual group consideration of a problem with teacher or one pupil at a time handling the development. This powerful method emphasized that the study of classroom group verbal behaviors by direct observation taps only part of the overall learning activities stimulated in the classroom, albeit a major one, particularly in this locale. In classrooms XI and XV some use of the same method of involvement of more than one pupil at a time was apparent in the midst of group discussion. When it was clear what behavior was underway e.g. several steps in solution of specific problems (mathematical, synthesizing, independence), it was so classified. Otherwise these behaviors were recorded as S.

Individual frame scores

The four types of classrooms selected by measures of the independent variables were also clearly separated by individual frame measures of the dependent variables. An overall view of differences of classroom scores in single categories within each frame, and further distinction in teacher and pupils components from consideration of proportions of pupils total behaviors both permitted this inference. Distinction by classroom individual frame scores in single categories was most marked in the Attitude frame followed by the Content frame. The least distinction was shown in Process. Further differences resulted in teacher and pupil components as noted by the varying pupils proportion of total behaviors in the four types. With the emphasis of the independent variables on subject matter and participation, greater differences in the two frames, Content and Attitude, seemed reasonable. Further it would be quite possible for similar emphasis of the several categories of Process to occur yet one classroom type might be solving particular mathematical applications while another type was developing relations. Certainly where systematic thinking patterns are well established--as at this age level--similar processes may well be employed in problems of varying importance and difficulty.

Distinction of the pairs of types RP and rP; rp and Rp by the proposed Index of Initiative based on the individual Attitude category scores served as a useful test of construct and concurrent validity of the index.

The considerable detail of results in the individual frame scores may usefully be summarized for discussion frame by frame.

Content

1. Order of category emphasis and underlining of significant differences in the four classroom types.

Classroom

RP	rP
mathematical	mathematical
deductive	deductive
structure	techniques
inductive	structure
techniques	inductive
other	other

rp	Rp
mathematical	deductive
techniques	inductive
inductive	mathematical
deductive	structure
structure	techniques
other	other

Teacher

RP	rP
mathematical	mathematical
structure	techniques
deductive	deductive
inductive	structure
techniques	inductive
other	other

rp	Rp
mathematical	deductive
techniques	inductive
inductive	mathematical
deductive	structure
structure	techniques
other	other

Pupils

Rp	rP
mathematical	mathematical
deductive	deductive
structure	= ┌ techniques
inductive	└ structure
techniques	inductive
other	other

rp	Rp
mathematical	mathematical
┌ techniques	deductive
│ deductive	┌ inductive
= │ structure	= │ techniques
└ inductive	└ structure
other	other

ii. Hypotheses to be entertained for classroom scores in single categories. Those confirmed in this sample starred. Those tentative (from analysis of variance) unstarred.

Structure varies jointly and directly as rigor and participation.

Techniques varies jointly but inversely as rigor and participation.

*Deductive varies directly as rigor.

Inductive varies directly as rigor and inversely as participation.

*Mathematical varies inversely as rigor.

Other varies directly and jointly as rigor and participation.

These hypotheses were generally suitable for teacher proportions also; however the comparison of $\frac{P_i}{P_v}$ were widely different from the classroom scores hypotheses.

iii. Hypotheses to be entertained for classroom scores in combinations of single categories. Those confirmed in this sample starred. Those tentative (from analysis of variance) unstarred.

*Theory (structure + deductive + inductive)
varies jointly and directly as rigor
and participation.
*Relations (deductive + inductive) varies
jointly and directly as rigor and
participation.
*Applications (mathematical + other)
varies directly as participation
and inversely as rigor.
*Practive in specific problems (mathematical + inductive) varies
jointly but inversely as rigor
and participation
p = 0.05 approximately.

These hypotheses also were suitable for teacher and pupil proportions with only one exception: Pupils' practice in specific problems. In this case: practice in specific problems varies inversely as rigor but directly as participation.

These single category classroom scores were useful in emphasizing the similarities as well as certain distinctions shown by this sample of the four classroom types. In addition to the categories distinguished by analysis of variance across the four types certain other categories are underlined where distinction of one type from the other three was noted, e.g. RP, structure.

Several single categories' levels were particularly representative of each type: e.g. RP· mathematical, deductive, structure. In the other three categories, inductive, techniques, and other, considerable variety in emphasis was observed so that the levels were not distinctive of this type. Thus the classrooms of type RP while very similar in mathematical, deductive, structure, were quite different in inductive, techniques and other. Any study limited to any one or all of these latter three categories would result in a common dilemma "Here are three classrooms that in broad terms are very similar; in the three aspects of our study they vary in marked degree on continua of inductive, techniques and other (if value judgments made even on continua good to bad)." This dilemma of explaining marked divergence in classrooms of equal success (e.g. by judgment of measures of pupil change) then, would be resolved in this instance by noting that only 3/6 categories of content had been noted and only 1 frame in 3 had been considered. In our consideration of the Content frame, only when a complete view of all categories could be taken would the apparent discrepancies in inductive, techniques and other be resolved, and this marked variation in single dimensions accepted as reasonable and perhaps desirable.

Additional support for this view was gained from the differences in generality of hypotheses linking single dependent categories or combinations of dependent categories with independent variables. 2/6 single category hypotheses firmly established. 3/4 combination of category hypotheses firmly established; the last established, p = 0.05 within our usual limits, though much less definite than in the other cases.

This broader view resolves apparent discrepancies in types of classrooms. Also the hypotheses entertained for category combinations of classroom scores also explained teacher and pupil results observed, while single category hypotheses for classroom were less descriptive of teacher and pupil observations.

The hypotheses supported both for single categories and for categories in combination indicate the strong emphasis of skill in solution of specific problems in the three high school types in contrast to the one type found in the university. With greatest interest in technical skills in type rp and least attention to theory or relations, an increase in participation, rigor constant, (rP), resulted in continuing high emphasis of technical skills but increased consideration of theory and relations; with both rigor and participation high, (RP), technical skills were still foremost but with somewhat less emphasis replaced by increased attention to theory and relations. In the high rigor, low participation type, (Rp), while new emphasis of theory and relations was apparent in the predominant deductive aspect of content, the combined effect of inductive and mathematical both providing practice in specific problems indicated recognition of the importance of technical skills.

DESCRIPTION AND COMPARISON

The teacher scores forming the major component of classroom scores generally, then, followed classroom emphasis. While greater variety of pupil pattern was apparent in single categories, this was generally absorbed in the broader combinations of categories of Content.

Process

i. Order of category emphasis and underlining of significant differences in the four classroom types.

Classroom

RP	rP
specializing	specializing
synthesizing	synthesizing
analyzing	relevant
relevant	generalizing
generalizing	analyzing
rp	Rp
specializing	specializing
synthesizing	synthesizing
relevant	generalizing
generalizing	analyzing
analyzing	relevant

Teacher

RP	rP
specializing	specializing
synthesizing	relevant
=┌ analyzing	synthesizing
└ relevant	generalizing
generalizing	analyzing
rp	Rp
specializing	=┌ specializing
synthesizing	└ synthesizing
relevant	generalizing
generalizing	analyzing
analyzing	relevant

Pupils

RP	rP
synthesizing	synthesizing
specializing	specializing
analyzing	relevant
generalizing	=┌ analyzing
relevant	└ generalizing
rp	Rp
=┌ synthesizing	specializing
└ specializing	=┌ analyzing
=┌ analyzing	└ relevant
└ relevant	=┌ synthesizing
generalizing	└ generalizing

ii. Hypotheses to be entertained for classroom scores in single categories. Those confirmed in this sample starred. Those tentative (from analysis of variance) unstarred.

*Analyzing varies jointly and directly as rigor and participation.

Synthesizing is independent of rigor and of participation.

Specializing varies directly as participation and inversely as rigor.

Generalizing no apparent relationship with rigor or with participation, though in type Rp this category differed significantly from the three other types.

Relevant varies jointly but inversely as rigor and participation.

These held generally for teacher proportions. Synthesizing, however, varied inversely as participation. As before the comparisons of $\frac{P_i}{P_v}$ were generally different from those of the classroom. In the case of synthesizing an appropriate hypothesis would be "synthesizing varies directly as participation".

iii. Hypotheses to be entertained for classroom, teacher and pupils scores in combinations of single categories. These confirmed in this sample starred. Those tentative, unstarred.

*Syllogistic reasoning (analyzing + synthesizing) varies jointly and directly as rigor and participation. (0.05)

Classificatory reasoning (specializing + generalizing) is constant.

For the teacher proportions a different hypothesis only for syllogistic reasoning. Syllogistic reasoning varies directly with rigor and inversely with participation.

For Pupils proportions both hypotheses differ.

Syllogistic reasoning varies directly
as participation.
Classificatory reasoning varies inversely
with participation.

The process frame in single categories, and then in category combinations was very stable in this sample. While differences considered between pairs of types permitted the above series of tentative hypotheses which might be further investigated in samples of larger size, only the single category analyzing could be inferred to be distinctive in classrooms varying in rigor and participation.

Thus while the several categories of process may have been employed in one class mainly in dealing with applications, and in another mainly in developing relations, the actual frequency of the single categories or categories in combination with differed significantly only in analyzing. As in Content, the classes within each type must vary widely in all single categories but analyzing. Even when larger combinations were considered, only analyzing + synthesizing (syllogistic reasoning) was noted as distinctive.

In Process, then, all classrooms without regard to type emphasized dissection of problems and forward movement toward solution. While generalizing occurred in the three high school types about equally, much higher frequency was apparent in the university type. This was in keeping with the emphasis of deductive development of relations noted above in Content. Relevant occurred with approximately equal frequency in all types. Confirming the hypothesis, however, analyzing increased from its least frequency in rp, through rP, Rp until its maximum in RP.

Again teacher and pupils scores permit similar description; only pupil order of emphasis, reversing specializing and synthesizing, indicates the pupil emphasis of problem solution or relation development.

Attitude

i. Order of category emphasis and underlining of significant differences in the four classroom types.

Classroom

RP	rP
independence	independence
receptivity	receptivity
curiosity	curiosity
rp	Rp
receptivity	receptivity
independence	independence
curiosity	curiosity

Teacher

RP	rP
independence	independence
receptivity	receptivity
curiosity	curiosity
rp	Rp
receptivity	receptivity
independence	independence
curiosity	curiosity

Pupils

RP	rP
independence	independence
receptivity	receptivity
curiosity	curiosity
rp	Rp
independence	receptivity
receptivity	independence
curiosity	curiosity

Neutral: all about the same.

ii. Hypotheses to be entertained for classroom scores in single categories. Those confirmed in this sample starred.

*Curiosity varies directly as rigor
*Independence varies directly
as participation.
*Recepetivity varies inversely
as participation.

These hypotheses generally were followed by teacher proportions. For pupils proportions the two hypotheses dealing with independence and receptivity generally were supported; however no apparent relationship of curiosity with rigor or participation.

In Attitude the distinction of the

pair RP and rP from rp and Rp was apparent in the independence and receptivity categories. The former pair in classroom, teacher and pupils scores emphasized independence; in the latter pair the three sorts of scores emphasized receptivity.

Within the latter pair, the pupils in rp having higher frequency of participation than in Rp showed greater emphasis of independence.

While curiosity received least attention from all types this was quite reasonable in that only a small number of such behaviors may fruitfully be used (though perhaps more than observed). This category separated the pairs in terms of rigor. Thus in the pair RP, Rp greater curiosity was observed than in the pair rP and rp. In addition, within the former pair it has been concluded that greater rigor was apparent in Rp type. Curiosity shown in Rp exceeded that in RP.

Triple frame scores

The separation of the four types of classrooms by means of dependent measures was further confirmed by comparison of the first ten ranks of the triple frame classroom scores. The additional distinction of pupils totals in the four types permitted, as in the individual frame scores, the use of the teacher and pupils component scores as well as the classroom totals in description and comparison.

From the three sets of scores it was apparent that the classroom type low rigor, low participation, (rp), concentrated on applications and technical skills: the first five classroom ranks; five of the first six teacher ranks and the first four pupils ranks; where relations were considered, these were usually approached inductively--limited development of these, inductively, occurring in the 9th classroom rank and 10th teacher rank. Otherwise whether inductive or deductive, relationships were stated and accepted rather than supported by logical development from earlier theory. The limited attention to structure (structure/specializing/independence), the pupils 7th and last rank, merely selects ideas from fundamentals rather than examining or reconsidering them. The categories of process met dealt largely with classificatory aspects including emphasis of relevant. Where syllogistic categories were employed these were limited to synthesizing directed to specific mathematical problems. Use of these powerful tools for relation development was almost completely absent. Finally the attitude, receptivity, was predominant in classroom and teacher scores. Where independence was encouraged by teacher, this was in terms of mathematical applications--abstracting or solving; or collecting material for later use; or applying technical skills. While the pupils responded to these encouragements, in addition their interest was also apparent in relations and structure. Altogether, then, the rp classroom emphasized teacher demonstration, statement and solution of mathematical applications, and statement of relations; pupil statement and solution of mathematical applications with tentative minor efforts to consider relations and structure.

Where participation was increased--with no intent to modify rigor--in type rP, applications and technical skills received prime attention again, though with somewhat less emphasis of techniques: five of the first six classroom ranks; six of the first seven teacher ranks; four of the first six pupils ranks. Relations, however, received considerably greater emphasis: deductive/specializing/independence, in 4th classroom rank, 5th teacher rank and 3rd pupil rank. Further not only are relations stated but also undergo some deductive development: deductive/synthesizing/independence, 10th classroom rank, 5th pupil rank. Attention to structure was essentially the same as in rp. The categories of process were essentially unchanged. However synthesizing of relations deductively now occurs. Finally as would be anticipated greater teacher encouragement and pupil demonstration of independence was observed. Thus a greater pupil involvement in a wider range of concerns was demonstrated.

This was even more dramatically asserted in the RP type. While applications again received highest emphasis it occurred explicitly only 3 times in the 1st six classroom, teacher and pupils ranks. In 2nd place in classroom and pupil ranks appeared deductive development of relations--largely a result of pupils activities. Relations occurred twice more in classrooms--each time specializing; 3 times in teacher; 6 times in the 13 pupil ranks recorded. It should be noted that relations inductive provided at the same time considerations of relations and further practice in dealing with mathematical application. Structure was emphasized in specializing, but also in the strong analyzing aspect of frames. In Process classificatory and syllogistic categories were both directed to all aspects of the subject matter. The attitude was even more strongly independent than in rP. Altogether the RP classroom and to a marked degree the RP pupils dealt with a wide variety of aspects of content and process demonstrating high independence.

The type Rp was fully described by the classroom scores, although the single pupil ranking emphasized the almost complete absence of pupil participation in this lecture situation. What behaviors may be expected then in this situation where pupil effort is expected to take place outside the classroom? The main emphasis was on establishment of relations deductively.

Great attention, however, was given to inductive examination of relations. Altogether 6/10 categories were directed to one or other of these two Content aspects. Solution of applications occurs in the 2nd rank with limited attention to techniques and applications occurring again at 7th and 9th rank. Where structure was considered it was directed to the strong syllogistic analyzing. Attitude throughout was limited to receptivity. Altogether the Rp classroom emphasized teacher demonstration of statement and development of relations without neglect of specific problems (the latter from both mathematical application and relations inductive).

In general, then, the triple frame scores lend body to the information available in the individual frame scores. For instance, while it was apparent by analyses of variance that the only category of process changing in overall emphasis from type to type was analyzing, the triple frame scores bring out the fact that while total synthesizing may not change across the four types it may be directed to widely different aspects of Content and with contrasting Attitude. Thus synthesizing in rp and rP was almost exclusively directed to applications; yet in RP and Rp it was directed not only to applications but also to deductive development of relations. As observed in the individual frame scores to some extent, the teacher and pupil components may contribute to the distinction of classroom types. In the triple scores this was particularly evident in pupils ranks where for

Rp: 1 was recorded with frequency ≥ 5
rp: 7 were recorded with frequency ≥ 5
rP: 10 were recorded with frequency ≥ 5
RP: 13 were recorded with frequency ≥ 5

Altogether, despite the simple method of ranking, the triple scores provided the best method yet determined with this instrument for rapid distinction of types and for provision of rounded description of all aspects of classroom behavior assessed with this instrument.

From both individual frame and triple frame scores, it was apparent that the Classes high rigor, high participation type characteristic of new experimental programs actually were achieving their objective of emphasis of these two independent variables. Further, the emphasis of theory as well as of applications may be successfully carried out by the pupils under very skilful guidance. Where rigor was low, when participation was increased, rigor also was more

marked. Finally in the desire to gain the advantages for learning of a structural and a pupil involved approach, the skills while receiving somewhat less emphasis in specific problems, mathematical applications and inductive relations, were not neglected and pupil activity in relation development involve similar skills in more difficult situations.

CHAPTER III

OBSERVER RELIABILITY

Determination of the levels of agreement of the classification by two experienced observers was directed to teacher and pupil components of totals of individual frame scores and rankings of triple frame scores. In addition, a broad consideration of specific categories susceptible to disagreement or to observer bias was approached by study of percentage agreement in individual categories for each classroom, and by examination of matrixes of cross classification of the entire 7735 behaviors classified jointly. The scores of this study--individual frame and triple frame--proved reliable generally, $p = 0.001$. The value of the study of percentage agreement was limited because of random variation in agreement from class to class and bacause of marked differences in the total n's. On the other hand, the comprehensive matrixes clearly supported the high level of observer reliability, $p = 0.001$, and sharply delineated the categories or definitions requiring further refinement.

INTRODUCTION

Determination of the level of agreement the two observers was an initial step in e assessment of the technical aspects e examples of behavior collected for de- ription of the classrooms of this study. e observers were well trained in mathe- tics generally and had directed close tention to the topics and to the text- oks that would be encountered during the servation period. Each had participated the development of the observational ol: one was the principal investigator o had begun the development in her doc- ral dissertation, continuing its refine- nt since then; the other had contributed a major revision in the content frame, lminating in a validation study prepared a master's thesis completed during this esent project.

Earlier formal studies of observer re- ability had been based on a previous proximation of the defined categories. ese tests had been twofold: first, the vel of agreement in classification hieved between two observers of single riods of classroom behaviors, and second, e level of agreement between repeated assifications by the same observer of ngle periods of classroom behavior from pescripts. The former had been assessed er a fourteen day training period of a novice observer by plotting of scattergrams daily, and by studying differences in proportions of behaviors by means of the binomial expansion. Each comparison was directed to the individual categories of each frame. Agreement between observers increased rapidly during the fourteen periods of observation until 50/60 categories were not significantly different $p = 0.70$. The test of stability, limited to scattergram comparison had shown overall small variability. Little use of the binomial expansion was found in the literature, and it was decided that later comparisons would include some indication of percentage agreement, for comparative purposes.

A second indication of the reliability that may be attained with this instrument is reported in Appendix B in discussion of a training sequence in which five secondary teachers of mathematics trained in the observational procedures over a ten-day period.

The current study presented the first opportunity of comprehensive and intensive comparison of the classifications of two experienced observers using this refined definition of the categories in a variety of classrooms. These comparisons were directed to the assessment of the level of agreement between the two observers.

OBSERVER RELIABILITY

Because both individual frame and triple frame scores were derived from the three frames of Content, of Process and of Attitude, a test of reliability for each type of score was necessary. Earlier forms of the instrument, being limited to description by individual frame scores, provided no indication of the reliability in the three frames viewed simultaneously. It was apparent that this would impose a stringent check on the reliability of the two observers.

The individual frame "representative" scores were derived from ten visits to a classroom and were expressed in standardized form as totals in each category of each of the three frames. These totals were compared by a chi square test of agreement across each entire frame.

The triple frame "representative" scores while expressed over the entire sample as totals standardized to a base 1,000 were sufficiently powerful for classroom description and comparison by simple ranking. Therefore the test of observer agreement was limited to a comparison of *rankings of the triple frame scores.*

Finally, we moved beyond the confirmation of the reliability of the socres immediate to the present study for intensive search of any categories still not clearly defined or subject to observer bias. Some indication of this would be gained by consideration of percentage agreement within individual classroom categories, but the main value was expected from a detailed cross checking of the classification by each observer of the entire set of some 8,000 behaviors jointly observed.

All of these tests were directed to the teacher and the pupils components of the classroom scores. Establishment of the reliability of these component scores would permit inference of adequate reliability for the classroom totals.

METHOD

Individual frame scores

Materials Frequencies of verbal behaviors of teacher and pupils for each category in the Content, the Process and the Attitude frames, and in Neutral, obtained for the entire set of periods classified jointly in fourteen classrooms provided the materials for the study of agreement between observers. In classrooms visited early in the study as many as nine of the ten periods had been classified jointly in: VII, IX, and XVII; later with seven periods in: I and II; and five periods in: VI, X-XVI, XVIII.

Analysis A matrix was determined for Teacher I by machine computation which detailed the cross classification of the two observers in each individual frame over the entire set of behaviors collected in the seven periods jointly observed. (Table 71) For the particular checks of agreement proposed, this made available the total number of behaviors which each observer separately had classified in a category C_x, say $(C_x)_1$ and $(C_x)_2$.

A comparison of the totals classified by each observer in each category within a single frame, C, was then made by applying chi-square to the $(C_x)_1$ and $(C_x)_2$-- x from 1 to i. However, before proceeding with such analysis, the statistical independence of the several categories within each frame was confirmed by plotting scattergrams in all possible pairs of categories using representative category scores from the entire set of classrooms. When no relationships were indicated, it was felt safe to proceed with the chi-square analysis across the several categories within a frame. (Table 71a)

$$X^2 = \sum \frac{(O-E)^2}{E}$$

Where O is observed frequency and E is expected frequency.

Table 71 **Matrix of cross classification by Observer 1 (column) and Observer 2 (rows) of 440 individual behaviors from classroom I in the Content frame for Teacher I components only.**

Content	**S**	**I**	**D**	**I**	**M**	**O**	**N**	**Total**
Structure	58	2	9	5	8	1	0	83
Techniques	1	39	9	0	12	5	1	67
Deductive	9	0	75	3	5	0	0	92
Inductive	1	1	10	30	3	0	1	46
Mathematical	8	6	9	5	88	1	1	118
Other	2	0	0	0	1	14	0	17
Neutral	0	1	0	1	2	0	11	15
Total	79	49	112	44	119	21	14	438

Table 71a **X^2 Comparison of totals of behaviors classified by Observer 1 (O_1) and by Observer 2 (O_2) within the Content frame for Teacher I.**

Content	**O_1**	**O_2**	**O_1+O_2**	**E**	**$\|O-E\|$**	**$\|O-E\|^2$**	**$\|O-E\|^2/E$**
Structure	79	83	162	81	2	4	0.049
Techniques	49	67	116	58	9	81	1.397
Deductive	112	92	204	102	10	100	0.980
Inductive	44	46	90	45	1	1	0.022
Mathematical	119	118	237	118.5	0.5	0.25	0.002
Other	21	17	38	19	2	4	0.210
Neutral	14	15	29	14.5	0.5	0.25	0.017

for each of O_1, O_2

$X^2 = 2(2.677) = 5.35$ **$df = 2(7-1) = 12$**

The number of degrees of freedom for this analysis equals $k(r - 1)$ where k represents the number of columns and r the number of rows (in this case, $k = 2$, $r = 7$). Because the observers necessarily view the same number of behaviors during joint classification, the column totals are fixed and equal. However, in moving to successive random samples of behaviors for joint classification where the row totals for individual categories cannot be expected to remain the same, no further restriction is required. Thus we do not follow the use of $(k - 1)(r - 1)$ indiscriminately.

The probability level of the occurrence of each X^2 was determined.

For Teacher I, in the test for independence of the totals obtained by Observer 1 ahd by Observer 2 in the categories of Content, a $X^2 = 5.35$ with df = 12 was determined. The probability of occurrence of such a value for X^2 was 0.99. (Fisher and Yates, 1957, p. 45). Thus the null hypothesis was not rejected, and it was concluded that the individual category totals obtained by the two observers were related.

This chi-square analysis was repeated for Pupils I and then for the entire set of classrooms where joint observation had occurred: II, VI, VIII--XVIII.

Triple frame scores

Materials The materials for this aspect of the study of reliability were provided by the frequencies of verbal behaviors of teacher and pupils for each triple combination in the Content, the Process and the Attitude frames, and in Neutral, for each of the fourteen classrooms and for each observer. Because of the very small frequencies in a limited number of the triple combination cells, it was decided to compare the cells only when mean observer classification $\geqslant 5$. This practice of eliminating cells < 5 was practiced throughout the analysis.

Analysis The triple frame scores of each of the two observer classifications of Teacher I were ranked (Table 73). The two sets of ranks were compared by means of the Spearman coefficient of rank correlation, r_s, and the significance level was noted. (Siegel, 1956, p. 284 or Kendall, 1948, p. 172)

For the case where ties occurred

$$r_s = 1 - \frac{S(d)^2}{1/6(n^3-n) - (T^1+U^1)}$$

Where: $S(d)^2$: sum squared deviations
n: number of ranks
T^1: $\sum_{t} 1/12\,(t^3-t)$ ties in rank one
U^1: $\sum_{u} 1/12\,(u^3-u)$ ties in rank two

(Kendall, p. 39: 3.10)

In the example of Teacher I with $n = 21$ the r_s correlation between the triple frame rankings of Observer 1 and Observer 2 was 0.86. From Siegel, p. 284 it was noted that a value as large as this was significant at the $p < 0.01$ level. Thus the null hypothesis, that the rankings of the two observers were unrelated, was rejected and an association between triple frame classifications of the two observers for Teacher I was concluded, $p < 0.01$. On the other hand for Pupils I, with $n = 6$, $r_s = 0.55$ has a probability of occurrence under H_0 of $p = 0.07$ (Kendall, p. 172). Thus the null hypothesis had to be retained and no association between the triple frame classifications of the two observers for Pupils I could be concluded, $p = 0.05$.

The entire analysis was repeated for classrooms II, VI-XVIII.

Delineation of category disagreement

Materials The matrixes determined from joint classification of verbal behaviors of teachers and pupils for each category in the Content, the Process and the Attitude frames including Neutral for the individual frame score analysis, (Table 71) and an entire aggregate of these for the fourteen classrooms

Table 73 Calculation of Spearman Coefficient of Correlation, r_s, between the rankings of triple frames scores for Observers 1 and 2 for Teacher I and Pupil I.

Numerical Symbolism of triple frame scores in Content, Process and Attitude, respectively.	Observer I frequency	Observer I rank	Observer II frequency	Observer II rank	d	d^2
			TEACHER I			
332	60	1	54	1	0	0.00
322	37	2	36	3	1	1.00
552	31	4	41	2	2	4.00
222	33	3	35	4	1	1.00
132	26	7	27	6	1	1.00
232	27	5.5	23	7	1.5	2.25
632	27	5.5	20	9.5	4	16.00
112	22	8	20	9.5	1.5	2.25
553	15	11	28	5	6	36.00
212	21	9	13	11.5	2.5	6.25
333	7	15.5	21	8	7.5	56.25
242	16	10	8	15	5	25.00
133	12	12	13	11.5	0.5	0.25
233	8	13.5	10	13	0.5	0.25
642	8	13.5	9	14	0.5	0.25
622	6	17	4	20	3	9.00
223	5	18.5	5	19	0.5	0.25
312	7	15.5	3	21	5.5	30.25
612	4	20.5	7	16	4.5	20.25
143	5	18.5	6	17.5	1.0	1.00
122	4	20.5	6	17.5	3.0	9.00
			PUPILS I			
322	38	1	45	1	0	0.00
332	31	2	17	4	2	4.00
222	23	3	21	2.5	0.5	0.25
132	11	4.5	21	2.5	2	4.00
232	11	4.5	5	6	1.5	2.25
242	7	6	7	5	1	1.00

Teacher I:

$S(d)^2$: 221.50

n: 21

T^1: 2.5

U^1: 1.5

$$r_s = 1 - \frac{221.50}{1539.50} = 0.86$$

significant $p < 0.01$

Pupils I:

$S(d)^2$: 11.50

n: 6

T^1: 0.5

U^1: 0.5

$$r_s = 1 - \frac{11.50}{34.00} = 0.66$$

$p = 0.07$

not significant, $p = 0.05$

permitted intensive study of the variation category by category.

Analysis

i.

The matrix determined for Teacher I in the individual frame scores analysis (Table 71) made available materials for percentage agreement in exact individual category classifications. If C_{xx} represented the number of behaviors which each observer had placed in category C_x and $(C_x)_1$ $(C_x)_2$ the total number of behaviors which each observer separately had classified as C_x, a simple index of agreement in C_x could be calculated:

$$\frac{2.C_{xx}}{(C_x)_1 \ (C_x)_2} \times 100$$

In particular, percentage agreement for Teacher I scores in $\text{Content}_{\text{Structure}}$ was:

$$\frac{2(58)}{79 + 83} \times 100 = 72$$

An index of percentage agreement across each entire frame also was obtained:

$$\frac{C_{xx}}{(C_x)_1 = (C_x)_2} \times 100$$

In particular, percentage agreement for Teacher I scores in Content was:

$$\frac{313}{438} \times 100 = 71$$

The single category and total frame indexes of percentage agreement were also calculated for Pupils I and for the entire set of remaining classrooms.

ii.

The aggregates of the teachers matrixes and of the pupils matrixes used in the individual frame scores analyses were obtained, thus providing a comprehensive Teacher matrix and a comprehensive Pupils matrix for each frame of Content, of Process, and of Attitude. As the behaviors classified as Neutral are neutral in each of the three frames individually, and as some of the behaviors classified by one observer as a triple score may be classified as neutral by the other, the Neutral category was included within each frame matrix. A typical aggregate matrix was that of Teacher Attitude:

Observer I

	Attitude	C	I	R	N	Total
o	Curiosity	117	31	27	0	175
b	Independence	19	1891	285	10	2205
s.	Receptivity	25	454	2336	38	2853
2	Neutral	1	10	28	891	930
	Total	162	2386	2676	939	6163

X^2 was determined most conveniently by a modification of $\frac{\leq (O-E)^2}{E}$ namely:

$$n \left(\leq \frac{n^2{}_{ij}}{n_i n_j} - 1 \right)$$

where n is overall total
n_j frequency of cell ij
n_1, n_j total frequency in row or column

The C coefficient of Contingency was then calculated: $\sqrt{\frac{X^2}{n+X^2}}$

In the case, $X^2 = 11.35 \times 10^3$; $n = 6163$; $C = 0.81$.

Because the maximum coefficient of contingency is not 1, but varies with the number of categories into which the distributions are divided, the maximum coefficient of contingency was stated so that the calculated one might be compared with it. This maximum coefficient may be determined from a square contingency table with t rows and to columns by $\sqrt{\frac{t-1}{t}}$ (Peters, and Van Voorhis, 1940, p. 393).

In this case, $C_{\text{Maximum}} = \sqrt{\frac{3}{4}} = 0.87$.

The level of significance of C was determined by reference to the X^2 distribution.

In the case of Teacher I, the relation between the classifications in Attitude of Observer 1 and Observer 2 was $C = 0.81$. In the course of computing C, X^2 was determined as 11.35×10^3. In order to test whether or not the classifications of the observers were related, $X^2 = 11.35 \times 10^3$, df = $(k-1)(r-1) = (4 - 1)(4 - 1) = 9$ was tested for significance (Fisher & Yates, 1957, p. 45). Such a X^2 has a probability of occurrence under H_0 of less than 0.001. Thus we rejected H_0 at the 0.001 level of significance and concluded that the classifications of the two observers were related for the Attitude frame.

This analysis was repeated for each teacher matrix and for the three pupils matrixes.

In addition to the determination of the coefficient of contingency, careful scrutiny was made of each matrix to determine the categories in which observer disagreement was focussed. This disagreement was considered for each observer in an attempt to determine if disagreement was random or related to some observer bias. For instance in the Teacher Attitude matrix, clear disagreement was noted between receptivity and independence in contrast to very little disagreement of receptivity with neutral or curiosity. In addition while both observers experienced some difficulty in deciding between receptivity and independence, Observer 1 was biassed towards independence and Observer 2 towards receptivity.

RESULTS

Individual frame scores

The comparison of the category totals classified by each observer in the fourteen classrooms was detailed in Table 76: chi-square, number of degrees of freedom, and the corresponding probability level. Of the fourteen comparisons in each of teacher and pupil components of the Content, the Process and the Attitude frames individually, only in classroom 9: Teacher Attitude was a probability beyond 0.05 found, namely, 0.01. In all other cases, the null hypothesis was retained and a significant association between the classifications of the two observers was noted.

Triple frame scores

The Spearman Rank Correlation comparisons between rankings by the two observers of triple category combinations in the fourteen classrooms was detailed in Table 77: number of days of joint classification; then for each of teacher and pupils components, number of ranks, r_s coefficient and the probability level. In all of the fourteen teacher comparisons r_s was significant, $p < 0.01$ and the null hypothesis was rejected. Of the twelve pupil comparisons possible, seven were not significant with probability levels: 0.06, 0.07, 0.10, 0.16, 0.21, 0.33, and the largest where $r_s = 0.00$. Two comparisons with a perfect r_s, namely, 1.00 were properly noted as statistically not significant. However, examination of the frequencies from which the triples were ranked emphasized that, practically, there could be no doubt about the relationship between the two classifications. Thus in seven of the twelve pupils comparisons the null hypothesis was retailed. In summary, for all of fourteen teacher comparisons and for five of twelve pupil comparisons the null hypothesis was rejected and a significant association between the classifications of the two observers was noted.

Delineation of category disagreement

i

Percentages of exact agreement in classification between Observers 1 and 2 in the individual categories were recorded in Tables 78, 79 for Teacher components, and in Tables 80, 81, for Pupils components. In addition, these tables included a total percentage of agreement across each frame. Totals for frame Attitude were included--slight variations for other frames resulted from minor discrepancies in machine sorting. Changes in level of agreement in the individual categories, seemed, on the whole to be random. That is, there seemed no consistent improvement in agreement during the course of the study; no clear-cut difference in teacher or pupil levels of agreement was apparent; within a single frame few

Table 76 **Chi-square comparison of joint classification by Observer 1 and Observer 2 from fourteen classrooms in terms of the totals in individual categories grouped in the frames of Content, of Process and Attitude, and in Neutral. Teacher and Pupils components separately. Degrees freedom and probability levels recorded.**

	CONTENT			**PROCESS**			**ATTITUDE**		
Class	χ^2	df	p	χ^2	df	p	χ^2	df	p
				TEACHER					
I	5.354	12	0.90	3.582	10	0.95	5.406	6	0.45
II	4.102	12	0.98	0.922	10	0.99	10.462	6	0.10
VI	1.154	12	0.99	4.426	10	0.90	1.882	6	0.90
VIII	2.022	12	0.99	0.792	10	0.99	1.780	4	0.75
IX	15.718	12	0.20	2.024	10	0.99	16.630	6	0.01*
X	0.340	12	0.99	1.396	10	0.99	0.142	6	0.99
XI	0.862	12	0.99	6.692	10	0.75	2.926	6	0.80
XII	17.894	10	0.06	9.404	10	0.50	3.828	4	0.40
XIII	0.350	10	0.99	4.994	8	0.75	0.472	4	0.99
XIV	0.678	8	0.99	0.716	10	0.99	1.806	4	0.75
XV	3.672	12	0.99	3.880	10	0.99	0.710	4	0.95
XVI	1.326	10	0.99	5.018	8	0.75	3.090	4	0.50
XVII	4.510	12	0.95	5.876	10	0.80	3.482	6	0.70
XVIII	1.414	12	0.99	1.406	10	0.99	1.598	4	0.99
				PUPILS					
I	2.980	6	0.80	2.002	8	0.98	2.168	4	0.90
II	0.532	6	0.99	4.268	8	0.80	1.062	2	0.50
VI	---	-	---	---	-	---	---	-	---
VIII	1.098	6	0.98	0.234	6	0.99	0.468	2	0.75
IX	0.992	6	0.98	2.792	8	0.95	0.432	2	0.80
X	0.348	4	0.98	3.120	6	0.90	0.444	2	0.80
XI	0.348	8	0.99	1.618	6	0.99	0.100	2	0.95
XII	---	-	---	---	-	---	---	-	---
XIII	0.252	6	0.99	0.226	6	0.99	1.476	2	0.50
XIV	0.244	4	0.99	0.468	4	0.98	0.594	2	0.75
XV	0.418	4	0.98	---	-	---	2.110	2	0.30
XVI	0.768	4	0.95	0.034	4	0.99	0.044	2	0.97
XVII	3.770	6	0.70	7.920	8	0.40	0.112	4	0.99
XVIII	0.430	8	0.99	0.816	6	0.99	0.736	2	0.65

- Except in region $p \leqslant 0.10$ approximation of p to nearest 0.05 from Fisher and Yates, p. 45.

-* significant $p = 0.05$ or beyond.
Reject H_0 at this level.

Table 77 Spearman Rank Correlation Coefficient r_s between rankings by Observers 1 and 2 of triple category combinations in the Content, the Process and the Attitude frames, and in Neutral for teacher and pupils in fourteen classrooms. Also recorded: number of days of joint classification, D; number of ranks in each correlation, n; and the probability level at which r_s significant, p.

CLASS	D	TEACHER			PUPILS		
		n	r_s	p	n	r_s	p
I	7	21	0.86	0.01	6	0.66	0.07*
II	7	23	0.83	0.01	6	0.83	0.05
VI	5	16	0.85	0.01	0	----	----
VIII	9	23	0.77	0.01	5	0.00	not significant*
IX	9	25	0.91	0.01	6	0.90	0.02
X	5	19	0.90	0.01	3	1.00	0.16?
XI	5	19	0.66	0.01	9	0.36	0.16*
XII	5	13	0.73	0.01	0	----	----
XIII	5	30	0.67	0.01	6	0.40	0.21*
XIV	5	12	0.74	0.01	4	0.72	0.10*
XV	5	9	0.95	0.01	3	1.00	0.16?
XVI	5	11	0.83	0.01	3	0.50	0.33*
XVII	9	21	0.77	0.01	8	0.63	0.06*
XVIII	5	16	0.69	0.01	6	0.99	0.01

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* Not significant $p = 0.05$
Reject H_0 at this level
? Practically if not statistically significant.

Table 78 Percentages of exact agreement in classification by Observers 1 and 2 of individual categories in each frame of Content, of Process and of Attitude, and in Neutral with corresponding n. Classrooms: I, II, VI, VIII, IX, X, XI. Teacher components.

CLASS	I	II	VI	VIII	IX	X	XI
Content							
Structure	71	67	68	49	65	81	64
n	82.0	89.0	31.0	65.5	90.5	107.5	45.0
Techniques	67	74	75	80	73	85	82
n	58.0	51.5	88.0	144.0	109.5	34.0	81.0
Deductive	73	61	85	66	61	85	82
n	102.0	88.0	164.0	106.5	83.5	33.0	52.5
Inductive	67	56	74	74	57	85	80
n	45.0	46.0	59.5	46.0	56.0	54.0	80.0
Mathematical	73	65	91	74	71	87	79
n	119.5	107.0	102.5	218.5	192.0	107.0	53.0
Other	74	57	80	73	72	87	83
n	19.0	21.0	30.0	15.0	23.5	11.5	23.0
Process							
Analyzing	61	56	64	30	55	61	61
n	51.0	46.0	40.0	27.0	46.0	62.0	33.0
Synthesizing	66	60	71	65	59	76	54
n	84.0	74.0	132.0	117.0	105.0	94.0	34.0
Specializing	72	69	64	73	72	66	63
n	174.0	163.0	166.0	236.0	262.0	119.0	177.0
Generalizing	54	64	49	66	57	72	64
n	39.0	48.0	39.0	59.0	58.0	39.0	33.0
Relevant	72	76	80	86	74	82	80
n	78.0	70.0	96.0	157.0	85.0	32.0	80.0
Attitude							
Curiosity	67	44	87	20	37	74	82
n	15.0	18.0	30.0	5.0	13.5	9.5	22.0
Independence	87	84	61	79	84	74	80
n	314.5	286.5	41.0	281.0	354.0	75.5	135.0
Receptivity	59	59	94	79	70	92	84
n	96.0	98.0	404.0	310.5	187.5	262.0	177.5
Totals							
Content	71	65	82	75	71	87	80
Process	67	67	68	76	70	74	72
Attitude	80	77	90	82	80	89	84
n for each frame	425.5	402.5	475.0	596.5	555.0	347.0	334.5 approx.
Neutral	76	87	83	95	97	100	96
n	14.5	19.5	23.0	120.5	61.0	60.0	35.5

Table 79 Percentages of exact agreement in classification by Observers 1 and 2 of individual categories in each frame of Content, of Process and of Attitude, and in Neutral with corresponding n. Classrooms: I, II, VI, VIII, IX, X, XI. Pupil components.

CLASS	I	II	VI	VIII	IX	X	XI	
ontent								
Structure	70	72	--	50	94	33	60	
n	28.5	18.0	--	8.0	17.0	12.0	10.0	
Techniques	71	89	80	80	64	78	67	
n	7.0	14.5	2.5	2.5	15.5	9.0	4.5	
Deductive	79	81	100	88	82	55	95	
n	30.5	42.0	2.0	12.5	25.5	5.5	52.5	
Inductive	60	67	67	67	66	--	85	
n	5.0	4.5	1.5	1.5	1.5	--	13.0	
Mathematical	84	83	50	87	87	81	95	
n	62.0	51.5	2.0	42.5	69.0	33.5	64.5	
Other	100	--	--	--	40	--	67	
n	1.0	--	--	--	2.5	--	6	
rocess								
Analyzing	56	45	--	73	70	63	53	
n	9.0	11.0	--	11.0	10.0	8.0	17.0	
Synthesizing	83	84	--	82	85	93	78	
n	63.0	64.0	--	22.0	53.0	22.0	88.0	
Specializing	75	68	--	85	82	73	58	
n	54.0	42.0	--	39.0	62.0	26.0	43.0	
Generalizing	74	75	--	100	75	33	50	
n	14.0	12.0	--	2.0	12.0	3.0	2.0	
Relevant	88	90	--	80	64	80	100	
n	8.0	10.0	--	5.0	11.0	5.0	4.0	
ttitude								
Curiosity	77	57	--	67	57	100	--	
n	6.5	3.5	--	4.5	3.5	2.0	--	
Independence	95	97	89	87	90	91	98	
n	128.0	131.0	4.5	49.5	105.0	51.5	131.0	
Receptivity	64	50	86	80	78	57	88	
n	12.5	4.0	3.5	25.0	38.5	10.5	24.0	
otals								
Content	75	80	75	82	70	70	89	
Process	78	76	--	82	80	78	70	
Attitude	92	94	88	84	85	86	96	
for each frame	147.0	138.5	8.0	79.0	147.0	64.0	155.0	approx.
Neutral	--	--	--	--	--	100	--	
n	--	--	--	--	--	2.0	--	

Table 80 Percentages of exact agreement in classification by Observers 1 and 2 of individual categories in each frame of Content, of Process and of Attitude, and in Neutral with corresponding n. Classrooms XII--XVIII, Teacher components.

CLASS	XII	XIII	XIV	XV	XVI	XVII	XVIII	
Content								
Structure	40	88	71	79	75	64	70	
n	15.0	129.0	39.5	14.0	8.0	52.0	35.5	
Techniques	50	72	76	71	86	62	94	
n	34.0	26.5	57.5	42.5	88.5	76.0	70.5	
Deductive	71	74	57	81	54	74	81	
n	38.0	52.5	3.5	18.5	13.0	143.5	73.0	
Inductive	76	84	50	58	74	63	68	
n	94.5	76.5	2.0	15.5	42.0	44.5	28.0	
Mathematical	72	89	82	83	83	74	87	
n	100.0	68.5	114.5	72.0	86.0	172.0	67.5	
Other	33	76	68	88	--	83	93	
n	3.0	22.5	19.0	16.0	--	6.0	7.5	
Process								
Analyzing	32	55	73	33	25	65	64	
n	25.0	65.0	22.0	9.0	8.0	50.0	28.0	
Synthesizing	57	65	76	64	70	61	70	
n	83.0	68.0	66.0	31.0	64.0	92.0	43.0	
Specializing	44	74	75	76	70	64	78	
n	116.0	164.0	88.0	86.0	112.0	194.0	129.0	
Generalizing	54	72	72	100	57	60	25	
n	24.0	46.0	14.0	6.0	7.0	60.0	8.0	
Relevant	58	76	84	67	75	74	74	
n	36.0	33.0	51.0	46.0	48.0	98.0	72.0	
Attitude								
Curiosity	33	86	67	100	--	61	100	
n	3.0	31.5	1.5	1.0	--	16.5	2.0	
Independence	42	85	67	71	69	87	87	
n	19.0	178.5	28.5	53.5	33.5	334.5	160.5	
Receptivity	92	83	94	85	94	71	82	
n	262.5	166.0	209.5	124.0	205.5	143.0	118.5	
Totals								
Content	75	84	86	86	84	73	84	
Process	67	69	87	82	76	64	79	
Attitude	89	84	94	88	92	83	85	
n for each frame	284.5	376.0	239.5	178.5	239.0	494.0	281.0	approx.
Neutral	91	83	98	98	98	96	93	
n	112.0	6.0	194.0	132.0	83.0	48.0	15.0	

Table 81 **Percentages of exact agreement in classification by Observers 1 and 2 of individual categories in each frame of Content, of Process and of Attitude, and in Neutral with corresponding n. Classrooms XII, XIII, XIV, XV, XVI, XVII, XVIII. Pupil components.**

CLASS	**XII**	**XIII**	**XIV**	**XV**	**XVI**	**XVII**	**XVIII**
ontent							
Structure	75	81	95	75	33	69	61
n	4.0	37.0	10.5	4.0	3.0	13.0	6.5
echniques	67	100	89	80	85	80	94
n	1.5	3.0	9.0	7.5	20.0	5.0	8.5
eductive	25	84	--	78	100	78	85
n	4.0	9.5	--	9.0	1.0	76.0	33.0
nductive	100	80	--	--	80	--	80
n	4.0	10.0	--	--	2.5	--	5.0
athematical	86	89	81	95	96	92	92
n	3.5	40.1	34.5	146.0	58.5	109.0	56.5
ther	--	100	100	55	--	--	50
n	--	2.0	2.0	5.5	--	--	4.0
ocess							
nalyzing	--	60	100	25	67	38	56
n	--	10.0	2.0	4.0	6.0	8.0	9.0
ynthesizing	--	88	69	96	100	82	85
n	--	41.0	16.0	125.0	48.0	118.0	47.0
pecializing	88	77	79	72	90	66	81
n	12.0	40.0	34.0	32.0	20.0	86.0	57.0
eneralizing	100	92	--	100	--	55	50
n	2.0	12.0	--	2.0	--	20.0	4.0
elevant	50	50	92	82	93	57	87
n	2.0	6.0	12.0	11.0	14.0	14.0	8.0
titude							
uriosity	57	75	100	80	100	62	100
n	3.5	4.0	2.0	2.5	1.0	6.5	3.0
ndependence	76	94	91	98	97	88	96
n	14.5	91.0	44.0	162.5	73.5	181.5	111.0
eceptivity	--	71	78	67	82	68	55
n	--	14.0	18.0	9.0	13.5	57.5	11.0
tals							
ontent	76	80	74	91	88	70	78
rocess	71	80	80	89	94	72	80
ttitude	67	91	88	96	94	83	92
for each frame	18.0	109.0	64.0	174.0	88.0	245.5	125.0 approx.
eutral	50	--	100	--	--	100	100
n	2.0	--	1.0	--	--	1.0	1.0

categories appeared to have consistently high or low agreement. The total percentages of agreement, the indices customary in other research, indicated some differences between the frames, with Attitude percentages ranging from 80 to 96, Content ranging from 70 to 89 and Process ranging from 64 to 89.

ii

In contrast to the very limited information for broad category assessment afforded by the percentage agreements, the comprehensive matrixes supported the overall agreement and in addition sharply delineated categories still in disagreement. The X^2 comparisons and C coefficients of contingency from joint classification by Observer 1 and Observer 2 of the 7735 behaviors, with the comprehensive matrixes from which they were derived were presented in Table 83. For each frame and for both teacher and pupils components, the null hypothesis was rejected, $p < 0.001$, indicating a definitely significant association between the classifications of the two observers. The maximum coefficients of contingency possible in the several sized matrixes were indicated, emphasizing the high level of the obtained C's. It was also noted that while all C's were significant at $p < 0.001$, the coefficient for pupils' Attitude was at the maximum, with teacher and pupils' Content still very close, and teacher Attitude and teacher and pupils' Process farthest from the maximum.

Detailed examination of the matrixes permitted selection of categories still wanting in refinement or susceptible to observer bias.

Considering first the Teacher Content frame: disagreements in structure were for the most part directed to the mathematical, the deductive and the inductive categories in that order; and more sharply in the classification of Observer 1. Disagreements in techniques were concentrated in the mathematical category. In addition, in total classification, Observer 2 emphasized techniques more than did Observer 1. Disagreements in deductive were accounted for in the inductive, the mathematical and the structure categories. Observer 1 favored the inductive over the mathematical in exact contrast to Observer 2: both classified about 50 disagreements in structure. Disagreements in inductive were concentrated in the deductive and mathematical categories for Observer 1, but in Observer 2 included also the structure and the techniques categories. In addition, in total classification, Observer 1 favored inductive much more than did Observer 2. In mathematical, disagreement was largely in structure and in techniques with considerably more weight given in techniques by Observer 2. Further variation was noted in Observer 1's emphasis of deductive and inductive, and Observer 2's emphasis of Other. The only marked disagreement in other was in Observer 1's emphasis of mathematical.

Reference to the Pupils Content frame showed no additional disagreements. As would be expected variability was much less in actual value and some of the variability apparent in the larger teacher scores was not apparent in those of the pupils.

In the Teacher Process frame most of the variability in agreement between the two observers was accounted for in the specializing category: with the exception of neutral, heavy disagreement was shown by all the categories with that of specializing. Consideration of the classifications of the two observers showed no particular bias as may be noted by reflection of row 3, column 3 across the main diagnoal; analyzing--127, 146; synthesizing--279, 257; generalizing-100, 106; relevant--117, 139. The only other disagreement of any size was between analyzing and synthesizing with slight bias of Observer 1 towards synthesizing, and of Observer 2 towards analyzing.

These considerations generally were confirmed in classification of the Pupils Process. In addition, the bias of Observer 2 towards analyzing was apparent

Table 83 X^2 comparisons and C coefficients of contingency from joint classification by Observer 1 (column) and Observer 2 (rows) of 7735 individual behaviors from 14 classrooms in the Content, the Process and the Attitude frames. Teacher and Pupils components separately.

TEACHER

Content	S	T	D	I	M	O	N	Total		
Structure	582	19	75	52	83	6	0	817	X^2	20.64×10^3
Techniques	29	732	57	37	118	16	26	1015		
Deductive	53	29	719	85	65	4	2	957	C_{Actual}	0.88
Inductive	26	11	49	503	37	10	1	637		
Mathematical	96	90	82	59	1230	7	17	1581	$C_{Maximum}$	0.92
Other	11	11	3	2	33	164	2	226		
Neutral	3	16	2	5	11	24	891	930		
Total	800	908	987	743	1577	209	939	6163		

rocess	A	Sy	Sp	G	R	N	Total		
Analyzing	292	62	146	18	10	3	531	X^2	11.42×10^3
Synthesizing	31	713	256	33	20	12	1065		
Specializing	127	279	1520	106	139	3	2174	C_{Actual}	0.81
Generalizing	35	35	100	296	22	1	479		
Relevant	13	13	117	27	771	29	984	$C_{Maximum}$	0.91
Neutral	1	1	16	1	17	892	930		
Total	499	1109	2155	481	979	940	6163		

ttitude	C	I	R	N	Total		
Curiosity	117	31	27	0	175	X^2	11.35×10^3
Independence	19	1891	285	10	2205		
Receptivity	25	454	2336	38	2853	C_{Actual}	0.81
Neutral	1	10	28	891	930		
Total	162	2386	2676	939	6163	$C_{Maximum}$	0.87

PUPILS

ontent	S	T	D	I	M	O	N	Total		
Structure	145	1	30	9	29	0	0	214	X^2	4.50×10^3
Techniques	3	91	7	2	18	0	1	122		
Deductive	5	1	279	6	26	1	0	318	C_{Actual}	0.86
Inductive	2	11	14	48	7	1	0	73		
Mathematical	15	9	20	26	751	5	0	802	$C_{Maximum}$	0.91
Other	1	2	0	0	8	24	0	35		
Neutral	0	0	2	0	0	0	6	8		
Total	171	105	352	67	839	31	7	1572		

rocess	A	Sy	Sp	G	R	N	Total		
Analyzing	61	21	35	4	0	0	121	X^2	2.94×10^3
Synthesizing	11	610	76	1	2	0	700		
Specializing	17	72	414	3	15	1	522	C_{Actual}	0.81
Generalizing	7	4	32	60	0	0	103		
Relevant	1	5	22	0	90	1	119	$C_{Maximum}$	0.89
Neutral	0	0	1	0	0	6	7		
Total	97	712	580	68	107	8	1572		

ttitude	C	I	R	N	Total		
Curiosity	31	10	0	0	41	X^2	3.10×10^3
Independence	13	1198	75	1	1287		
Receptivity	0	58	176	0	234	C_{Actual}	0.81
Neutral	0	4	0	6	10		
Total	44	1270	251	7	1572	$C_{Maximum}$	0.81

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etters across the top of the columns correspond to the categories at the side.

in the totals in that category. However, the totals in synthesizing emphasized that Observer 1's bias was limited to teacher behaviors. An additional bias of Observer 2 towards generalizing (or Observer 1 towards specializing) became apparent.

An earlier discussion of the Teacher Attitude frame during its use as an example of the analysis proposed, indicated that disagreement here was limited to receptivity and independence with a slight bias of Observer 1 towards independence, Observer 2 towards receptivity. If any bias may be inferred from the disagreements between these categories in the Pupils Attitude matrix, it would be just the reverse. Thus Observer 1 would be biased towards the teacher, Observer 2 towards the pupils.

DISCUSSION AND INFERENCES

Individual frame scores
Triple frame scores
Delineation of category disagreement

A high level of association between classifications by Observer 1 and Observer 2 was apparent in the individual frame scores in each classroom both in teacher and in pupils components. The scores used in this study derived from mean observer totals during joint observation combined with single observer totals from remaining observation provided even better values than those of either observer separately, in seeking reliable scores.

Similarly, rankings of triple frame scores from classifications by the two observers separately, showed a high level of association in the teacher component. Only one-half of the pupils components were highly comparable. The practice of deriving mean observer frequencies during joint observation and combining these with single observer frequencies from remaining observation before determining the final ranking would provide reliable scores for the teacher component and probably quite adequately reliable scores for the pupils component.

With the high reliability of individual scores and of teacher triple frame scores, adequate reliability of both types of classroom scores, so heavily dependent on heavy teacher weighting, was inferred.

Though total percentages showed favorabl levels of agreement compared to other studie percentage study was of very limited use to During the course of a study or of a trainin period, plotting scores in individual categories on binomial probability paper would have provided the same rapid assessment of observer agreement. In addition, this clear indication of the significance level of agreement would be in keeping with our other analyses of individual category differences based on the binomial distribution. In the broader consideration of focussing attention on categories wanting in refinement or susceptible to observer bias, it was felt that the accumulation of classroom percentages of agreement might provide useful indicators. Random variations from class to class and in particular marked differences in the total n's prevented any clearcut selection of categories for further refinement. On the other hand, the comprehensive matrixes derived from all behaviors jointly classified had a sufficiently large n to absorb sharp differences such as occurred in the percentages both because of small n and random classroom differences. The high level of significance of each coefficient of contingency based on such detailed check of agreement gave further support to the reliability of both individual frame and triple frame scores.

The more interesting aspect of the study of the matrixes was their sharp focussing of attention on needed refinements and possible biasses. During the course of current observation one clearcut difficulty of classification of Content had been apparent: the appropriate category for statement of new relations. Perhaps a series of particular examples had suggested an interesting characteristic to investigate; after its statement perhaps a logical proof of this characteristic followed.

Some conflict occurred in such a case between deductive and inductive. With the added emphasis of this disagreement in the matrix analysis, it was suggested that the relations category might be extended fruitfully to include: statements as well as deductive and inductive. Another disagreement with the statement of relations was actually in a restatement of old relations used in deductive or inductive development of theory. Where this was pulled out for close examination in terms of overall structure, it should be classified as structure. It was felt that some of the disagreement shown between the relations categories and that of structure was in determining when an old relation was merely used in development, or when examined for itself within such development. The two major differences with mathematical, occurred in techniques and in structure. These probably occurred during discussion of specific problems when one observer felt that the discussion had paused either for emphasis of technique or for connecting a point under discussion with understanding of its place in the overall structure. The very frequent occurrence of discussion centered on specific problems--consequently classified mathematical--probably resulted in a certain inertia in moving from that category while main attention was still with the problem solving area.

A second conflict often quite difficult to resolve was between mathematical and inductive. In each case discussion was centered on specific problems. The difference was in the intent--to gain skill in applying principles primarily, or to use facility in problems, well known to pupil but carefully selected by the teacher, to elicit new relationships. Finally some disagreement between mathematical and other resulted from Observer 2's practice of classifying the first reading of a problem phrased in language of some other field as other with Observer 1 noting the superficial quality of such application to other fields in the rapid abstraction of the ideas that could be treated mathematically. This could occur in a "word problem" exercise following development of some specific mathematical idea such as solution of simultaneous equations in two unknowns. While the problems might be expressed in a variety of other fields: physics, chemistry, business, building, etc., all would clearly provide e.g. two sets of conditions relating two variables...no more, no less. Thus this would ensure the mathematical model to be applied and provide limited indication of the use of such models in other fields. It was decided in the light of this disagreement to continue the policy of Observer 2 noting the potential even in such unimaginative use of work problems but recognizing Observer 1's emphasis of the superficial treatment by the practical fact that at most one behavior would be classified as other in such a case.

The main disagreement in Process, occurring in specializing emphasized the difficulties met during classification by each observer in "selection of specific attributes" one of the behavioral indicators of specializing. Often the selection of a specific proposition was followed rapidly by its use in a syllogism within analyzing or synthesizing. Judgment was then necessary on the "most important aspect of the behavior of the speaker". It was decided that for future classification where a proposition is selected and then promptly used in the course of problem solving that the problem solving aspect should be considered more important, and the behavior then classified in the appropriate category--either analyzing or synthesizing. However other differences were noted between specializing and generalizing, and between specializing and relevant. It was felt that much of the disagreement with generalizing could be traced to responses in which theorems or definitions were stated. No difficulty was apparent when a generalization was stated at the beginning or end of its development; rather, when earlier established generalizations were restated. This might occur as a preliminary to their use in a particular problem--such as "what relationship among trigonometric functions may be applied here?" or when the interest is more toward emphasizing the relationships among a small set of similar functions "Just what facts about trigonometric functions have we now?" Many times such distinctions were not as clear-cut and probably caused disagreement. No clear recommendation could be made to ensure improved agreement, but it was hoped that recognizing the above distinction would be of some use. Finally the disagreement between specializing and relevant was not entirely unexpected, for the relevant category had been derived largely from less active aspects of an earlier definition of specializing. The only clear-cut area of disagreement that was recalled was the feeling during early parts of this study that the triple behavior other/relevant/curiosity occurring during historical reference was not only an unusual behavior but potentially a very strong one. The fact of its strength in the Content and the Attitude frames seemed to bring some pressure on the observers to classify it in the strong specializing category of Process. Recognition of the importance of relevant as a holding place for potentially very powerful ideas helped during later observation.

In the Attitude categories only one area of disagreement was noted: between receptivity and independence. This disagreement was different in the teacher and in the pupils matrixes. A bias of Observer 1 towards independence or Observer 2 towards receptivity in the teacher behaviors was apparent. It was suggested that this resulted from two tendencies: in Observer 1 to classify all teacher open questions as encouraging independence regardless of the amount of thinking that would be required in answer, and Observer 2's intent to classify all behaviors in terms of the apparent interpretation by the listener (an early decision in the development of the instrument). Observer 1's practice if viewed in the light of speaker's intent or listener's interpretation would be emphasis of speaker's intent. While recognizing the reliable classification that would be ensured by Observer 1's method, it was felt that in terms of the broad aims of the instrument to classify behaviors in terms of their effect on the listener, the approach of Observer 1 should be more dependent on the level of demand of the question--which would often have to be decided in context of the previous and following behaviors. The slight bias in the opposite direction in the Pupils behaviors resulted, it was felt, because of Observer 1's equally precise approach to classification of pupil one-step responses. It had been decided when a pupil response was limited to the single answer required by the teacher's question the attitude should be receptivity...however that some leeway should be permitted when this one-step reply required considerable thinking before its expression as a single step. Apparently Observer 2 included more leeway. It was felt that in Attitude, the level of agreement was markedly improved over earlier studies largely due to such precise practices as this one of Observer 1. Clarification of the biasses still apparent, in conjunction with precise classification directions should permit the leeway important in specific behaviors without markedly changing either individual frame or triple frame scores.

All of these considerations were incorporated in the modification of the Observers' Manual Appendix. Many of them are discussed with specific examples in section "Marginal Difficulties".

CHAPTER IV

SAMPLE STABILITY

The adequacy of the chosen ten day sample is confirmed from consideration of the stability of the successive proportions of the individual frame scores and of the rankings from partial and total samples of triple frame scores, $p = 0.05$. The possibility of reduction of sample size to as few as six days is raised because of the small number of significant differences in successive proportions following day six. Direct comparison of individual frame scores derived from the six day and from the ten day samples indicate few major differences in the categories of Process and Attitude and in Neutral. The greater variability in the categories of Content seem related to adequate sampling of the phases of a topic. Direct comparison of rankings of triple frame scores from partial samples as small as five days with the entire sample indicate very high agreement in all teacher scores and in about half of the pupils scores. It is inferred, then, that the ten day sample certainly provides sound scores for interpretation of classroom discussion. In addition, the sample size may be reduced to six days if the several phases of a topic are completed or if the study is directed only to Process or Attitude.

INTRODUCTION

Knowing the classifications of the two observers to be reliable, it was necessary next to confirm the adequacy of the sample before proceeding with the description and the comparison of the classrooms by means of the scores derived from the ten day sample of behaviors. Two tests of minimum sample size for stability of individual category scores had been carried out with earlier approximations of frame and category definitions. In one case fourteen algebra classrooms in the high school had been visited for a complete topic--varying in length from five to eight days. Estimates of the number of visits necessary to provide stable samples in each category ranged from ten to twenty-three. The second case was a semester-long intensive study of one algebra classroom based on the refined frames and categories resulting from investigations of case one. Here ten days proved sufficient for all categories. These ten days could be derived either in order or at random. The present study gave the first opportunity for confirming in a variety of classrooms the choice of ten visits with a resulting sample of approximately one thousand behaviors. (All significance levels $p = 0.05$)

Because behaviors are classified across the three frames of Content, Process and Attitude, simultaneously, and scores subsequently derived both in individual frame and in triple frame, a test of stability was necessary for the two types of scores. Earlier studies had used only the individual frame score. The individual frame "representative" scores are derived over the total sample from proportions of behaviors falling into each category within each of the three frames, and into Neutral, for the entire Classroom interaction as well as for the Teacher and the Pupils components. The stability of these basic proportions was therefore essential. This was tested by the well established method of noting significant differences between corresponding category proportions in the successive steps of an accumulating sample.

The triple frame "representative"

scores are derived over the total sample as proportions of behaviors falling into a particular combination of categories during simultaneous classification across the three frames, or into Neutral, for the entire Classroom interaction as well as for the Teacher and Pupils components. The triple frame scores are merely ranked in providing strong complete views of the behaviors occurring in the classroom. The study of their stability therefore was limited to comparison by rank correlation methods of the partial and the entire sample in each classroom where joint classification occurred.

These two tests were directed to the Teacher and the Pupils components of the classroom scores. Establishment of the stability of these component scores would permit inference of adequate stability for the classroom totals.

METHOD

Individual frame scores

Materials For each classroom, frequencies of verbal behaviors of teacher and pupils in each category of the Content, the Process and the Attitude frames, and in Neutral, obtained for each period classified by each observer provided the materials for this study of stability. For the periods where simultaneous classification of behaviors by two observers had occurred, the mean scores of each of the categories and of neutral were obtained and used from then on as the frequencies for that period. Thus for classrooms I--III, V, VI, VIII--XVII there were ten sets of scores for each of teacher and pupils. Classification in classroom XVIII had occurred during only nine periods, thus there were only nine sets of scores for each of Teacher XVIII and Pupils XVIII.

Analysis The ten sets of individual frame scores for Teacher I were combined to provide successive samples of behaviors collected in one, two, three...nine periods culminating in a total sample set for the ten-day period. (Table 89).

This was repeated for Pupils I.

The same scores were derived in each of the Classrooms II through XVI.

Noting that each verbal behavior had been classified once in each of the three frames or in neutral the total behavior for each sample could be calculated by combining teacher and pupil category scores in any one frame and in neutral:

Content: $C_i + N$ (i from 1 to 6) equals
Process: $P_i + N$ (i from 1 to 5) equals
Attitude: $A_i + N$ (i from 1 to 3)

These ten sets of cumulative raw scores for Teacher I in category of Content, of Process and of Attitude, and in Neutral, together with the ten successive values of total classroom verbal interaction were used to determine ten corresponding sets of proportions. (Table 90) This was repeated for Pupils I and then the entire calculation was repeated for each of the Classrooms II through XVI. Thus:

Proportion Teacher I--Structure =
(3 day sample)

$$\frac{\text{Cumulative raw score Teacher-I--Structure}}{\text{Cumulative raw total verbal interaction}}$$

$$\frac{30.0}{271.5} = 0.110$$

(call this p3)

For each sample proportion the corresponding standard error was determined (Table 91):

Standard error of
Proportion Teacher I--Structure =
(3 day sample)

$$\frac{p(1 - p)}{n} =$$

$$\frac{0.110 \times 0.990}{271.5} = 0.019$$

The ten sets of proportions were then assessed for stability. The simple difference of a successive pair of proportions in a single category was obtained and this difference was compared with the standard

Table 89 Cumulative frequencies in each category of Content, of Process and of Attitude, and in Neutral for Teacher I and Pupils I derived from ten periods of classification.

DAYS GORY	1	2	3	4	5	6	7	8	9	10
					TEACHER I					
ent										
ucture	15.0	26.0	30.0	34.0	45.5	45.5	55.5	77.5	92.5	109.5
hniques	15.0	22.0	30.5	37.5	47.5	56.5	63.5	69.5	73.5	81.5
uctive	14.0	17.0	36.0	53.5	63.0	73.0	86.0	112.0	150.0	162.5
uctive	0.0	12.0	20.5	31.0	37.0	39.0	53.0	55.0	56.0	61.0
hematical	18.5	40.5	62.0	82.5	100.0	107.0	126.5	136.5	153.5	170.5
er	0.0	0.0	0.0	3.0	11.0	22.0	28.5	30.5	30.5	30.5
ess										
lyzing	7.5	15.5	21.0	26.5	36.0	37.0	49.0	61.0	76.0	80.0
thesizing	12.5	27.5	47.0	58.5	63.5	69.5	76.0	104.5	129.5	138.5
cializing	16.5	31.5	52.0	78.5	109.0	115.0	149.5	167.5	186.5	220.5
eralizing	8.5	11.5	18.5	27.0	30.0	35.0	45.5	47.0	56.0	59.0
evant	17.5	32.5	41.5	53.0	67.5	88.5	98.0	108.0	116.0	125.0
tude										
losity	1.0	1.0	1.0	4.5	8.5	15.5	18.5	22.5	22.5	22.5
ependence	48.5	96.5	136.5	181.5	231.5	256.5	311.5	362.5	420.5	468.5
eptivity	13.0	21.0	42.5	57.5	66.0	73.0	88.0	103.0	121.0	133.0
ral	2.0	11.0	11.0	12.5	12.5	12.5	13.5	15.0	15.0	18.5
					PUPILS I					
ent										
cture	7.5	7.5	9.5	9.5	11.5	11.5	14.0	23.5	26.5	30.5
niques	0.5	1.5	2.5	4.0	4.0	5.0	6.5	6.5	8.5	12.5
ctive	2.0	2.0	8.0	22.5	26.5	28.5	32.5	39.5	46.5	51.0
ctive	0.0	7.0	7.0	7.5	9.0	11.0	16.0	16.0	16.0	17.0
ematical	13.5	28.5	46.5	54.5	66.5	70.5	73.5	75.0	76.0	89.5
r	0.0	0.0	0.0	0.0	1.0	1.0	1.0	1.0	1.0	1.0
ss										
yzing	1.5	2.5	4.0	5.5	6.5	6.5	7.5	10.0	14.0	15.0
hesizing	16.0	30.0	47.0	57.5	66.0	71.0	75.0	83.5	88.5	95.5
ializing	4.0	13.0	20.0	28.0	40.5	43.5	51.5	59.5	63.5	79.5
ralizing	3.5	3.5	5.0	9.0	10.5	11.5	15.5	16.5	18.5	19.5
vant	1.0	3.0	5.0	7.5	7.5	9.5	10.5	11.0	12.0	16.0
ude										
osity	1.0	1.0	1.0	4.5	6.5	6.5	7.0	7.0	7.0	7.0
pendence	22.0	46.0	71.5	92.0	112.5	122.5	140.0	159.5	175.5	200.0
ptivity	3.0	5.0	8.5	11.0	12.0	13.0	13.0	14.0	14.0	18.5
al	0.0	0.0	0.0	0.0	0.5	0.5	0.5	0.5	0.5	0.5
T & P	90.5	181.5	271.5	363.5	449.5	499.5	591.5	683.5	775.5	867.5

---

ional scores result from averaging joint classification by two observers.

Table 90 Cumulative proportions in each category of Content, of Process and of Attitude, and in Neutral for Teacher I and Pupils I derived from Table 89. All decimals omitted in proportions.

DAYS CATEGORY	1	2	3	4	5	6	7	8	9	10
					TEACHER I					
Content										
Structure	166	143	110	094	101	091	094	113	119	126
Techniques	166	121	112	103	106	113	107	102	095	094
Deductive	155	094	133	147	140	146	145	164	193	187
Inductive	---	066	076	085	082	078	090	080	072	070
Mathematical	204	223	228	227	222	214	214	200	198	197
Other	---	---	---	008	024	044	048	045	039	035
Process										
Analyzing	082	085	077	073	080	074	083	089	098	092
Synthesizing	138	152	173	161	141	139	128	153	167	160
Specializing	182	174	192	216	242	230	253	245	240	254
Generalizing	094	063	068	074	067	070	077	069	072	068
Relevant	193	179	153	146	150	177	166	158	150	144
Attitude										
Curiosity	011	006	004	012	019	031	031	033	029	026
Independence	536	532	503	499	515	514	527	530	542	540
Receptivity	144	116	157	158	147	146	149	151	156	153
Neutral	022	061	041	034	028	025	023	022	019	021
					PUPILS I					
Content										
Structure	083	041	035	026	026	023	024	034	034	035
Techniques	006	008	009	011	009	010	011	010	011	014
Deductive	022	011	029	062	059	057	055	058	060	059
Inductive	---	039	026	021	020	022	027	023	021	020
Mathematical	149	157	171	150	148	141	124	110	098	103
Other	---	---	---	---	002	002	002	001	001	001
Process										
Analyzing	017	014	015	015	014	013	013	015	018	017
Synthesizing	177	165	173	158	147	142	127	122	114	110
Specializing	044	072	074	077	090	087	087	087	082	092
Generalizing	039	019	018	025	023	023	026	024	024	022
Relevant	011	017	018	021	017	019	018	016	015	018
Attitude										
Curiosity	011	006	004	012	014	013	012	010	009	008
Independence	243	253	263	253	250	245	237	233	226	231
Receptivity	033	028	031	030	027	026	022	020	018	021
Neutral	---	---	---	001	001	001	001	001	001	001
Total T & P on which proportions based:	90.5	181.5	271.5	363.5	449.5	499.5	591.5	683.5	775.5	867.5

· In each sample if teacher and pupil proportions combined in each category, then

$$\Sigma C_1 + N = \Sigma P_1 + N = \Sigma A_1 + N = 1.000$$

Table 91 **Standard errors of cumulative proportions in each category of Content, of Process and of Attitude, and in Neutral for Teacher I and Pupils I derived from Table 90. All decimal points omitted.**

DAYS	1	2	3	4	5	6	7	8	9	10
EGORY										
tent					TEACHER I					
ructure	039	026	019	015	014	013	012	012	012	011
chniques	039	024	019	016	015	014	013	012	011	010
ductive	038	022	021	019	016	016	014	014	014	013
ductive	---	018	016	015	013	012	012	010	009	009
thematical	042	031	025	022	020	018	017	015	014	014
her	000	000	000	005	007	009	009	008	007	006
cess										
alyzing	029	021	016	014	013	012	011	011	011	010
nthesizing	036	027	023	019	016	015	014	014	013	012
ecializing	041	028	024	022	020	019	018	016	015	015
neralizing	031	018	015	014	012	011	011	010	009	009
levant	041	028	022	019	017	017	015	014	013	012
itude										
riosity	011	006	004	006	006	008	007	007	006	005
dependence	052	037	030	026	024	022	021	019	018	017
ceptivity	037	024	022	019	017	016	015	014	013	012
tral	015	018	012	010	008	007	006	006	005	005
tent					PUPILS I					
ructure	029	015	011	008	008	007	006	007	007	006
chniques	008	007	006	005	004	004	004	004	004	004
ductive	015	008	010	013	011	010	009	009	009	008
ductive	---	014	010	008	007	007	007	006	005	005
thematical	037	027	023	019	017	016	014	012	011	010
her	---	---	---	---	002	002	002	001	001	001
cess										
alyzing	014	009	007	006	006	005	005	005	005	004
nthesizing	040	028	023	019	017	016	014	013	011	011
ecializing	022	019	016	004	013	013	012	011	010	010
neralizing	020	010	008	008	007	007	007	006	005	005
levant	011	010	008	008	006	006	005	005	004	005
itude										
riosity	011	006	004	006	006	005	004	004	003	003
dependence	045	032	027	023	020	019	017	016	015	014
ceptivity	019	012	011	009	008	007	006	005	005	005
tral	---	---	---	002	001	001	001	001	001	001

error of the proportion in the smaller sample (Table 93).

Simple difference in Teacher I--Structure =
(from 3 day sample to 4 day sample)

$$|p_3 - p_4| =$$

$$|0.110-0.094| = 0.016$$

Standard error of $p_3 = 0.019$

In this example a null hypothesis of no difference between successive proportions would be confirmed $p = 0.05$. It is noted that the difference between the two sample values of the category proportion is less than one standard error.

It is noted that where both np and n(1 - p) are 5 or more the normal distribution provides a satisfactory approximation to the sampling distribution of a proportion. The values of the proportions and the sample n's generally were such to fulfill these conditions throughout the sets of cumulative proportions. (Wallis and Roberts, 1956, p. 379). Because of the large numbers of proportions involved in this test of stability this simple approximation proved very advantageous.

The entire set of simple differences and corresponding standard errors were tabulated for teacher and pupils proportions in classrooms I through XVI. Differences less than one standard error were unmarked; those greater than one and less than 1.96 were marked with one asterisk; those greater than 1.96 and less than 2.58 with two asterisks; and those differences greater than 2.58 standard errors with three asterisks. This enabled rapid summary of three levels of significance of these simple differences:

$p = 0.34, 0.05, 0.01$

It was hoped that a stable sample could be derived in less than the ten days by a clear-cut decrease in the number of significant differences ($p = 0.05$) at, for instance, the 6 day sample. If such clear-cut decrease were to occur at the 6 day sample, the simple difference of the 6 day proportion and the 10 day proportion in a single category would be obtained and expressed in standard error units of the smaller sample proportion.

Simple difference in Teacher I--Structure from 6 day sample to 10 day sample

$$|p_6 - p_{10}| = 0.035$$

Difference in standard error units =

$$\frac{|p_6 - p_{10}|}{s.e._6} =$$

$$\frac{0.035}{0.013} = 2.70$$

In this example a null hypothesis of no difference between the six and the ten day samples would have to be rejected. It is noted that the difference between the two sample values of the category proportion is greater than 2.58 standard errors in size, and thus is significant beyond the 0.01 level.

Triple frame scores

Materials Because of the large number of triple frame combinations possible--ninety in all--and the usual daily sample not exceeding one hundred behaviors, it was felt that comparison of successive steps, as in the individual frame score case, would include too many extremely small values for sound analysis. Convenient samples less than those derived from the ten day period were available from joint study in eleven of the sixteen classrooms: I, II, VI, X--XVI, and XVIII. The number of days included in the joint classifications ranged from a maximum of seven in I and II to a minimum of five in VI, X-XVI and XVIII. Three classrooms, VIII, IX, and XVII visited early in the assessment of reliability had been jointly classified through nine periods and it was felt that the final single day would not appreciably change the frequency of the individual cells. Two classrooms, III and V, had not been studied jointly, in any period. These

Table 93 Simple differences of successive cumulative proportions in each category of Content, of Process and of Attitude, and in Neutral for Teacher I and Pupils I. All values stated as absolute values; no decimal points included.

DAYS	1-2	2-3	3-4	4-5	5-6	6-7	7-8	8-9	9-10
ATEGORY									
ontent					TEACHER I				
tructure	023	033*	016	007	010	003	019*	006	007
echniques	045*	009	009	003	007	006	005	007	001
eductive	061*	029*	014	007	006	001	019*	029**	006
nductive	066	010	009	003	004	012	010	008	002
athematical	019	005	001	005	007	000	014	002	001
ther	---	---	008	016***	020**	004	003	006	004
ocess									
nalyzing	002	008	004	007	006	009	006	009	006
ynthesizing	014	021	012	020*	002	011	025*	014	007
pecializing	008	019	024	026*	012	027*	008	005	014
eneralizing	031	005	006	007	003	007	008	003	004
elevant	014	026	007	004	027*	011	008	008	006
titude									
uriosity	005	002	008*	007	012**	000	002	004	003
ndependence	004	029	004	016	001	013	003	012	002
eceptivity	028	041*	001	011	001	003	002	005	003
utral	039***	020*	007	006	003	002	001	003	002
ntent					PUPILS I				
tructure	042*	006	009	000	003	001	010	000	001
echniques	002	001	002	002	001	001	001	001	003
eductive	011	014*	033***	003	002	002	003	002	001
nductive	039	013	005	001	002	002	005	002	001
athematical	008	014	021	002	007	017*	014	012	005
ther	---	---	---	002	000	000	001	000	000
ocess									
nalyzing	003	001	000	001	001	000	002	003	001
ynthesizing	012	008	015	011	005	015	005	008	004
pecializing	028*	002	003	023*	003	000	000	005	010
eneralizing	020	001	007	002	000	003	002	000	002
elevant	006	001	003	004	002	001	002	001	003
titude									
uriosity	005	002	008**	002	001	001	002	001	001
ndependence	010	010	010	003	005	008	004	003	005
eceptivity	006	003	001	003	001	004	002	002	003
utral	---	---	001	000	000	000	000	000	000

---

* Differences between 1 and 2 s.e.

** Differences between 2 and 3 s.e.

*** Greater than 3 s.e.

five classrooms could not be included in this study, but it was felt that the remaining eleven would provide sufficient evidence for consideration of the triple frame scores in terms of stability.

The materials for this aspect of the study were provided by the frequencies of verbal behaviors of teacher and pupils in each category combination of the Content, the Process and the Atttiude frames, and in Neutral, for each of the eleven classrooms and for each observer. In each class, a partial matrix was obtained from the periods where simultaneous classification of behaviors by two observers had occurred by summing the mean values of each period in each of the triple frame scores and in neutral. A corresponding total matrix was obtained by combining this partial matrix with the totals of triple frame and neutral scores from the remaining periods classified by only one observer.

Thus there was a partial matrix and a total matrix for both teacher and pupils in each of eleven classrooms.

Analysis The triple frame scores in each of the partial and total matrix of Teacher I were ranked (Table 95). The two series of ranks were compared by means of the Spearman r_s coefficient of rank correlation and the significance level noted.

For the case where ties occur:

$$r_s = 1 - \frac{S(d^2)}{1/6\ (n^3 - n) - (T^1 + U^1)}$$

$S(d^2)$: sum squared deviations

n: number of ranks

T^1: $\frac{1}{12} \sum_t (t^3 - t)$ ties in rank one

U^1: $\frac{1}{12} \sum_u (u^3 - u)$ ties in rank two

(Kendall, 1948, p. 39: 3.10)

It was decided that five should be the smallest cell value acceptable in the total matrix. In the total sample of approximately 1,000 raw behaviors this would permit errors of 0.005. When ranking the partial matrix, it was decided to continue the use of 5 as the minimum. The exception to this is illustrated in the Teacher I rankings where the triples 332 and 323 have frequencies 1 and 4 in the partial matrix. This permits inclusion of these two triples having ranks 15.5 and 20 in the 23 ranks of the total matrix. Inclusion of small numbers of such small frequencies would tend to depress r_s and thus to provide a more rather than a less stringent test.

In the example of Teacher I with $n = 23$ the r_s correlation between the triple frame rankings of Observer 1 and Observer 2 was 0.97. From Siegel, 1956, p. 284 a value as large as this was significant at the $p < 0.01$ level. Thus the null hypothesis, that the rankings derived from the two matrixes were unrelated was rejected and an association between triple frame classifications from partial and entire sample for Teacher I was concluded. A similar conclusion was reached for partial and total sample of Pupils I.

This analysis was repeated for the classrooms II, VI, X--XVI and XVIII.

RESULTS

Individual frame scores

Tabulation of the ten sets of cumulative raw scores, cumulative proportions and corresponding standard errors for Teacher I and for Pupils I was made for purposes of example in Tables 89, 90 and 91, respectively. Similar preparation for obtaining simple differences between successive proportions in each category was made for every classroom. However, only this example for classroom I was included in this report.

Simple differences between successive proportions in individual categories for Teacher I and for Pupils I were recorded earlier in Table 93. Differences significant at 34%, 5% and 1% level were noted. Corresponding tables for the remaining fifteen classrooms were included in the Appendix A: 216 to 231.

Table 95 Calculation of Spearman Coefficient of Correlation, r_s, between the rankings of the total matrix and the partial matrix for Teacher I and Pupils I.

Numerical symbolism of triple frame scores in Content, Process and Attitude, respectively.	Total matrix frequency rank		Partial matrix frequency rank		$\lvert d \rvert$	d^2
			TEACHER I			
332	68	1	56	1	0	0.00
322	53	2	36	2.5	0.5	0.25
222	49	3	34	4	1	1.00
552	48	4	36	2.5	1.5	2.25
232	36	5	24	6	1	1.00
112	32	6	21	8.5	2.5	6.25
132	31	7	25	5	2	4.00
632	28	8.5	23	7	0.5	0.25
553	28	8.5	21	8.5	0	0.00
212	23	10	17	10	0	0.00
242	18	11	12	12.5	0.5	0.25
333	15	12	14	11	1	1.00
233	13	13	9	14	1	1.00
133	12	14	12	12.5	1.5	2.25
642	11	15.5	8	15	0.5	0.25
352	11	15.5	1	23	7.5	56.25
622	9	17	5	18.5	1.5	2.25
223	7	20	5	18.5	1.5	2.25
323	7	20	4	22	2	4.00
312	7	20	5	18.5	1.5	2.25
612	7	20	5	18.5	1.5	2.25
143	7	20	5	18.5	1.5	2.25
122	6	23	5	18.5	4.5	20.25
			PUPILS I			
322	55	1	41	1	0	0.00
332	29	2	24	2	0	0.00
222	28	3	21	3	0	0.00
132	16	4	15	4	0	0.00
232	15	5	13	5	0	0.00

Teacher I:

$S(d^2) = 111.50$ $\quad r_s = 1 - \dfrac{111.50}{1996.80}$

$n = 23$

$T' = 11.00$ $\quad = 0.97$ significant $p < 0.01$

$U' = 18.20$

Pupils I: $\quad r_s = 1.00$ significant $p < 0.01$

$S(d^2) = 0$

SAMPLE STABILITY

From these difference tables it was apparent that a clear drop in the number of significant differences occurred about day six. Accordingly the simple differences between the proportions in individual categories for all teachers and pupils determined in a six day sample and the total ten day sample were recorded and the three significance levels noted. (Tables 97 and 98)

Summary of these significant differences for the entire group of sixteen classrooms was made in Table 99. The numbers of significant differences were recorded for the nine sets of successive differences in the entire sample and for the four sets of differences from day six to day ten. These numbers were separated into teacher and pupil components, and in addition their frequencies in the several categories within each frame and by total frame were detailed. This table also recorded the numbers of differences between corresponding category values derived from six days and from the total sample significant at the three specified levels.

A second summary was made in Table 100 of the percentage of differences within each frame significant at each level, in the teacher and pupils components.

It was noted that the summary tables were so constructed that numbers or percentages of the category differences significant at the 34% level include those significant at both the 5% level and the 1% level; and the numbers or percentages significant at the 5% level include those significant at the 1% level.

Even in the totals of the nine sets of differences throughout days one to ten a high percentage of the differences was seen to cluster within one standard error of the theoretical proportion--ranging from a maximum 100% for Pupils Neutral to the minimum 72% for Teachers Content. This was more marked in the four sets of differences throughout days six to ten--ranging from a maximum 100% for Pupils Neutral to the minimum 81.5% for Teachers Content. At the 5% level (the one basic to this study) quite satisfactory levels were attained: the percentage of differences within 1.96 standard errors of the theoretical proportion ranging from 100% (Pupils Neutral) to 91.1% (Teacher Content) for the entire ten days; and from 100% (Pupils Neutral) to 93.7% (Teacher Neutral) from days six through ten.

The direct comparison of the proportion for day 6 and day 10 shows many high percentages of significant differences at each specified level. Thus at the percentage of differences clustering within 1.96 standard errors of the proposed theoretical proportion ranges from 100% (Pupil Neutral) to as low as 69.8% (Teacher Content).

While overall percentages for the entire set of Teacher and Pupils in all three frames and neutral could readily be calculated and would in part absorb the effect of the Teacher Content and Teacher Neutral, it was noted that this would contribute to over optimistic conclusions about decreasing sample size, and would hide the interesting distinctions in stability apparent in the three frames.

From the sets of successive differences both for days 1--10 and days 6--10 the frames of Process and of Attitude were remarkably stable. A heavy clustering even within one standard error of the theoretical proportion was observed: i. for days 1--10 Teacher: 85.0% and 82.3%; Pupils: 90.3% and 89.1%; in Process and Attitude, respectively. ii. for days 6--10. Teacher: 92.4% and 88.4%; Pupils: 96.8% and 93.7%; in Process and Attitude respectively. Then withi 1.96 standard errors: i. for days 1--10. Teacher: 97.6% and 95.8%; Pupils: 96.2% a 95.8%; in Process and Attitude respectively ii. for days 6--10. Teacher: 99.0% and 97 in Process and Attitude, respectively; Pupil 98.7% and 99.0% in Process and Attitude, respectively. In the direct comparison of day 6 and day 10, Process and Attitude were agai quite stable: Teacher 88.8% and 91.7%; Pupils 95.0% and 91.7%; in Process and Attitude, respectively, within 1.96 standard errors of the theoretical proportion.

Table 97 Simple differences in eight classrooms, between the proportions derived from a six day sample and from a ten day sample in each category of Content, of Process and of Attitude and in Neutral for all classrooms. All values absolute; no decimal points included. Dash occurs when proportion equals zero.

CLASS CATEGORY	I	II	III	V	VI	VIII	IX	X
				TEACHER				
Content								
Structure	035***	049***	020*	004	003	025**	017*	036*
Techniques	019*	015*	008*	000	036*	018	018*	026*
Deductive	041**	034**	015	013	039**	017*	040***	014*
Inductive	008	022*	041**	082***	018*	025**	028**	032***
Mathematical	017	031*	053***	093***	022*	077***	001	020
Other	009	006	003	008***	023**	007	009*	009
Process								
Analyzing	018*	017*	002	016*	011	014**	011*	024*
Synthesizing	021*	027*	029*	034*	007	030*	011	004
Specializing	024*	025*	039**	065***	010	020	006	008
Generalizing	002	002	004	006	004	000	002	008
Relevant	033*	010	008	015*	011	001	004	032**
Attitude								
Curiosity	005	002	016*	002	029**	012***	014***	014*
Independence	026*	009	009	012*	007	023*	032*	022*
Receptivity	007	010	004	016*	045**	016	039**	012
Neutral	004	002	013***	006***	014*	002	000	007
				PUPILS				
Content								
Structure	012*	007	000	001	000	008	001	001
Techniques	004	002	000	000	003***	000	005	011*
Deductive	002	013*	002	004	002	001	004	005
Inductive	002	001	001	000	002	002	003	003***
Mathematical	038***	026*	008	009*	004*	008	023*	004
Other	001	000	---	---	---	003	001	002
Process								
Analyzing	004	006	000	003	001	007*	000	005
Synthesizing	032	019*	---	001	001	007	013	008
Specializing	005	005*	010	006	004	001	010	007
Generalizing	001	001	---	001	000	001	006*	003
Relevant	001	000	001	002	002	002	006	020**
Attitude								
Curiosity	005	---	004	004	001	005***	001	002
Independence	014	013	003	006*	001	007	023*	006
Receptivity	005	002	008	003	006**	015*	000	020*
Neutral	000	001	---	---	---	---	---	000

---

* Differences between 1 and 1.96 s.e.
** Differences between 1.96 and 2.58 s.e.
*** Greater than 2.58 s.e.

Table 98 Simple differences in eight classrooms, between the proportions derived from a six day sample and from a ten day sample in each category of Content, of Process and of Attitude and in Neutral for all classrooms. All values absolute; no decimal points included. Dash occurs when proportion equals zero.

CLASS CATEGORY	XI	XII	XIII	XIV	XV	XVI	XVII	XVIII
				TEACHER				
Content								
Structure	005	001	041**	018*	011*	007	028***	021*
Techniques	016*	011	001	028*	014	030*	000	001
Deductive	016*	053***	005	004	028***	000	011	037**
Inductive	042***	043*	003	012***	024	052***	025***	032***
Mathematical	022*	023	020*	016	002	039*	010	015
Other	014*	008*	010*	015*	001	006	001	010**
Process								
Analyzing	017*	011	015*	017*	002	007	012*	018*
Synthesizing	010	026*	036***	054***	020**	001	013	007
Specializing	012	015	011	003	016	008	022*	025*
Generalizing	013*	004	002	002	007*	005	014*	005
Relevant	012	016	008	006	014	043**	010	037**
Attitude								
Curiosity	005	004	008	002	002	001	000	000
Independence	008	004	017	007	010	027*	036*	010
Receptivity	014	010	021*	035*	037*	008	016	068***
Neutral	005	000	003	018*	004	011*	023**	026***
				PUPILS				
Content								
Structure	002	001	004	002	003	008	008**	005
Techniques	001	004	001	017*	009	015	001	009*
Deductive	054***	007*	037***	---	014***	001	010	049***
Inductive	001	006	006	---	001	011***	002	017***
Mathematical	035**	003	003	005	023*	075***	038**	008
Other	003	002	006*	001	003	002	001	003
Process								
Analyzing	009*	001	020**	001	008	000	003	003
Synthesizing	029**	000	008	017*	017	090***	016	027*
Specializing	011	007	019*	010	004	024	014	009
Generalizing	002	002	004	---	004	002	006	005*
Relevant	001	004	001	003	004	013	007*	009*
Attitude								
Curiosity	000	005	001	001	002	001	005	002
Independence	003	010	049***	013	021	061***	013	037*
Receptivity	009*	016***	001	022	001	011	010	009*
Neutral	---	001	---	003*	001	002	001	004*

---

* Differences between 1 and 1.96 s.e.

** Differences between 1.96 and 2.58 s.e.

*** Greater than 2.58 s.e.

Table 99 Numbers of simple differences of cumulative proportions significant at p = 0.34, p = 0.05, p = 0.01 in each category of Content, of Process and of Attitude, and in Neutral for all teachers and pupils. In the nine sets of differences for days 1-10; in the four sets of differences for days 6-10; between day 6 and day 10.

	DAYS 1-10			DAYS 6-10			DAY 6 - DAY 10		
ignificance Levels	0.34	0.05	0.01	0.34	0.05	0.01	0.34	0.05	0.01
ATEGORY									
ontent					**TEACHER**				
Structure	35	4	3	11	1	0	11	5	3
Techniques	25	3	3	3	0	0	9	1	0
Deductive	53	17	10	19	4	1	10	7	3
Inductive	45	26	17	13	7	6	13	10	7
Mathematical	63	19	10	22	2	0	8	3	3
Other	14	8	5	2	1	0	8	3	1
Total	235	77	48	70	15	7	59	29	17
rocess									
Analyzing	17	2	0	3	0	0	11	1	0
Synthesizing	29	3	1	8	2	0	9	3	2
Specializing	27	3	2	7	0	0	6	2	1
Generalizing	20	3	2	4	0	0	3	0	0
Relevant	14	6	3	2	1	1	5	3	0
Total	107	17	8	24	3	1	34	9	3
ttitude									
Curiosity	13	7	3	2	0	0	5	3	2
Independence	22	3	2	7	1	1	7	0	0
Receptivity	41	8	2	13	3	1	7	3	1
Total	76	18	7	22	4	2	19	6	3
utral	21	10	8	6	4	4	7	4	3
ontent					**PUPILS**				
Structure	14	5	1	4	3	0	2	1	0
Techniques	10	4	3	3	2	1	4	1	1
Deductive	28	15	9	11	5	4	6	4	4
nductive	12	5	4	6	2	2	3	3	3
athematical	37	13	6	11	3	2	9	4	1
Other	5	4	2	3	2	0	1	0	0
Total	106	46	25	38	17	9	25	13	9
ocess									
nalyzing	9	2	0	2	1	0	3	1	0
ynthesizing	29	12	6	5	3	2	5	2	1
pecializing	15	6	2	0	0	0	2	0	0
eneralizing	7	4	3	2	0	0	2	0	0
elevant	9	3	1	1	0	0	3	1	0
Total	69	27	12	10	4	2	15	4	1
titude									
uriosity	3	2	0	1	0	0	1	1	1
Independence	32	12	7	8	1	0	5	2	2
eceptivity	12	4	3	3	1	1	6	2	1
Total	47	18	10	12	2	1	12	4	4
utral	0	0	0	0	0	0	0	0	0

Table 100 Percentages of cumulative proportions clustering within 1, 1.96 and 2.58 s.e. in each frame Content, Process and Attitude, and in Neutral for all teachers and pupils: in the nine sets of differences days 1-10; in the four sets of differences days 6-10; between day 6 and day 10.

	DAYS 1-10				DAYS 6-10				DAY 6 - DAY 10			
s.e.	1.00	1.96	2.58	n	1.00	1.96	2.58	n	1.00	1.96	2.58	n
Frame					TEACHERS							
Content	72.9	91.1	94.5	858	81.5	96.0	97.4	378	38.5	69.8	82.3	96
Process	85.0	97.6	98.9	715	92.4	99.0	99.7	315	57.5	88.8	96.3	80
Attitude	82.3	95.8	98.4	429	88.4	97.9	99.0	189	60.4	91.7	93.8	48
Neutral	85.3	93.0	94.4	143	91.5	93.7	93.7	63	56.3	75.0	81.3	16
					PUPILS							
Content	87.8	94.7	97.1	858	90.0	95.5	97.6	378	74.0	86.5	91.6	96
Process	90.3	96.2	98.3	715	96.8	98.7	99.4	315	81.3	95.0	98.8	80
Attitude	89.1	95.8	97.7	429	93.7	99.0	99.5	189	75.0	91.7	91.7	48
Neutral	100.0	100.0	100.0	143	100.0	100.0	100.0	63	87.5	100.0	100.0	16

While Content also showed consistent clustering, it was somewhat less pronounced: i. for days 1--10. Teacher: 72.9% and 91.1%; Pupils: 87.8% and 94.7% for 1.00 and 1.96 standard errors, respectively. ii. for days 6--10. Teacher: 81.5% and 96.0%; Pupils: 90.0% and 95.5% for 1.00 and 1.96 standard errors, respectively. iii. for days 6--day 10: Teacher: 38.5% and 69.8%; Pupils: 74.0% and 86.5% for 1.00 and 1.96 standard errors, respectively. Similarly, Teacher Neutral was somewhat more widespread than the Process and the Attitude.

A closer examination of the individual category differences summarized in Table 99, permitted selection of the categories within the Teacher Content frame contributing most to the variability. Relations, both deductive and inductive, and Applications, mathematical, stood out particularly over the entire sample. In the differences derived from 6 to 10 days, and in the differences between day 6 and day 10, the Applications, mathematical, no longer contributed to the variability at the 5% level. Reference to the corresponding Pupils categories indicated most variability in the Pupils Content frame in two of the above three: Relations, deductive, and Applications, mathematical.

Triple frame scores

A summary of the Spearman r_s coefficients of correlation between the rankings of the triple frame scores in the partial and the total matrixes for all eleven classrooms was made in Table 102 for teachers and for pupils separately. The significance level of each r_s was noted. In addition, the number of days contributing to each partial matrix, and the numbers of ranks in each comparison was recorded.

Most of the r_s coefficients were significant, < 0.01. Of the eleven teacher coefficients, no level greater than 0.01 was recorded. Of the nine pupils coefficients five levels greater than 0.05 were recorded: Pupils X with 0.08; Pupils XIII with 0.06; Pupils XIV with 0.21 and Pupils XV with 0.16 (despite $r_s = 1.00$). All of these coefficients with significance levels greater than 0.05 were derived from comparisons of the five day with the total samples.

DISCUSSION AND INFERENCES

Individual frame scores

The assessment of simple differences between proportions derived from successive samples showed a high level of stability, $p = 0.05$, over the entire ten day period of observation. Stability was particularly marked in the successive samples collected from six days on. The direct comparison of the sample collected during six days and of the total sample derived from the ten days of observation, while showing a higher proportion of significant differences than did the comparison of successive proportions, still indicated a very high clustering within our chosen confidence interval. Thus a classroom visited for six days would receive essentially the same description as if visited for the entire ten days of this experiment. It was apparent, then, that by the end of the ten day period the scores in the individual categories of the three frames of Content, of Process and of Attitude, and in Neutral, were quite stabilized and the choice of the ten day sample to provide "representative behaviors" of a classroom was sound.

The clear-cut increase in the clustering of the proportions apparent about the sixth day of observation raised the possibility of reducing the sample size even below that collected during ten days. Certainly in the categories of Process and of Attitude, both for teachers and for pupils, and in the category of Neutral, for pupils, such a reduction was supported in all comparisons made. While such a reduction was quite adequately supported for the remaining categories (Content and Neutral, teacher) in the comparison of successive proportions, some caution was imposed by higher proportion of differences apparent between the samples derived from six days and from ten days.

Table 102 Spearman Rank Correlation Coefficients r_s between rankings in partial and total samples of triple category combinations in the Content, the Process and the Attitude frames, and in Neutral for teachers and for pupils in eleven classrooms. Also recorded: members of days in partial samples, D; number of ranks in each correlation, n; and the probability level at which r_s significant, p.

CLASS	D	TEACHER			PUPILS		
		n	r_s	p	n	r_s	p
I	7	23	0.95	0.01	6	1.00	0.01
II	7	26	0.89	0.01	6	0.96	0.01
VI	5	20	0.93	0.01	0	----	----
X	5	21	0.86	0.01	5	0.67	0.08*
XI	5	26	0.77	0.01	10	0.89	0.01
XII	5	15	0.91	0.01	0	----	----
XIII	5	36	0.77	0.01	10	0.53	0.06*
XIV	5	13	0.82	0.01	4	0.40	0.21*
XV	5	13	0.75	0.01	3	1.00	0.16?
XVI	5	14	0.83	0.01	3	0.87	0.33*
XVIII	5	17	0.66	0.01	7	0.91	0.01

---

* Not significant $p = 0.05$
Reject H_0 at this level.
? Practically if not statistically significant.

The concentration of these greater differences mainly in the categories of Content led to consideration of changes in general aspects of content apparent at a gross level in the classroom. An obvious unit for such patterns was the small topic, lasting perhaps from six to twelve days. In the several phases of a topic some or all of the following may be expected: introduction of the new topic, examination of well established concepts in preparation for their connection with the new problem; emerging relationships suggesting a new concept; statement of relationships; establishment of these relationships; application of these relationshipsl application of these to particular problems. It was recognized that these general changes during the course of a topic would be largely related to development and application of new relations. It was in just these categories that the least stability from day to day and from the six day to the ten day samples was observed. Further evidence of this idea was gained from examination of the varying incidence of these differences, class by class. It was apparent that in the classrooms where the full range of phases of a topic (as described above) was encountered, the greatest incidence of these differences occurred.

It was suggested, then, that the ten day sample, probably provided quite an adequate sample of the Content, because it was derived over a period of time sufficient to cover most of a topic. Decision about reduction of the number of days of observation from ten towards six might well be based on completion of an adequate number of phases of a topic.

It may be inferred, then, that an indence in the "representative behaviors" derived from ten visits to a classroom. Where interest is particularly directed to the Attitude or to the Process frames, fewer days--with six as a minimum--would be quite satisfactory. On the other hand, caution should be used in reducing the number of visits for accurate measurement in Content, recognizing that much of the variability may be directly related to the several phases of a developing topic.

Triple frame scores

From the evidence of the coefficients of correlation between rankings of triple frame scores derived from partial and from complete samples, even five visits provided an impressive indication of the results of the entire ten visits.

It may be noted that a decision was reached much later in this study, during the comparison of the four types of classrooms, to use only the first ten ranks (provided frequencies $\geqslant 5$) for classroom description. In this analysis of stability, all scores in which the average of joint classification equalled or exceeded five were included. Further when the triple scores were quite small--say less than twenty--any unreliability might well affect the remaining rankings. The favorable results of this test of stability permitted the use of all ranks with frequency $\geqslant 5$ throughout the remaining parts of the experiment until the final step when the decision to select the first ten ranks was made. It may be inferred that where fewer ranks than used in this test were selected for final descriptive purposes, the judgment of stability of these fewer ranks was even more affirmative. Such strengthening of the stability was generally limited to the teacher rankings, for the number of pupils ranks rarely exceeds ten.

The ten days of observations, then with a few exceptions in pupils scores, provided a stable ordering of the triple frame scores. In addition, such similarity in ordering was apparent even in five days of observations that where a reduction in days of observation is feasible for the individual frame scores, significant changes in ordering would be expected in only a limited number of pupil triple frame scores. For each of individual and triple frame scores, it was noted that the success of the tests directed to teacher and pupil components of classroom results permits inference of stability of classroom scores.

CHAPTER V

VARIABILITY OF VERBAL BEHAVIORS DURING PROBLEM SOLVING IN THE TYPICAL SUBGROUPS OF A HIGH SCHOOL GEOMETRY CLASSROOM

Patterns of verbal behaviors during problem solving by the entire classroom, small groups of four pupils with teacher, small groups of four pupils, individual pupils with teacher and individual pupils are examined by the Wright-Proctor Observational Instrument in its categories of Attitude, Process and Content. These are illustrated in tabular and graphic form. A much greater continuity of discussion is observed in the first three subgroups than in the last two resulting from the directive and checking role of the teacher or from the wealth of ideas available from the several participants. Similarity of the classroom and the small group with teacher is noted and the possibility of the adequacy of the small group with teacher in simulating the classroom situation for laboratory purposes receives some support. In contrast the remaining subgroups differ quite sharply from the classroom patterns, particularly in Process and in Attitude. The individual working alone is perhaps of most interest: while all the subjects of this study show remarkable persistance, when working alone the number of ideas stated which might stimulate association of desired conclusion with the established structure is quite small; the checking of false starts by teacher or fellow student is lacking; and rarely are clear-cut systematic procedures of attack noted. Procedures are generally limited to dissecting the problem, considering remembered analogous problems, and, particularly in the novel problems, to trial and error. Similar though less marked distinctions between individuals and small groups or small groups with teacher are noted. The great similarity of individual with teacher and individual patterns emphasizes the small amount of assistance asked for by the pupils. These considerations permit inference of group and classroom factors in addition to a simple combination of effects that would be expected by bringing several individuals together.

INTRODUCTION

Studies with the Wright-Proctor Instrument to date have been concerned only with patterns of verbal behaviors shown in discussion in the entire mathematics classroom. Because the individual pupil is the ultimate concern in the classroom learning situation it seemed important to consider how the patterns dominant in the discussion periods of the entire classroom were related to those which occur in individual work. As illustrated in Chapter II from both individual frame and triple frame scores, pupils activities in all types of classrooms concentrated on solution of specific problems--in some types to the exclusion of most other behaviors. Typically then individual work will be directed to application of principles developed in classroom discussion and to exploring fairly carefully limited novel problems suggested by this discussion. The individual pupil may solve problems alone, with the aid of the teacher, in small groups of pupils and in small groups with the aid of the teacher. This study was limited then to comparison of verbal behaviors occurring during problem solving in the subgroups typical in the classroom.

It was considered likely that when the teacher was present even though not actively participating in the solution of problems his presence would affect certain aspects of pupils behaviors. For instance in the teacher's presence the pupil might show greater persistance or might attack a problem more systematically.

Also, it was possible that a small group of four pupils and teacher might simulate the situation in the entire classroom where discussion generally takes place among the teacher and a few pupils at a time. It has been noted that the number of pupils involved in the discussion of a particular problem or part of a problem varies from two to five. Close agreement of patterns in the entire classroom and in the small group of pupils with teacher would permit consideration of the use of the small group with teacher for laboratory studies of variables diffiult to manipulate in the classroom. If lose agreement should occur between class nd small group with teacher in this study hich is limited to the problem solving spect of classroom activities, then conideration of agreement of teacher and upils behaviors in the variety of aspects f the lesson taught in a classroom of ome twenty pupils and in a small group of ay 4-6 pupils might lead to establishment f an appropriate "laboratory" class for lose study of verbal discussion.

METHOD

aterials used

Subjects

The subjects for the study were teacher nd twenty-five pupils of high school eometry classroom in a suburban district f St. Louis. This classroom was in the irst track of a three track geometry proram at the junior level. Pupils were atched in fours on the basis of school chievement in combined algebra grades in irst two courses (A), scores on the Caliornia Algebra Aptitude (CA), present chool achievement in geometry (G), general ptitude (HN-Henmon Nelson) and sex (F or M). our teams of four were selected for the bgroups: small group and teacher, small oup, individual and teacher, and individual, facilitate the assignment of problems irly. The teacher and all twenty-five pils made up the subgroup: entire classroom. The teacher, a woman, was experienced in teaching all mathematics topics at the high school level, had a thorough preparation in undergraduate mathematics and had recently completed a year's study in mathematics, science and mathematics education in an Academic Year Institute supported by the National Science Foundation. The composition of the teams of pupils was summarized in Table 106.

Problems

Eight comparable problems of two parts each were prepared so that part A was concerned with direct application or modification of class work in geometry, and part B while novel was amenable to treatment by pupils of this class. Because it was necessary to equate these eight problems, the following procedure for selection and judgment was used.

From within the subject matter to be covered in a two week period the teacher selected eight first parts which she considered comparable in difficulty. One investigator selected the eight second parts. These two problem sets A_{1-8} and B_{1-8} were given to a mathematics club in another high school whose members were students in their senior year. Seven club members were chosen as judges for set A, and seven others as judges for set B. Using A_1 and B_3 as model problems, each judge solved the model problem and one other. Group discussion of the eight problem solutions prepared the judges for individual rating of the eight problems of their set. With the model as 0, easier problems were rated -2, -1 and more difficult problems 1,2. The ratings were converted to ranks for each judge and comparison of the standards used by the seven judges in each team was made by the Kendall Coefficient of Concordance W. (Siegel, 1956, pp.229-238). W for set A was 0.46 with X^2 equal to 22.54, significant beyond $p = 0.01$. W for set B was 0.59 with X^2 equal to 28.91 significant beyond $p = 0.001$. This indicated that the judges were using essentially the same standard in making their rankings, and justified a selection of the mean ratings

Table 106 Scores of four teams of four pupils each, matched on school achievement in algebra (A), California Algebra Aptitude (CA), school achievement in geometry (G), general aptitude (HN) and sex.

Pupil	Team	A	CA	G	HN	Sex
a	I	98	90-99	96	166	F
	II	100	88-99	96	140	M
	III	98	92-99	99	161	F
	IV	97	92-98	99	162	M
b	I	95	85-99	91	142	M
	II	89	86-95	92	136	F
	III	98	89-100	93	138	M
	IV	95	89-100	90	145	M
c	I	95	81-99	83	130	M
	II	90	80-85	86	127	F
	III	95	87-95	82	130	F
	IV	91	82-99	82	138	F
d	I	94	81-99	98	132	F
	II	98	85-95	78	128	M
	III	91	86-95	81	136	F
	IV	93	88-99	96	140	F
mean	I	95.5	81-99	92.0	142.5	n.b. teams rather
s.d.		1.5		6.6	14.3	than individual
mean	II	94.3	80-99	88.0	143.8	matching on sex;
s.d.		4.8		4.8	12.3	teams, I, II, IV:
mean	III	95.5	86-100	89.0	141.3	FFMM; Team III:
s.d.		2.9		7.5	11.8	FFFM.
mean	IV	94.0	82-100	91.8	146.3	
s.d.		2.2		6.5	9.4	
			outside range in CA			

as the best value in seeking an estimate of the "true" ratings. In set A, only problems A_5, A_6 and A_7 differed from the zero rating of the model: -8/7, -11/7, 5/7 respectively. Adjustments were made to bring these more into line. In set B all problems but B_5 were rated somewhat easier than the model B_3. The model was written more clearly and B_5 was adjusted. The revised set of problems is included in Tables 108-110.

Administration

Problems 1_{AB}, 2_{AB}, 3_{AB}, 4_{AB} were preented for class discussion during two eriods in two successive weeks as the eveloping theory required application in he A parts. This occurred during the eeks of November 17 and 24, 1960. While iscussion was recorded on tape, details of lackboard development were carefully noted. he remaining four problems 5_{AB}, 6_{AB}, 7_{AB}, $_{AB}$ were presented to the other subgroups ccording to the plan outlined in Table 111a. his occurred in two half-hour periods fter school during the week of December 1 s indicated in Table 111b.

Subsequently typescripts of these apes were obtained and classification rom the typescripts made by the two obervers. Where disagreements occurred, he issue was resolved and a final "best lassification" was obtained. The indiidual frame and triple frame scores from he "best classification" of the typecripts provided the materials for comarison of the several subgroups.

nalysis

Approximately 240 behaviors were obained for consideration of each subgroup. n the classroom, the four problems 1_{AB}, $_{AB}$, 3_{AB} and 4_{AB} required about fifteen mintes of discussion for each part. Thus a otal of 120 minutes of discussion or 240 ehaviors occurred. In the subgroups "small roup and teacher", and "small group", the 40 behaviors were derived in a similar way. o avoid possible bias in problem selection he sixteen pupil subjects all were included n the "individual and teacher" and "individual" subgroups; thus 960 behaviors were obtained from which a mean team result was determined to provide the 240 behaviors for the two subgroups. These were recorded as individual frame and triple frame scores and the former were illustrated in bar graphs.

Comparison of subgroups was begun by assessment of the amount of time spent in discussion and in silent behaviors classified as supervised study. For this purpose proportions of the non-verbal behaviors in the total number were calculated with corresponding standard deviations. The null hypothesis of no significant difference in emphasis of supervised study across the five subgroups was tested by means of chi square in the sense of goodness of fit. If rejected, this was to be followed by comparison of the subgroups in pairs to determine the occurrence of the significant difference.

In Chapter II, it became apparent that one of the best ways to gain a comprehensive view of the classroom types for clear distinction was by reference to the triple frame scores; another required a broad consideration of single frame scores in each frame. Accordingly in study of the several subgroups our attention was first directed to rankings of the triple frame scores. For each of the five subgroups these scores were ranked for classroom and for pupils scores. In each subgroup where the teacher behaviors occurred in sufficient quantity these scores also were ranked. The following comparisons were made:

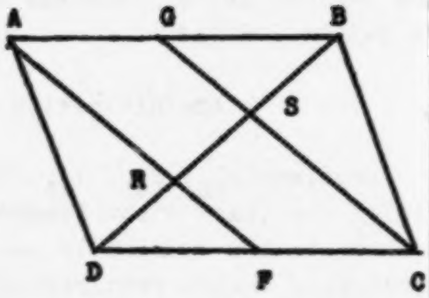
I. By the Kendall Coefficient of Concordance, W,
  i. classroom and pupils scores--for entire set of five subgroups
  ii. pupils scores--for the four laboratory subgroups
  iii. classroom and pupils scores--for the three subgroups with the teacher included
II. By the Spearman Coefficient of Ranked Correlation, r_s,
  i. classroom and pupils scores, and where possible teacher scores--of the ten pairs of subgroups.

Table 108 Revised problem set A

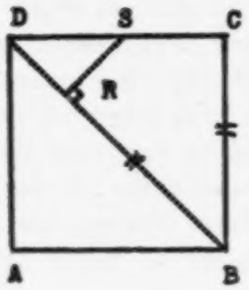
CLASS A_1: Construct triangle ABC, given AB, AC and the median onto AB.

A_2: Construct a right triangle, given the hypothenuse and the sum of the other two sides.

A_3: Given a parrallelogram ABCD with the mid-points G and F of AB and CD, respectively, AF and CG are joined, the diagonal BD cuts AF at R and GC at S. Prove that RS equals SB.



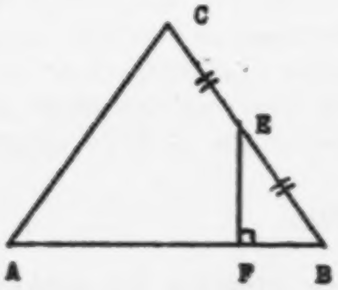
A_4: Given a square, ABCD, and a point R on the diagonal BD so that RB equals CB; further given S on DC so that SR is perpendicular to DB. Prove that DR, RS, and SC are all equal.



OTHER SUBGROUPS A_5: Construct a triangle, having given one side, c, the altitude to side a, h_a, and the sum of the two angles B and C.

A_6: Construct a triangle, having given the bisector of angle A, t_A.

A_7: Given a triangle ABC with AC equal to BC. On BC a point E is chosen so that BE equals EC. EF is drawn perpendicular to AB. Prove that FB equals one-quarter (1/4) the length of AB.



A_8: If the mid-points of the sides of a quadrilateral, taken in order, are joined, a parallelogram is formed within a perimeter equal to the sum of the diagonals of the original quadrilateral. If you establish this, can you also show that the perimeter of the parrallelogram is equal to one-half that of the original quadrilateral?

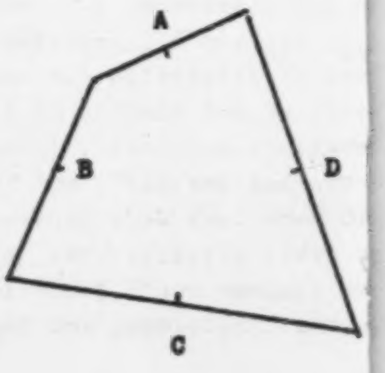
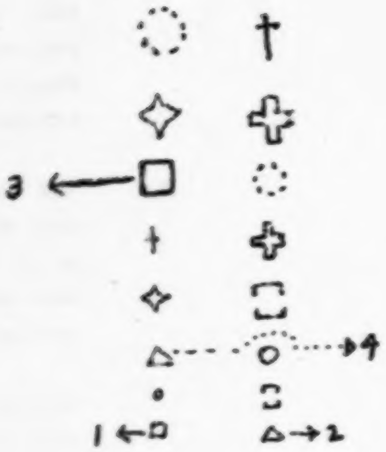


Table 109 Revised problem set B

LASS **B_1:** This can be regarded as an arrangement of shapes which changes by a series of steps into another arrangement of the same shapes. Four shapes are moved into their new positions 1, 2, 3, 4 in that order as indicated by the dotted lines. Consider the evidence given in the moves indicated and continue the process by steps until a satisfying final arrangement is reached. What principles do you think are involved?



B_2: Three persons are to divide among themselves 21 equal casks, of which 7 are full, 7 are half full and 7 are empty. How can an equable division be made without pouring the wine from any cask into any other? Each person should receive the same amount of wine and the same number of casks. Is there only one solution?

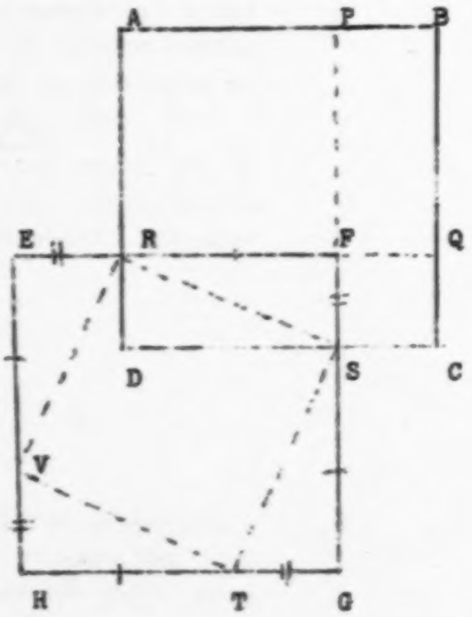
B_3: Given a substitution operation, a, in which A → B, B → C, C → A is written concisely: $a = \begin{pmatrix} ABC \\ BCA \end{pmatrix}$

Further given five other comparable operations:

$b = \begin{pmatrix} ABC \\ CBA \end{pmatrix}$; $c = \begin{pmatrix} ABC \\ BAC \end{pmatrix}$; $d = \begin{pmatrix} ABC \\ CAB \end{pmatrix}$; $e = \begin{pmatrix} ABC \\ ACB \end{pmatrix}$; $1 = \begin{pmatrix} ABC \\ ABC \end{pmatrix}$

Show that d followed by e is not the same as e followed by d. Would this be true for all pairs chosen from the six operations?

B_4: Given equal squares ABCD and EFGH overlapping as indicated, so that FS = GT = HV = ER and RF // DS, show that the area of RSTV = area of APFR + FQCS.



THER UBGROUPS **B_5:** Change the position of the items in the design along the lines indicated in arrangement 1, 2, e, until the final arrangement #7 is obtained. What modification of this problem might you suggest?

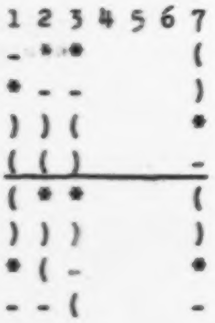
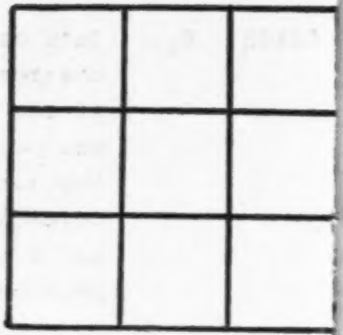


Table 110 Revised problem set B, continued

B_6: Place the numbers 1, 2, 3, 4, 5, 6, 7, 8, 9 once and only once in the 3 x 3 square so that each row, column and diagonal has the same sum 15. If you solve this, state a somewhat related problem. Would you have certain principles in mind in attempting to solve it?



B_7: The use of the accompanying addition table for the set of four numbers a, b, c and d may be illustrated: a a a; b c d, etc. Note that a unique member of the set may be inserted in the square to make the following statement true.

△c + □ = ⓑ

Further note that regardless of the pair of numbers chosen for the triangle and circles the above assertion may be made.

+	a	b	c	d
a	a	b	c	d
b	b	c	d	a
c	c	d	a	b
d	d	a	b	c

Consider: From a close study of the multiplication table for these same numbers, can you decide that a unique member of the set may be inserted in the square to make the following statement true?

△c + □ = ⓑ

If any other pair of numbers replaces c and b, may a unique member of the set be inserted in the square to make the statement true? Discuss any variation you see.

x	a	b	c	d
a	a	a	a	a
b	a	b	c	d
c	a	c	a	c
d	a	d	c	b

B_8: A staircase is 60 steps high. Each step is 1 foot deep and rises 1 foot. A carpenter plans to place wood tiles one foot square completely covering the wall on the side of the staircase. How many tiles would he need? Could you solve this for any number of steps?

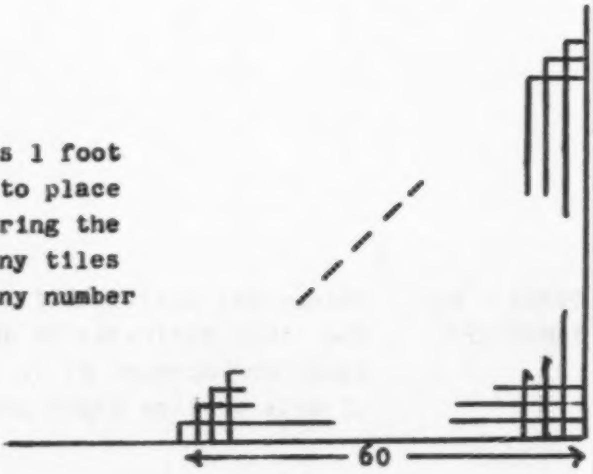


Table 111a **Plan of presentation of problems 5-8 by teams**

Problem	**Small group & Teacher**	**Small group**	**Individual & Teacher**	**Individual**
5_{AB}	**Team IV**	**III**	**II**	**I**
6_{AB}	**III**	**II**	**I**	**IV**
7_{AB}	**II**	**I**	**IV**	**III**
8_{AB}	**I**	**IV**	**III**	**II**

Table 111b **Administration of problems 5-8 by teams**

Time	**Period**	**Equipment**	**Subgroup**	**Day 1**	**Day 2**	**Day 3**	**Day 4**
3.10	**1**	**4 tapes**	**Individual**	**I,5**	**IV,6**	**III,7**	**II,8**
		conference	**Group & T**	**IV,5**	**I,8**	**II,7**	**III,6**
3.45	**2**	**4 tapes**	**Indiv. & T**	**IV,7**	**I,6**	**II,5**	**III,8**
		conference	**Group**	**I,7**	**IV,8**	**III,5**	**II,6**

I,5 specifies Team I, assignment 5_{AB}

The individual frame scores were then compared by the chi square test in all the cases listed above for triple score assessment. This test was applied across each frame of Content, Process and Attitude. If this test appeared too stringent when applied across each frame or if differences required further pin-pointing, consequent tests would be carried out category by category as had been necessary in Chapter II.

RESULTS

The individual frame scores in categories of Content, of Process and of Attitude, and in Neutral for classroom, teacher and pupil components were recorded for the five subgroups in Table 113. These were illustrated in Figures 114-17. The triple frame scores were recorded in classroom, teacher and pupil components in Tables 118-20. In addition rankings of the major triples were recorded in Tables 121, 122.

The proportions of non-verbal behaviors in the total determined for each subgroup were of the level 5/100 in the class, 8/100 in the small group with teacher and the small group, with a sharp rise to 22/100 for individual with teacher or individual.

	P	s.d.
Class	0.047	0.014
Group & Teacher	0.082	0.018
Group	0.076	0.017
Individual & Teacher	0.218	0.014
Individual	0.225	0.014

These proportions were shown to differ significantly, $p < 0.001$ across the five subgroups. However neither in the first three subgroups of the above list, nor in the last two subgroups were the proportions of non-verbal behaviors significantly different, $p = 0.05$, Table 123a.

While noting the distinction of the first three subgroups from the last two in non-verbal behaviors, the verbal scores were standardized in each subgroup to a total of 240 to avoid bias toward any particular subgroup in subsequent analysis.

The significant differences noted in the broad comparisons of triple frame scores or of individual frame scores across five, four and three subgroups were recorded in Table 123b. No differences in rankings were observed across the entire five subgroups, across the four laboratory subgroups, nor across the three subgroups including the teacher. On the other hand, the individual frame scores did indicate some distinctions. Across the five subgroups, significant differences were noted in (T & P) and (P) scores in Process and (T & P) scores in Attitude. This distinction in Process occurred throughout all other comparisons. In addition, the four laboratory subgroups differed in category emphasis in Content. Finally both (T & P) and (P) scores in the categories of Attitude differed across the three subgroups with the teacher present.

Clear-cut pin-pointing of the differences apparent in individual frame scores was noted in Table 124. In addition distinctions which do occur between pairs of subgroups in triple frame scores were evident. Comparisons of each laboratory subgroup with the classroom indicated where most of the differences occurred. In small group and individual subgroups only pupil scores were available. These were compared with (T & P) scores as well as with (P) scores in several comparisons and included in the table under (T & P) as well as (P) columns.

i. The small groups with teacher was like the classroom in (T & P) and (T) triples but differed in (P) triples: alike in all content and Process scores; alike in (T) Attitude but differed so sharply in (P) Attitude that the (T & P) scores were also significantly different. Only in this comparison were (T) scores large enough to be considered separately.

ii. The small group differed from the classroom in (T & P) and (P) triples, Process and (T & P) Attitude. The two groups were alike in (T & P) and (P) Content and in (P) Attitude.

Table 113 Individual frame scores of the five subgroups in all categories of Content, of Process and of Attitude and in Neutral. Classroom, teacher pupil components; adjusted scores.

	CLASS			GROUP & TEACHER			GROUP	INDIV. & TEACHER			INDIV.
Content	**(T&P)**	**T**	**P**	**(T&P)**	**T**	**P**	**P**	**(T&P)**	**T**	**P**	**P**
S	6	3	3	3	1	2	6	3	0	3	2
T	30	26	4	33	12	21	31	28	2	26	32
D	11	9	2	3	1	2	4	3	0	3	6
I	0	0	0	5	0	5	12	0	0	0	0
M	185	116	69	195	42	153	186	204	10	194	197
O	5	5	0	1	0	1	1	2	0	2	3
Process											
A	12	9	3	11	3	8	16	6	1	5	6
Sy.	52	30	22	41	9	32	28	35	8	27	32
Sp.	127	80	47	124	23	101	138	152	3	149	153
G	11	10	1	19	6	13	18	6	0	6	7
R	35	30	5	45	15	30	40	41	0	41	42
Attitude											
C	8	6	2	7	4	3	4	4	0	4	2
I	159	92	67	202	29	173	220	216	6	210	227
R	70	61	9	31	23	8	16	20	6	14	11
Neutral	3	3	--	--	--	--	--	--	--	--	
Totals	240	162	78	240	56	184	240	240	12	228	240

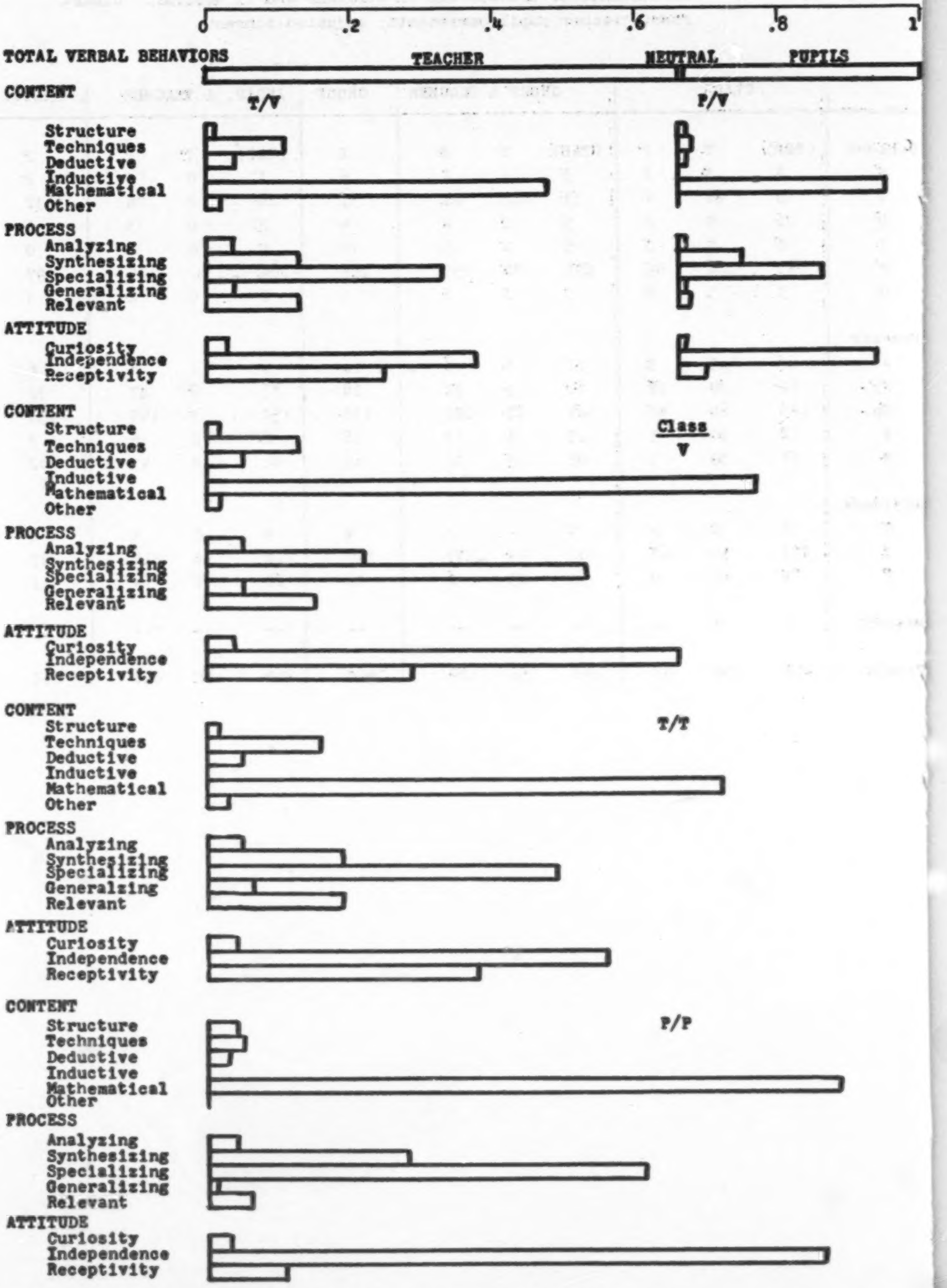


Figure 114 Subgroup comprising entire class. Individual frame scores in Content, Process and Attitude for classroom (T&P)/V, teacher T/V, pupil P/V; and teacher proportions T/T, pupil proportions P/P.

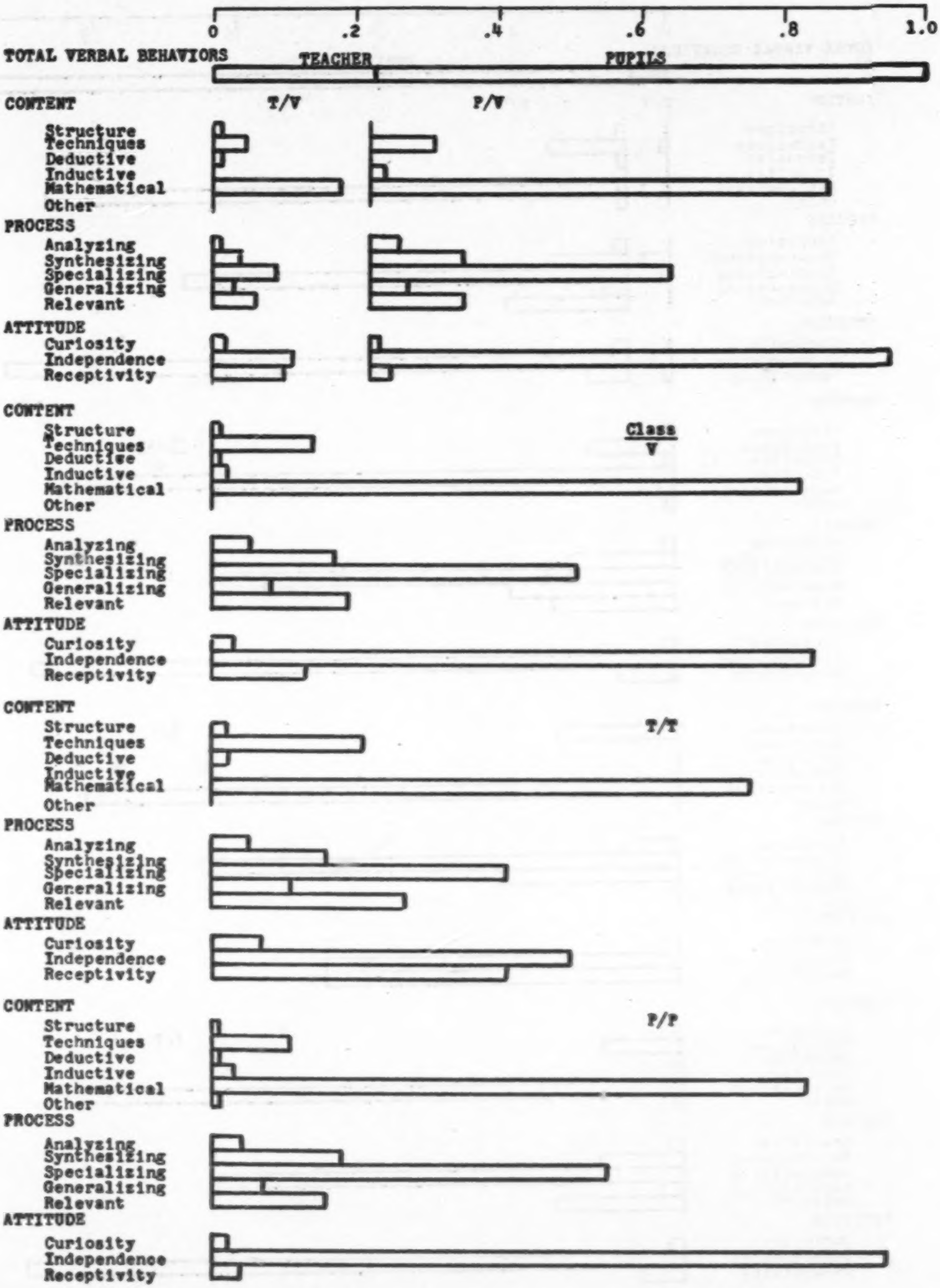


Figure 115 Subgroup comprising small group with teacher. Individual frame scores in Content, Process and Attitude for classroom (T&P)/V, teacher T/V, pupil, P/V; and teacher proportions T/T, pupil proportions P/P.

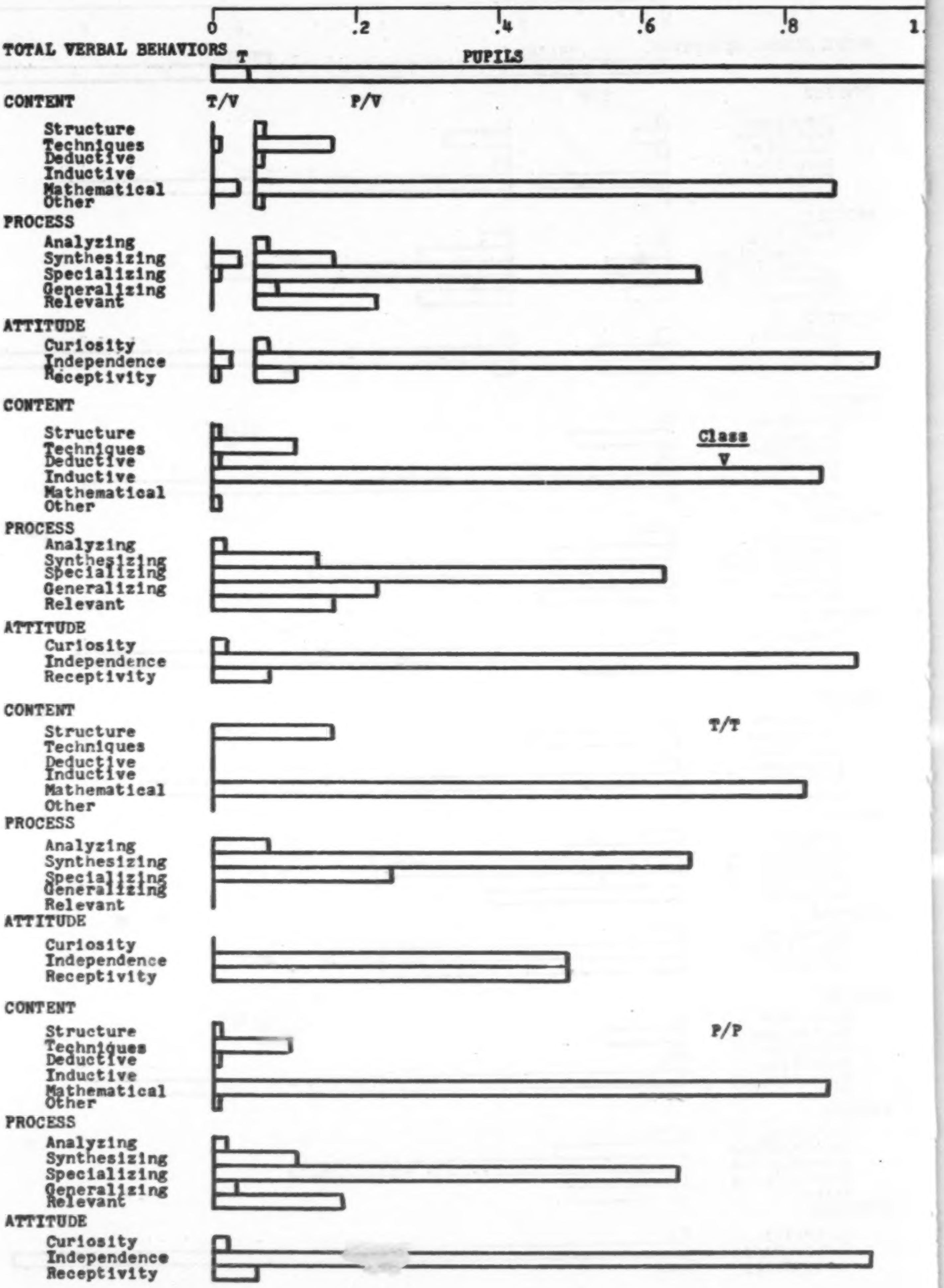


Figure 116 Subgroup comprising individuals with teacher. Individual frame scores in Content, Process and Attitude for classroom (T&P)/V, teacher T/V, pupil P/V; and teacher proportions T/T, pupil proportions P/P.

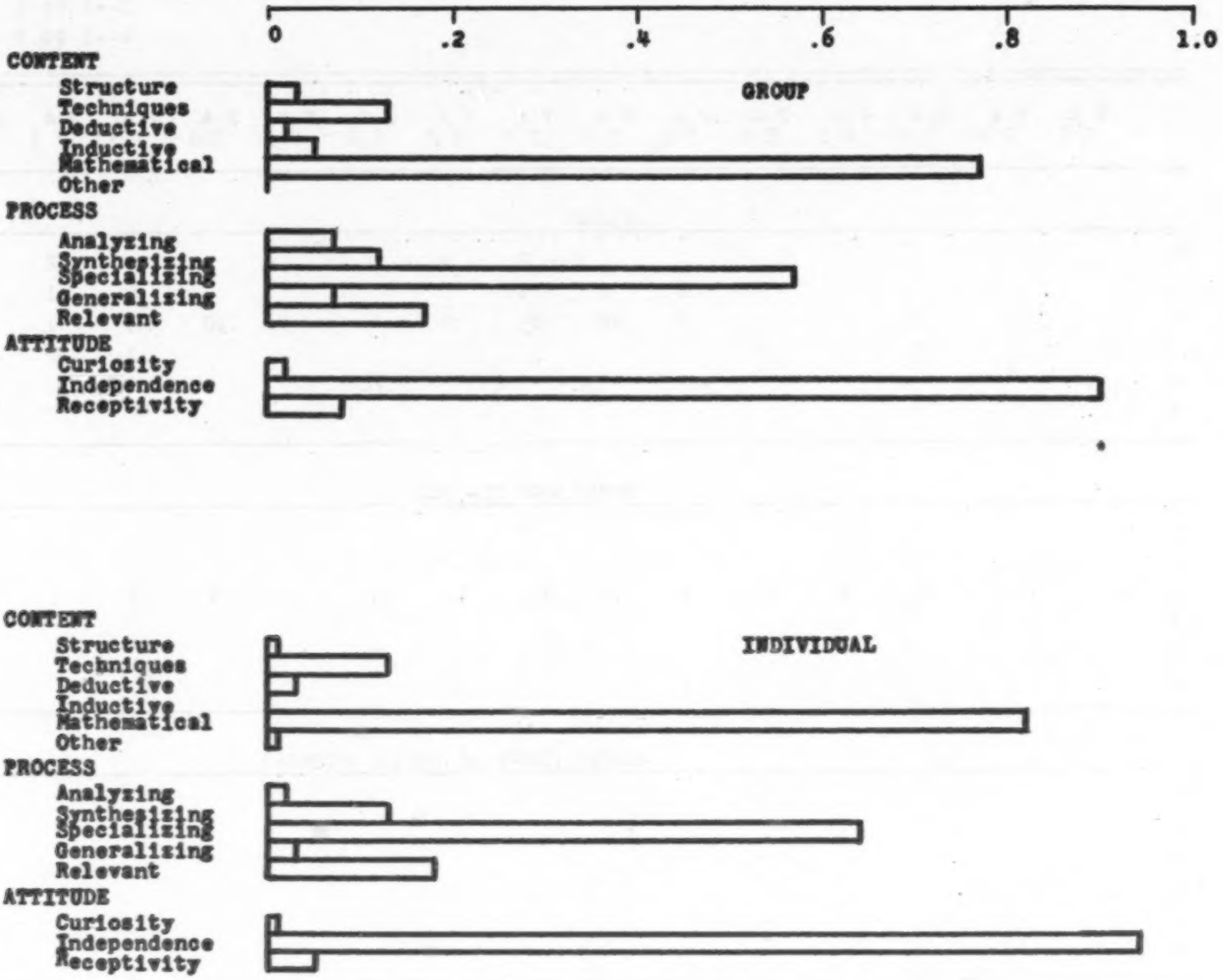


Figure 117 Subgroups comprising small groups and individuals. Individual frame scores in Content, Process and Attitude for classroom (T&P)/V, teacher not participating, pupil equivalent to classroom scores.

Table 118 Triple frame classification of the five subgroups in Content, Process and Attitude categories. Classroom adjusted scores (T & P). Negatives disregarded. Symbolically: $C_iP_jA_k$ i--1 to 6
j--1 to 5
k--1 to 3

CLASS

	P A 1 1	P A 2 1	P A 3 1	P A 4 1	P A 5 1	P A 1 2	P A 2 2	P A 3 2	P A 4 2	P A 5 2	P A 1 3	P A 2 3	P A 3 3	P A 4 3	P A 5 3
C_1		1						2	1					2	
2						2	1	3					4	1	
3			3			7	40	85	6		3	10	27	1	3
4			2		2								1		
5										12					18
6															

GROUP AND TEACHER

	P A 1 1	P A 2 1	P A 3 1	P A 4 1	P A 5 1	P A 1 2	P A 2 2	P A 3 2	P A 4 2	P A 5 2	P A 1 3	P A 2 3	P A 3 3	P A 4 3	P A 5 3
C_1								2			1				
2								3							
3			3	2	1	9	37	101	11	10		4	15	1	1
4				1											
5										24					9
6						1			4						

GROUP (SAME AS PUPILS SCORES)

	P A 1 1	P A 2 1	P A 3 1	P A 4 1	P A 5 1	P A 1 2	P A 2 2	P A 3 2	P A 4 2	P A 5 2	P A 1 3	P A 2 3	P A 3 3	P A 4 3	P A 5 3
C_1															
2															
3															
4															
5															
6															

INDIVIDUAL AND TEACHER

	P A 1 1	P A 2 1	P A 3 1	P A 4 1	P A 5 1	P A 1 2	P A 2 2	P A 3 2	P A 4 2	P A 5 2	P A 1 3	P A 2 3	P A 3 3	P A 4 3	P A 5 3
C_1						1		1	1						
2							1	1	1						
3			3			4	26	146	4	14	1		4		2
4								2							
5					1					14					13
6															

INDIVIDUAL (SAME AS PUPILS SCORES)

	P A 1 1	P A 2 1	P A 3 1	P A 4 1	P A 5 1	P A 1 2	P A 2 2	P A 3 2	P A 4 2	P A 5 2	P A 1 3	P A 2 3	P A 3 3	P A 4 3	P A 5 3
C_1															
2															
3															
4															
5															
6															

Table 119 Triple classification of the three pertinent subgroups in Content, Process and Attitude categories. Teacher adjusted scores. Negatives disregarded. Symbolically: $C_iP_jA_k$

i--1 to 6
j--1 to 5
k--1 to 3

	P_1A_1	P_2A_1	P_3A_1	P_4A_1	P_5A_1	P_1A_2	P_2A_2	P_3A_2	2	P_5A_2	P_1A_3	P_2A_3	P_3A_3	P_4A_3	P_5A_3
								CLASS							
C_1									1					2	
2						2		2					4	1	
3			2			4	20	48	5		3	10	21	1	2
4			2		2								1		
5										10					16
6															
								GROUP AND TEACHER							
C_1											1				
2								1							
3			2	1	1	2	5	7	4	1		4	13	1	1
4															
5										9					3
6															
								INDIVIDUAL AND TEACHER							
C_1															
2															
3								5			1		3		1
4															
5										1					1
6															

Table 120 Triple classification of the five subgroups in Content, Process and Attitude categories. Pupils adjusted scores. Negatives disregarded. Symbolically: $C_iP_jA_k$ i--1 to 6
j--1 to 5
k--1 to 3

	P_1A_1	P_2A_1	P_3A_1	P_4A_1	P_5A_1	P_1A_2	P_2A_2	P_3A_2	P_4A_2	P_5A_2	P_1A_3	P_2A_3	P_3A_3	P_4A_3	P_5A_3
								CLASS							
C_1		1						2							
2							1	1							
3			1			3	20	37	1				6		1
4															
5										2					2
6															
								GROUP AND TEACHER							
C_1								2							
2								2							
3			1	1		7	32	94	7	9			2		
4				1											
5										15					6
6						1			4						
								GROUP							
C_1				2		1		3							
2								1	2	1					
3						14	26	130	8	8					
4															1
5								1		15					15
6			2			1	2	1	6						
								INDIVIDUAL AND TEACHER							
C_1						1		1	1						
2							1	1	1						
3			3			4	26	141	4	14			1		1
4								2							
5					1					13					12
6															
								INDIVIDUAL							
C_1						1		1							
2						1	1	2	2						
3				1		4	31	147	3	10			1		
4				1				2							
5										22					10
6															

Table 121 First ten ranks of Classroom behaviors in Content, Process and Attitude in the several classroom subgroups: class; small group with teacher; small group; individual with teacher; individual. Numbers indicate frequencies per 240

Class	Group & Teacher	Group	Indiv. & Teacher	Indiv.
Mathematical Specializing Independence	101 Mathematical Specializing Independence	130 Mathematical Specializing Independence	145 Mathematical Specializing Independence	147 Mathematical Specializing Independence
Mathematical Synthesizing Independence	37 Mathematical Synthesizing Independence	26 Mathematical Synthesizing Independence	26 Mathematical Synthesizing Independence	31 Mathematical Synthesizing Independence
Mathematical Specializing Receptivity	24 Techniques Relevant Independence	15 Techniques Relevant Independence	14 Mathematical Relevant Independence	22 Techniques Relevant Independence
Techniques Relevant Receptivity	15 Mathematical Specializing Receptivity	15 Techniques Relevant Receptivity	14 Techniques Relevant Independence	10 Mathematical Relevant Independence
Techniques Relevant Independence	11 Mathematical Generalizing Independence	14 Mathematical Analyzing Independence	13 Techniques Relevant Receptivity	10 Techniques Relevant Receptivity
Mathematical Synthesizing Receptivity	10 Mathematical Relevant Independence	8 Mathematical Generalizing Independence	4 Mathematical Analyzing Independence	4 Mathematical Analyzing Independence
Mathematical Analyzing Independence	9 Mathematical Analyzing Independence	8 Mathematical Relevant Independence	4 Mathematical Specializing Receptivity	3 Mathematical Generalizing Independence
Mathematical Generalizing Independence	9 Techniques Relevant Receptivity	0 Mathematical Synthesizing Receptivity	4 Mathematical Generalizing Independence	1 Mathematical Specializing Receptivity
Mathematical Relevant Independence	4 Mathematical Synthesizing Receptivity	0 Mathematical Specializing Receptivity	0 Mathematical Synthesizing Receptivity	0 Mathematical Synthesizing Receptivity

Table 122 First ten ranks of Pupils behaviors in Content, Process and Attitude in the several classroom subgroups: class; small group with teacher; small group; individual with teacher; and individual. Numbers indicate frequencies per 240

Class	Group & Teacher	Group	Indiv. & Teacher	Indiv.
37 Mathematical Specializing Independence	94 Mathematical Specializing Independence	130 Mathematical Specializing Independence	141 Mathematical Specializing Independence	147 Mathematical Specializing Independence
20 Mathematical Synthesizing Independence	32 Mathematical Synthesizing Independence	26 Mathematical Synthesizing Independence	26 Mathematical Synthesizing Independence	31 Mathematical Synthesizing Independence
6 Mathematical Specializing Receptivity	15 Techniques Relevant Independence	15 Techniques Relevant Independence	15 Mathematical Relevant Independence	22 Techniques Relevant Independence
3 Mathematical Analyzing Independence	9 Mathematical Relevant Independence	15 Techniques Relevant Receptivity	13 Techniques Relevant Independence	10 Mathematical Relevant Independence
2 Structure Specializing Independence	8 Mathematical Generalizing Independence	14 Mathematical Analyzing Independence	12 Techniques Relevant Receptivity	10 Techniques Revelant Receptivity
2 Techniques Relevant Independence	7 Mathematical Analyzing Independence	8 Mathematical Generalizing Independence	4 Mathematical Analyzing Independence	8 Deductive Specializing Independence
2 Techniques Relevant Receptivity	6 Techniques Revelant Independence	8 Mathematical Revelant Independence	4 Mathematical Generalizing Independence	4 Mathematical Analyzing Independence
1 Deductive Specializing Independence	2 Structure Specializing Independence	3 Structure Specializing Independence	1 Structure Specializing Independence	3 Mathematical Generalizing Independence
1 Mathematical Generalizing Independence	2 Deductive Specializing Independence	1 Deductive Specializing Independence	1 Deductive Specializing Independence	1 Structure Specializing Independence
1 Mathematical Relevant Independence	2 Mathematical Specializing Receptivity	0 Mathematical Specializing Receptivity	1 Mathematical Specializing Receptivity	1 Mathematical Specializing Receptivity

Table 123a Comparison Supervised Study.
Similarity of groups noted, p = 0.05.

5 groups	No
Class Small group & T. Small group	Yes
Individual & T. Individual	Yes

Table 123b Comparison of triple frame and individual frame scores* across several subgroups: (1) all five (2) the four other than the class (3) all three with teacher a member. Similarity of groups noted, p = 0.05.

	Five Subgroups		Four lab. Subgroups	Three subgroups with teacher	
	(T&P)	P	P	(T&P)	P
Triple	Yes	Yes	Yes	Yes	Yes
Content	Yes	Yes	No	Yes	Yes
Process	No	No	No	No	No
Attitude	No	Yes	Yes	No	No

---

* Probability levels recorded in Appendix A, Table 233

Table 124 Comparison triple frame and individual frame scores* of subgroups taken in pairs (10 comparisons) p = 0.05.

		Class			Small group & T		Small group		Indiv. & T	
		T&P	T	P	T&P	P	T&P	P	T&P	P
Small Group & T	Triple	Yes	Yes	No						
	C	Yes	Yes	Yes						
	P	Yes	Yes	Yes						
	A	No	Yes	No						
Small Group	Triple	No		No	Yes	Yes				
	C	Yes		Yes	Yes	Yes				
	P	No		No	Yes	Yes				
	A	No		Yes	Yes	Yes				
Individual & T	Triple	No		No	Yes	Yes	Yes	Yes		
	C	No		No	Yes	Yes	No	No		
	P	Yes		No	No	No	No	No		
	A	No		Yes	Yes	Yes	Yes	Yes		
Individual	Triple	No		No	Yes	Yes		Yes	Yes	Yes
	C	Yes		No	Yes	Yes		Yes	Yes	Yes
	P	No		No	No	Yes		No	Yes	Yes
	A	No		No	No	Yes		Yes	Yes	Yes

---

* Probability levels recorded in Appendix A, Table 232
Yes indicates similarity

1. Comparison of subgroup individual with teacher and classroom showed additional differences. Only in (T & P) Process and in (P) Attitude were they alike.

. For class and individual subgroups duly in the (T & P) comparison in Content were these two subgroups alike.

Comparison of the small group with eacher with the other laboratory groups enerally indicated similarity of behaviors.

. All comparisons between small groups were alike.

. Comparisons between small group with teacher and individual with teacher were alike in all cases but the Process individual frame scores.

. The small group with teacher resembled the individual except in (T & P) Process and Attitude.

While the small group

. was like the individual with teacher in the triples and in the Attitude, clear distinction was apparent in Content and Process.

. On the other hand the small group resembled the individual in the triples and the Content but differed in Process and Attitude.

Finally, similarity was apparent in ll aspects of individuals with teacher and ndividuals comparison.

Details of the single category levels, ecorded in Table 113 were compared in igures 126-9. In addition excerpts from tape ecordings--one for each subgroup--are inluded. Tables 130-134.

Discussion and Inferences

The first distinction apparent among the ive subgroups was the greater continuity verbal behaviors in the classroom, small oup with teacher and small group over the o subgroups where the individuals were reonsible singly for development. Both the esence of the teacher--in a situation where she felt drawn to keep discussiom going--and the greater variety of ideas available from the classroom or the team contributed to the fewer pauses in the first three subgroups. While the teacher presence might have affected some greater continuity in individual with teacher over individual, because the teacher in this study gave assistance only when asked, pressure from teacher was not comparable with that in classroom or small group. In the change in proportion of supervised study from classroom to small group with teacher, it seemed that teacher pressure was somewhat less; indeed no difference was apparent between small group with teacher and small group. Altogether, the teacher presence seemed most affective in the classroom; otherwise the continuity of verbal behaviors seemed to vary with the availability of a variety of ideas.

Following standardization of verbal behavior totals to 240 to avoid undue emphasis of differences because of long silent periods in individual subgroups, the comparison of triples and individual frame scores again permitted consideration of the three subgroups: classroom, small group with teacher and small group from the two individual subgroups. The clear distinction of the non-verbal behaviors' comparison was no longer apparent.

The major distinction observed was between the classroom and three subgroups other than the small group with teacher. Because of the similarity of the patterns in the classroom and small group with teacher in this problem solving aspect, future studies considering patterns resulting from complete lessons should be planned in working towards a convenient "laboratory classroom". The differences observed were all related to Attitude. It was quite apparent from general observation that the teacher was following her usual role in the classroom subgroup, but in the small group entered less often into the discussion, volunteered fewer suggestions and encouraged pupil activity at each step. We had purposely offered no instruction such as "Act just as you would

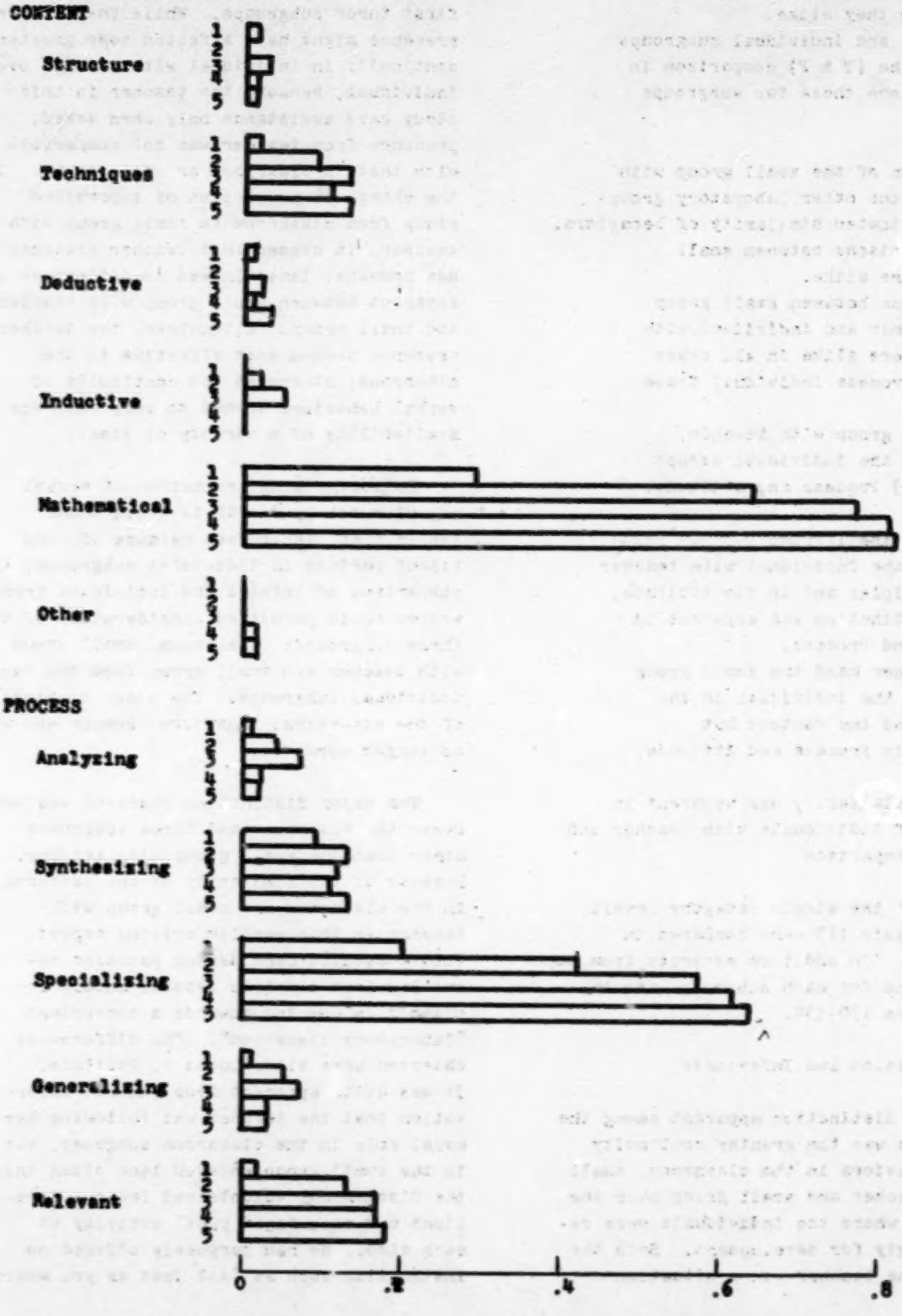


Figure 126 Comparison of single categories in pupils scores, Pupil/ Verbal, in the five classroom subgroups: class (1), subgroup with teacher (2), subgroup (3), individual with teacher (4) and individual (5). Content and Process frames only.

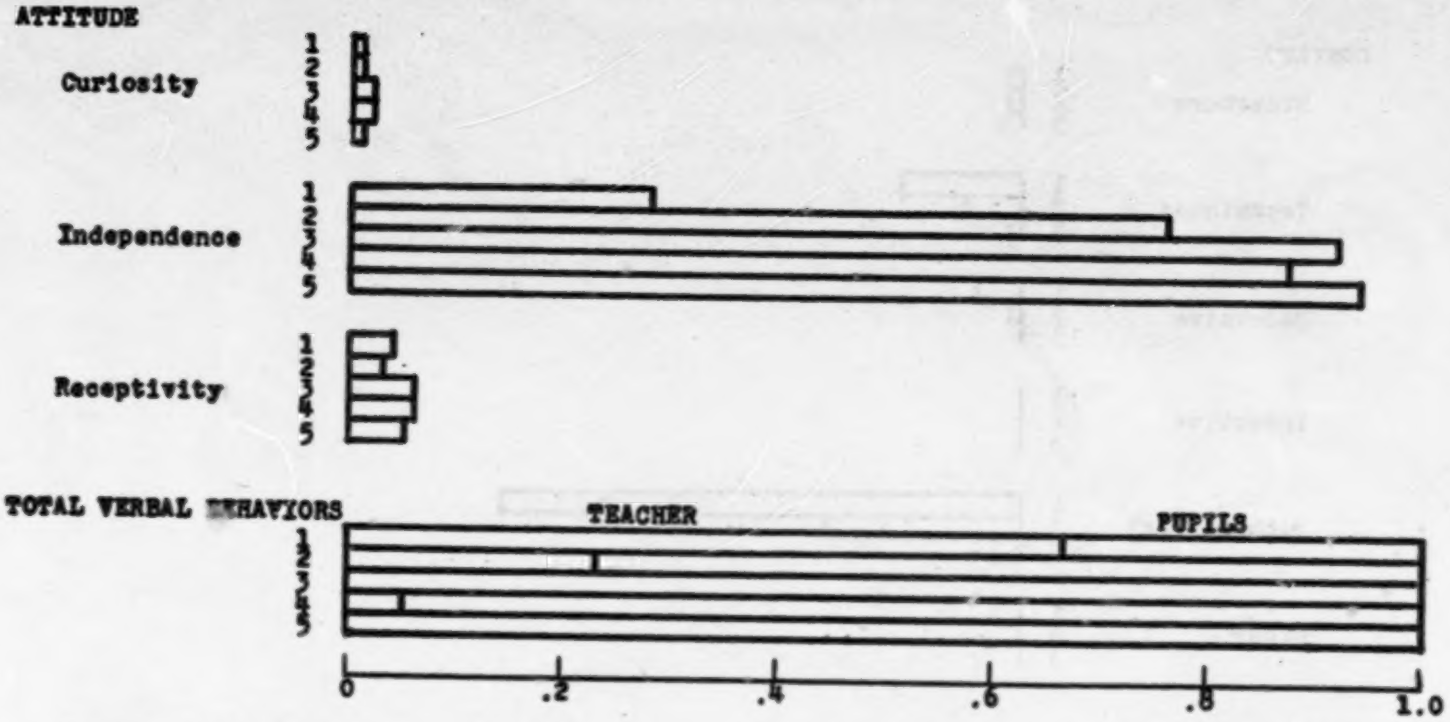


Figure 127 Comparison of single categories in pupils scores, Pupil/ Verbal; also division of total verbal behaviors. In the five classroom subgroups: class (1), subgroup with teacher (2), subgroup (3), individual with teacher (4) and individual (5). For the single categories, Attitude only.

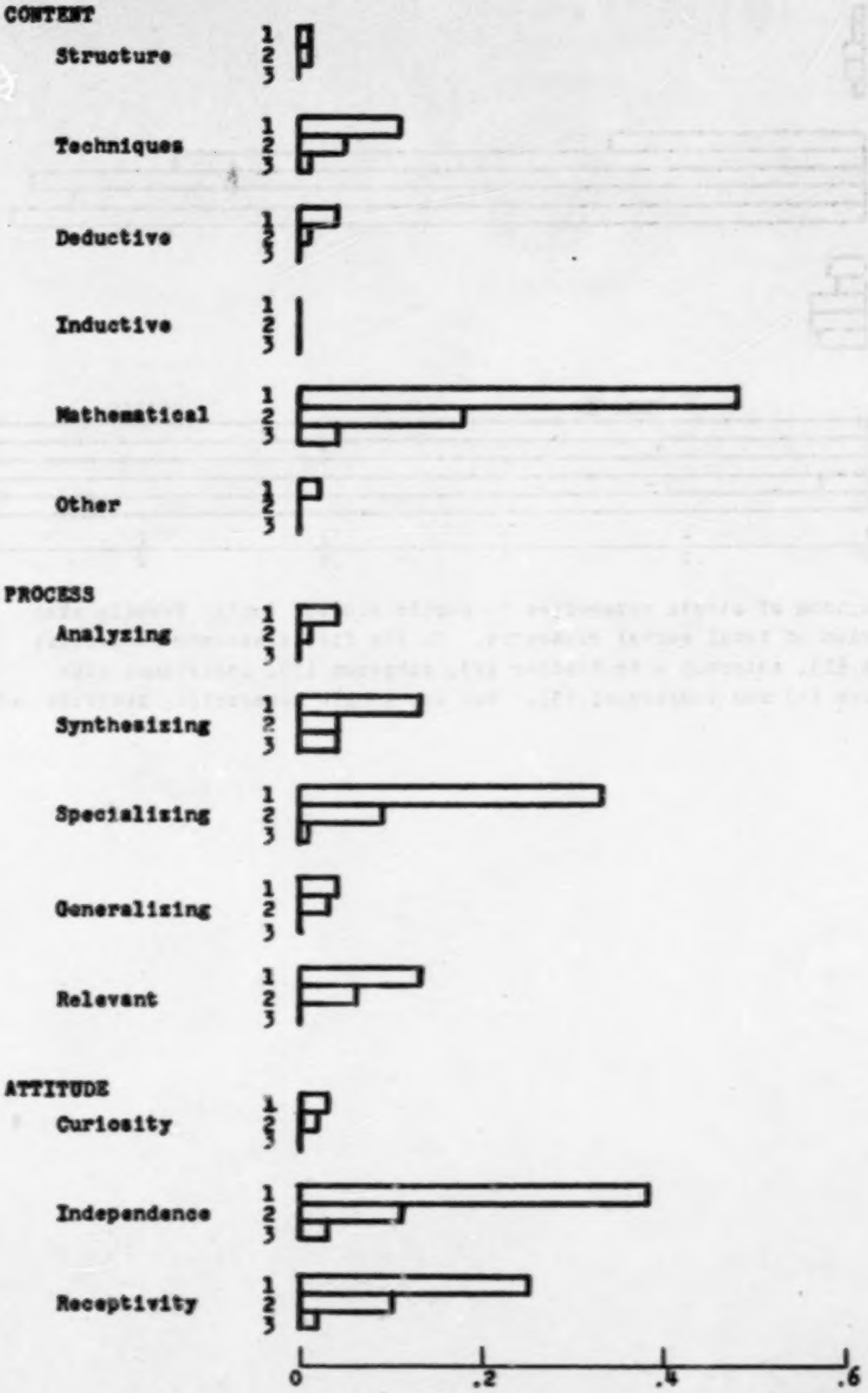


Figure 128 Comparison of single categories in teachers scores, Teacher/ Verbal, in the three classroom subgroups with teacher present; class (1), subgroup with teacher (2) and individual with teacher (3).

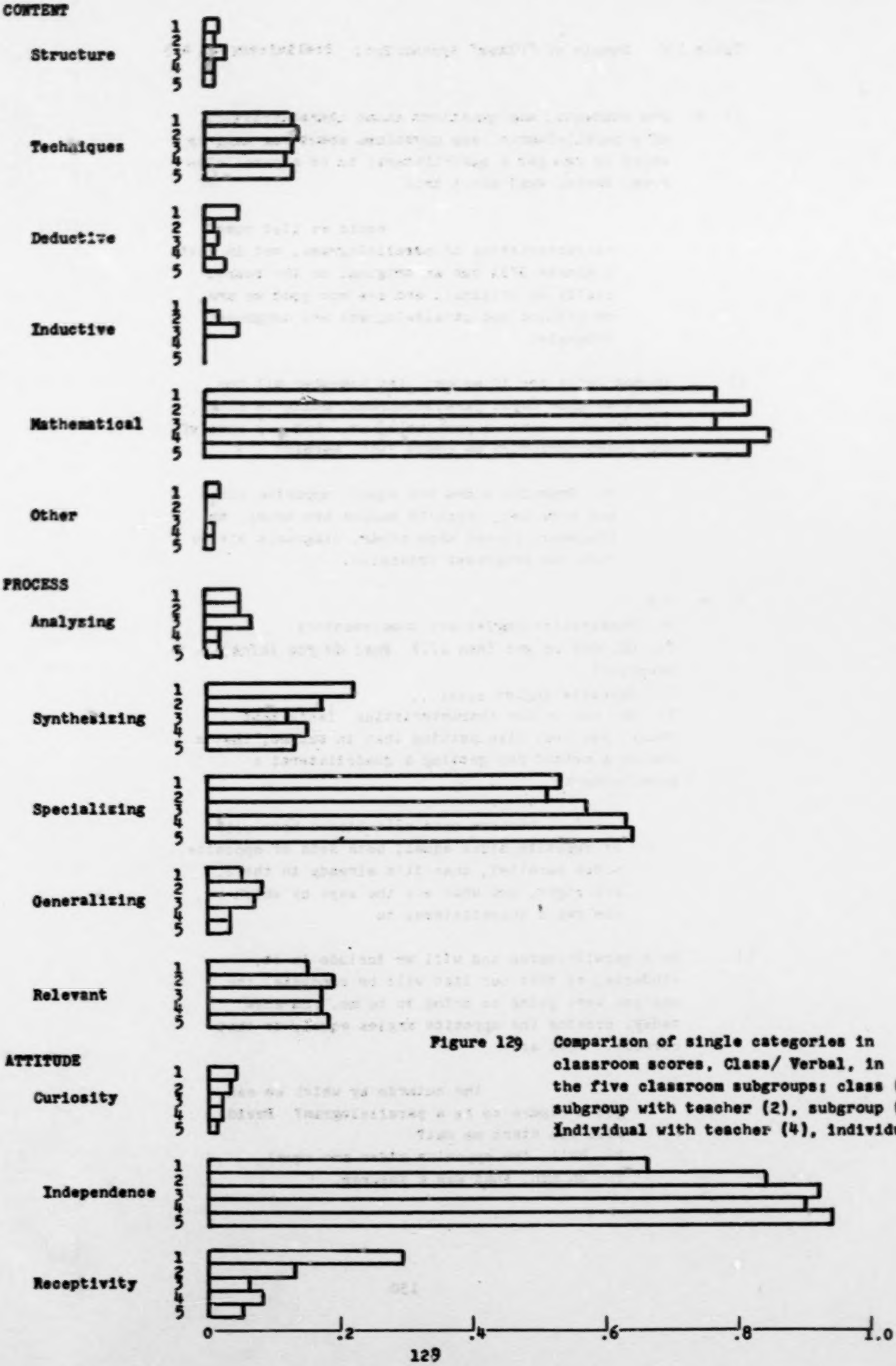


Figure 129 Comparison of single categories in classroom scores, Class/ Verbal, in the five classroom subgroups: class (1) subgroup with teacher (2), subgroup (3) individual with teacher (4), individual (5)

Table 130 Sample of "Class" typescript: Preliminary to A 3

1) T: Now students, any questions about characteristics of a parallelogram, any questions about the ways by which we can get a quadrilateral to be a parallelogram, David, what about that

could we list some characteristics of parallelograms, and in just a minute I'll put an original on the board, really an original, and see how good we are, on nothing but parallelograms and congruent triangles

2) so now let's see if we can list together all the facts we know about parallelograms, going in both directions. Given a parallelogram, what are some of the characteristics we could list, Harold?

P: Opposite sides are equal, opposite sides are parallel, opposite angles are equal, the diagonals bisect each other, diagonals divide into two congruent triangles.

3) T: O.K.
P: Consecutive angles are complementary.
T: OK, did we get them all? What do you think, Beverley?
P: Opposite angles equal...
T: Oh, now in the characteristics, isn't that funny, you feel like putting that in but no, that's really a method for getting a quadrilateral a parallelogram

because once he's listed both sets of opposite sides equal, both sets of opposite sides parallel, then it's already in there, all right, now what are the ways by which we can get a quadrilateral to

4) be a parallelogram and will we include in it, students, so that our list will be complete, the one you were going to bring in to me, you know today, proving the opposite angles equal, is that correct. What are

the methods by which we can get a figure to be a parallelogram? David, will you start me out?
P: Well, the opposite sides are equal.
T: Uh huh, that was a theorem.

Table 131 Sample of "Teacher and Group" typescript; Problem B 8

42) T: What were the three methods you used, we have a little time, we'll just throw this in.
B: We only had two didn't we?
B: We had one by a formula.
B: What was your formula?
T: What was the formula by the way?
B: x times x plus one over 2, am I right?
G: Yes

x times -
B: the formula for adding consecutive numbers.
B: My x's look like plusses

43) G: x times x-1, so its x squared plus 1.
B: No, x squared plus x.
G: x times
G: Oh, let's see.
T: Where did you get that formula, in algebra?
B: No

I think I saw it about 5th or 6th grade, somehow
T: Oh, really, in a regular class?
B: No, I was reading some book, I just thought it was an interesting formula, and I thought since I'd seen some mathematical tricks I thought it might involve that.

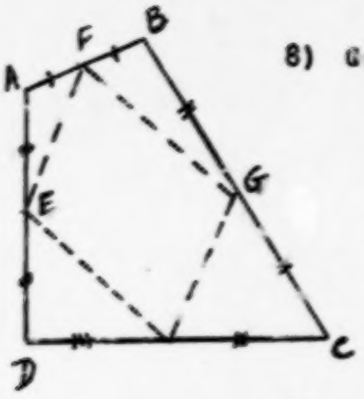
44) B: What does x stand for?
B: x is any number, one through x, one through x, you try to add up the numbers from one through x say x is 5, you want to add the numbers 1 through 5, so x, 5 times

x 1 is 6, over 2 would be 15.
G: Is there a more general formula for that?
G: Yes I do, yes, it's the number of terms

45) times the first term plus the last term divided by 2.
B: Oh, I see, that's like adding, you want to add 23 and 29, something like that.
G: Your x 1 would only work if it's 1, 2, 3.
B: Uh huh, I see. I was wondering if there were

a formula for multiplying consecutive numbers, is there one?
B: That's right.
B: It jumps by 2
T: What you did was add.
B: Yeh, if you're just adding.
T: What you're talking about is an arithmetic progression where the first number is 1, the

Table 132 Sample of "Group" typescript: Problem A 8



8) G: You also know that these angles are equal too, to
360° too, but
B: You have to prove that the opposite, if you
prove that the opposite angles are equal, I'm thinking
it doesn't do much with the sides.

B: It
would also be a parallelogram.
G: What have we got, AF = FB

9) BG = CG, DH = CH, and AE = DE

10) /

B: if

11) that thing is a parallelogram the opposite side
would have to be equal, and the opposite angles would
have to be equal, if we can just prove that it's a
parallelogram

B: Sure we can get the midpoints
for something.
B: We just had a theorem on that, with a triangle
B: Yes.

12) B: But then you'd just have to draw a lot of extra lines
B: I'm just thinking.

G: What was the theorem about
the midpoints of the sides of a triangle?
B: If you connect the line connecting the
midpoints of the sides of a triangle it's

13) parallel to the base and equal to one-half the base.
G: Well, here's a triangle, ABD.
B: Hey, that's right, we do have a triangle, B and
G, yes, yes. This triangle here, this goes to

B: That's right, we should prove the opposite
sides are parallel.
B: Hey, they're parallel to the same line so
they're parallel, parallel to AC.
G: Yes
B: And EF is parallel to HG, I knew.
G: So it's the same way with the other one.

14) B: So FG is parallel, so it's a parallelogram
G: And it works both ways
G: So it is a parallelogram

Table 133 Sample of "Teacher and Individual" typescript: Problem A 7

13) P: if I could get... angle A equal to angle EGB, since angle A is equal to angle B.

then angle EGB would be equal to angle B..and if in some way these lines were parallel then that would be corresponding angles

14) ...Now is there any way I could...make a parallelogram here...hum

...if I drew FC parallel to AG it would have two sides of a parallelogram

15) ...don't see any way to get 'em equal though...hum

/

16)

/

17)

there must be some way I can use that CE equals BE...I don't see what good that would do me though

18) ...hum.

could you tell me if I was on the right track or anything...well, I was trying to get, I drew this line, and I was trying to get these triangles congruent.

19) um...all right ought I have the right triangles... and I have one arm, now I'm trying to get these angles equal and this is equal to this so if there was some way I could get these lines parallel, then this

would be equal to this, and this would be equal to this.

T: Um hum

P: that's as far as I've got

T: Genie, do you know anything about these points?

20) You said if you could get this line parallel to this...

P: All that I know is that this equals this and this equals this

Table 134 Sample of "Individual" tyepscript: Problem A 6

1) P: Construct a triangle, having given the bisector of angle A, t_A. The side c and the sum of the two angles A and C. First thing is to draw the figure as it will look in the end

...Well first draw a working line, then from point A..from one end set an arc equal

2) to the length of the bisector--with the radius equal to the length of the bisector and the centre, and the centre at one end of the working line, and call the centre point A.

Now you're also given Also mark off the working line c..

3) um After

you've done this, draw another working line, and on that line construct the given angle--the given sum of angles A and C.

4) The angle which is supplementary to that will be the third angle of the triangle since the sum of the angles of a triangle is 180 degrees. um..

on tne end of line c which you have marked off on your working line, construct, copy this angle which is angle B

5)

/

6) Now after you've done this, this line which is the angle B will intersect the arc which is equal to t_{AA}.

at some point--it will intersect at two points. I believe there is two triangles, but I'm not sure. Draw a line from A to the first intersection point closest to the vertex of the angle

7) (? E MJW) This is t_A, Double.. you will have an angle...call the point of intersection with the arc

D, and call the vertex of your copied angle B. Angle DAB will be equal to 1/2 angle A, in the classroom" anticipating that with the very small group (of four in contrast to twenty in classroom) some such modification would naturally occur. It would be reasonable to suppose that with some such instruction, or by increasing the group size to six--where some direction of who should talk next might be needed--that the small group with teacher might prove a convenient laboratory classroom.

The differences between the classroom and each other small group were quite marked--all the triple rankings were distinct. Examination of the individual frame scores indicated similarity in content with the small group, but otherwise quite general disagreement in both classroom and pupil scores. Examination of the single category emphasis and typescripts points up the more active qualities of classroom and small group with teacher in moving to and from premises and problem solution, in consideration of structure useful for application, and in the classroom the ready teacher demonstration or assistance. In contrast the other subgroups: small group, individual with teacher and individual moved much more slowly, emphasizing classificatory aspects of the specific problems and showing high pupil initiative. This was also emphasized by the greater number of solutions completed in the classroom and small group with teacher.

Most other pairs of comparisons did not show significant difference. However where this occurred it was concentrated in process--small group with teacher or small group versus individual with teacher or individual. One additional difference to be noted was in Content between small group and individual with teacher. Again these differences seem related to the greater activity towards solution of problems in the two small group subgroups.

It was apparent, then in this instance, that the question of small group with teacher simulating the classroom should be pursued in seeking a convenient "laboratory classroom". While the teacher classroom role had been quite encouraging of pupil initiative, even more pupil participation was invited in the small group. This sometimes took the form of permitting longer silent periods but more often was indicated by such remarks as "We want to hear your thinking about that". This seemed a direct result of confidence in sustaining interest in the smaller number of pupils. Purposely, no instructions of teacher role in any of the situations had been given. It was considered likely that asking the teacher to continue her classroom role, or increasing the small group even to six would result in even greater similarity of small group with teacher and the classroom. In addition, the individual working on his own differed significantly in speed, in quantity of ideas and in variety of methods of attack from the individuals contributing to pupil behaviors in small group or in classroom. However, the relative value to the individual may well differ little. In the small group of pupils, it was apparent that much of the burden of solution was carried by one or two pupils, the others serving more as checks or interrogators than as contributors. While this also occurred in small group and classroom with teacher present, she usually paused to determine by questions or by inviting broader participation that all individuals could ensure their understanding of the problem. This observation with small groups was also made by Hudgins (1958) and is receiving further study by Smith and Hudgins (1961). Finally in this experiment very marked similarity between individual and individual with teacher patterns was shown.

While this latter subgroup apparently corresponded to a supervised study situation, the pupils of this classroom were all so capable in mathematics and so highly motivated both in general and in particular for this experiment, that only occasionally was teacher help requested. It was not offered at other times. However, this was much in keeping with our general observations for little time was spent in supervised study in 18/20 classrooms initially visited in this project. Generally when observed it took place irregularly for a few minutes at the end of a period with

the teacher waiting to be asked for help and then often giving it by demonstration of problem solution. This was in marked contrast to 2/20 classrooms and in the personal experience of the investigators where in consistent use of supervised study the teacher not only responded on request but also sought the opportunity of examining individual work in detail, developing individual skill in questioning, encouraging brighter students to go beyond immediate problem solution and in assignment development of significant material (at theorem level) with teacher available for specific assistance.

RESUME

SUMMARY AND CLASSROOM IMPLICATIONS

Current variability in mathematics classrooms resulting from renewed interest in mathematical rigor and student involvement in all phases of system development, together with the validity associated with direct observation of classroom discussion for seeking immediate answers to questions of success of experimental programs stimulated the use of the Wright-Proctor instrument in comparison of the effects in mathematical discourse of varying conditions of rigor and student involvement.

Distinctive patterns in the areas of Content, Process and Attitude were established for the four types of classrooms investigated: high rigor, high participation; low rigor, high participation; low rigor, low participation; and high rigor, low participation. These distinctions were noted in terms of single categories within each area. by combinations of categories within Content and Process, and by triple combinations of categories across the three areas simultaneously. It was sharply apparent that if our study had been limited to one or two dimensions of the classroom, e.g. to a few categories within one area, or to one area, that wide variations in behaviors would have suggested major distinctions between teachers and classrooms quite unjustified by the independent variables and by any one of the complete views of the dependent variables. By whichever of the three means a complete view of the classroom variables was taken, certain apparent discrepancies in earlier limited assessment were resolved, and marked variation in a particular dimension could be accepted as reasonable and indeed as quite desirable. It was apparent that in the high rigor, high participation type where the teachers were committed theoretically to both these viewpoints, in practice this concern took form as they would want in emphasis of structure and in successful pupil assistance in construction of the mathematical system. In addition, both an increase of participation, rigor constant and an increase or rigor, participation constant, resulted in this greater emphasis of structure without lack of attention to technical skills. It was also noticed that despite the reasonable concern of many teachers that marked student participation will limit unduly the amount of subject matter that may be presented, at this senior high school level the high participation groups were moving more rapidly than was the low participation group. The three classes forming the low participation group at the university level, however, were covering more material in the given time than any of the high school classes. This was in part related to the practice of assignments being treated outside the regular lectures in "help sessions". While it is very likely true that a class will move more slowly in early stages of particular subject matter or where student involvement is newly encouraged (e.g. first year in high school), by the senior year any disadvantage in terms of quantity of subject matter covered is probably more than overcome.

While this intensive study in the categories of Content, Process and Attitude was of particular interest to the investigators, other mathematics teachers and the general observer would notice differences in the four types of classrooms at a more functional level. In the low participation, low rigor type (rp), a routine was quite apparent (supported formally by more rapid stability of the independent variable measures). This might take such form as: each class beginning with reading of answers to home work, followed by teacher explanation of difficulties; development of new material by the teacher often from specific examples without supporting general proof or by statement of important relations without development; assignment of applications, usually many of the same form, with the teacher working one or two examples or with the limited student activity taking place here; some time for supervised study for beginning homework problems.

With an increase of participation to type low rigor, high participation (rP) much greater variety of classroom organization was customary. While a class might begin as in type rp, it often would be modified slightly, with the pupils joining in discussion of assignment difficulties, or actually taking considerable responsibility with the pupils writing out assignment solutions on board and presenting solution to entire class. Greater emphasis of strategy in problem solving was evident. New work, while generally presented with the teacher as discussion leader, was developed, for the most part in heuristic fashion with the teacher presenting leading questions and the students responding. At times new work would be developed by several students; at such times the student rarely acted successfully as leader, apparently seeing himself as a demonstrator not as a questioner. However, occasional halting attempts at following the teacher's role were seen and were assisted by the teacher from the back of the room. Development of new work was generally from specific examples, though the habit of participation left the students feeling free to question when logical connections were not sufficiently apparent at least on an intuitive basis. While systematic development of the mathematical system was still limited there was greater evidence of logical argument from one point to another. In working specific problems the students took the major responsibility in this rP type. One popular part of the lesson in one classroom was the question-answer period met early in the class when by rapid fire questioning requiring short answers the teacher would check round and about the subject matter covered recently noting the student's preparation and sharpening attention to important details. This sometimes took the form of estimation of answers to specific problems which would then receive support by appeal to a recently established principle. While of deeper significance, this had much of the fascination of the spelling bee or the mental arithmetic contest of the elementary school.

Much this same variety was seen in the high rigor, high participation type (RP). On the whole, less time was needed for treatment of specific problems, for reference by teacher or other pupils to a principle back of a difficulty often resulted in a nod of understanding or gasp of mild embarrassment from the student with difficulty. Here throughout development of new work and treatment of applications, brief but sharp attention was frequently given to accuracy, precision, aspect of logic, strategies of problem-solving, etc. Development of new work occurred in class discussion or in mass individual work at desks with teacher or a student acting as leader. During search for solution longer pauses in overt behavior occurred without loss of attention and without the teacher providing the needed step. More concern with presentation of a variety of solutions probably resulted from the several students who would find a method and wish to present their case. When a relation had been arrived at, proof generally followed. This was supported by basic postulates and later theorems, definitions, etc. Criticism by other students and teacher of errors or inefficient reasoning was rapid and accepted in good form. A particularly good score in this game was achieved when the teacher was caught in a careless statement or unsupported generalization. Again students took part in treatment of applications, though these were often more sophisticated and requiring greater variety of attack than in the low rigor types.

Finally, the high rigor, low participation type (Rp) was found almost entirely in the university. Here the main emphasis throughout the lesson was on discovery, statement and proof of new relations; this was supplemented by discussion of specific problems at varying times and in varying degrees by the several lecturers, but opportunity was given every day or every second day for raising of difficulties, and to treatment of application at times appropriate in lesson development. Almost all aspects of the lesson were treated exclusively by the teacher; student participation was intentionally limited in these classes of size 60-100. Participation was expected outside of class and was stimulated in assignments consisting of applications and theoretical problems. In the development of new work, the observers were quite surprised to note the high incidence of intuitive treatment either preceding statement of relations, in the course of discovering a series of plausible relations to be investigated, or immediately following logical demonstration of relations. This practice provided emphasis of specific problems and technical skills although the apparent intent was in support of the theory. Where mathematical applications were discussed, these were always placed in the context of the theoretical development. Here more than in any other type historical reference was apparent, and humor was related to the subject matter. Particularly to high school teachers and members of university departments concerned with technical application, knowledge of the practice of inclusion of intuitive and application aspects in the university lectures as well as the expected rigorous treatment would seem of importance.

In addition to comparison of several classroom types, this extensive observation contributed to further evidence of technical refinement of the Wirght-Proctor Instrument. Certainly the support of the hypotheses relating the categories of Content, Process and Attitude with the independent variables of rigor and participation logically supported the appropriateness of the design of the instrument. The clear separation of participation types by the Index of Initiative proposed on logical grounds, supported the interpretation of the continuous aspect of the categories in the Attitude frame by the empirical test. In addition to the satisfactory level of agreement between two experienced observers for the particular scores used in this study, the overall intensive study of cross-classification by the two observers clarified weaknesses still present in definitions and categories susceptible to observer bias. On the other hand, for overall assurance of reliability of the scores obtained with this instrument good levels of correlation must be sought in the future with observers using this instrument but not trained at this laboratory, with other direct tests, and with some related indirect measures. Some support to reliability was, of course, given by the assessment of the stability of the chosen sample of behaviors. Earlier estimates of ten days as providing a sound minimum sample were supported, though in the areas of Process and Attitude as few as six days seemed adequate. The greater variability in Content seemed to occur in classrooms of the high rigor type where a greater number of phases occurred in the course of development and application of a new relation. For this reason, the reduction of sample size below the ten days here established as valid, should be largely on the basis of treatment of subject matter. It is felt that the six days suggested as the minimum for Process and Attitude, and on occasion, Content, is surely the minimum desirable in making such intensive study of a classroom. In broader terms the teacher, class and administration are likely to be more confident if the investigator is prepared to spend ample time in collecting information that will form the basis for classroom description, comparison, etc. all aspects, in one sense, of evaluation.

The exploratory study of patterns of verbal behaviors in the several subgroups typical of a classroom was very productive. Many practical difficulties of problem selection, treatment of data from individuals and small groups, use of tape recorders in recording of verbal solution to problems, working with a teacher and group of pupils with a minimum of class interruption and after school time, transcription of recorded discussions, classification from typed transcripts or carefully timed tapes, have been met head on--occasionally anticipated--and, on the whole, solutions determined. In the order listed some of our present considerations are these. A major difficulty in problem selection is the obtaining of problems that have been used successfully in other studies of problem solving for comparative purposes. Few problem solving studies report the exact problems used, the exact scoring methods followed, any concern with strategy, whether an investigator is present or not, what specific role the investigator plays, a time limit, any step-by-step variation of conditions in given problems, any concern with problem complexity or subject matter context. Our own initial and naive selection of problems was so novel that the students regarded the problems in the same light as parlor games and treated them, after very brief systematic attempts, by trial and error. While this effect was modified to some extent by drawing of certain examples from topics related to fairly recent classroom experiences our immediate turn to related research was largely unrewarding. Even such brilliant work as that of F. Bartlett and of K. Duncker indicated little reason for the choice of problems or systematic modification of conditions or complexity that might indicate their rationale. Of equal concern was the lack of methodology developed for sound comparison of problem solution of individuals and groups of varying sizes. All these concerns which gradually became apparent to us were summarized by Ray (1955) and in a personal communication by Vanderplas (1960). We therefore embarked again on the exploratory work described in Chapter V with many warnings and a main intent to record our procedures in detail. Our difficulties in task selection (avoiding complete trial and error treatment and seeking comparable difficulty) were met as follows. Two-part problems were selected: the first part related to current class development established a systematic approach; the second part, novel yet amenable to treatment by these students providing means of comparison in later studies. Then, in addition to teacher and investigator judgment the entire problem set was worked out by other high school students just one year in advance of the experimental class and judged by them. However no detailed logical assessment of task conditions or complexity was made.

In comparison of individual with small group and class solution the effects of the individuals participating in group and class discussion were sought in individual discussion despite the larger number of individual behaviors that would result. This discrepancy in total number of behaviors in individual, group and class solutions was met by further use of the teams selected for balanced design purposes in obtaining team mean scores for the four individual problem solutions. In this way the combined effect of individuals in group and class was attained in the individual solution, and at the same time the total number of behaviors was consistent for comparative purposes. The whole problem of the effect of systematic variation of group size of individuals of known problem solving ability is of itself of considerable interest to us. E.g. would problem solving success of groups of size 2, 3, 4, 5, etc. differ from that which theoretically can be inferred from known probabilities of success of the individuals participating. If a group factor (which might be inferred from our studies) exists over and above the theoretical expectancy does this taper off with groups over some optimum size such as four or five? With these considerations, the exploratory quality of this particular study becomes even more marked to us. The other practical considerations will be passed over quickly for they vary largely with the particular recorder used or the conditions peculiar to a given school. One point bears comment, however. That is the formidable task of complete transcription from tape to typescript of recorded material. While the problem of recording the complete class and group discussion at audible levels was readily handled by use of a sensitive recorder and two microphones, the rapidity of the discussion and technical language combined to scare off typists trained in transcription of usual office tasks. Our own long-suffering secretary was willing to persevere but the task was in general extremely time consuming and the technical language a definite problem. For the purposes of obtaining materials for training, we continued with our transcription. However, for purposes of study of recorded behaviors it was apparent that joint classification by two observers supplemented by replay of intervals of disagreement was the more efficient approach. The only concern was in arranging a convenient signal on the tape for starting the stop-watch at an exact position of the tape so that on rechecks the same division of fifteen second intervals would be obtained.

Our assessment of the verbal behaviors of the five subgroups emphasized the greater continuity of discussion in the class, small group with teacher and small group, as a result of the teacher's directive and sustaining role and the greater variety of ideas available from the larger population in a given time. Despite the ability of the students acting as subjects evidenced by school mathematics achievement, general intelligence measures and teacher judgment, continual opportunity in class for participation, and the remarkable persistence shown in working on the problems, few clear-cut strategies of attack were apparent in individual problem solving. Even students quite productive in the classroom were hesitant when the onus of direction was theirs alone. Two quite marked exceptions to this occurred. One student was obviously very keen on working problems of all sorts and indicated that he had met two of the novel problems and could rapidly solve one of the class-related problems by analogy. Instead of asking for direction or a new problem, he proceeded to invent a generalization of each of these three and to solve this by referring to specific problems, stating generalizations apparent, and testing these with specific problems. The second student had shown little productive activity in class though invited by the teacher; his mathematics grades were the lowest in the class. Occasionally he contributed in the small group development. However, in individual problem-solving he consistently confirmed all the facts and conditions of the given problem, noted the remaining aspects to be treated before solution achieved, noted the effects of variation of given conditions and stated the family of problems that would be met within the variety of possible conditions. There was rarely any hesitation in his discussion. It was interesting to note that each of these students turned to the teacher at least once for confirmation of their broader approach, which was not recognized in the close attention of the teacher to the specific problem at hand (and without the knowledge gained from entire record on tape) yet pursued own argument despite non-encouragement until time completed. In this context where these gifted students were relatively ineffective, Suchmann's work in developing in students a method of inquiry by requiring student development of consistent productive strategy by learning to ask pertinent questions seems particularly significant.

The opportunities for this extensive observation and the materials made available by all the studies discussed in this report made possible an extensive revision of the Observer's Manual with a view to self-training away from this laboratory and provided additional evidence for validity and reliability and limited data for comparative purposes.

SUGGESTED STUDIES

The considerations of this report provide many ideas for direction of subsequent studies. The one of most general interest for the school, I believe, is the success of this direct establishment of patterns and comparison of experimental and non-experimental mathematics programs, for both the immediacy and the validity of results.

The availability of the new Manual should make possible the use of this instrument in methods courses in mathematics teacher training programs in sharpening attention to aspects of content, process and attitude significant in the classroom and then by direct observation to emphasis of behaviors of the teacher that stimulate and of the pupils that demonstrate a particular desired end.

In terms of instrument refinement two ideas are perhaps most pressing: comparison of results of observation with this instrument with other relevant measures and broader tests of observer reliability. Empirical validation by comparison of results with other direct measures or with indirect measures of related variables would be consequent on the availability or development of comparative measures that could reasonably be expected to relate to all or part of this complex instrument. The Attitude frame might well be related to some of the refined instruments now available emphasizing types of teacher-pupil interaction; the Process frame to studies in strategy in problem-solving; the Content frame compared with paper and pencil tests graded for development as well as for final answers. For these purposes, in the field of paper and pencil tests the materials of Guilford should receive early intensive consideration. Observer agreement has so far been limited in assessment by the training of all observers at this laboratory. The self training manual may facilitate more general conclusions about this type of reliability.

The exploratory work of the problem-solving in subgroups within a classroom supports further investigation of a suitably-sized small group with teacher to simulate the classroom discussion situation. Laboratory studies or laboratory studies supplementing field work offer many advantages, in such detailed observation, over cumbersome field studies alone. The idea of applying this instrument further in the study of problem-solving is quite enticing. In particular a basic approach would be establishment of theoretical expectancies of problem solving ability, by individual paper and pencil pre-test of each subject, followed by consideration of actual with theoretically expected problem solving success in groups of systematically increasing size. Concurrent recording of verbal behaviors for classification of results by this instrument would provide detailed information of procedures in problem solution supplementing information about complete success in achieving answers.

Finally we are hopeful that our model for the study of mathematical discourse may be related with a broad learning model that includes the excellent aspects of current interaction models when both teacher and pupil are involved, but places these all in appropriate perspective with the ultimate raw encounter of learner and subject matter.

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Appendix A

Table 146 Frequency of behaviors in each category of Content, of Process and of Attitude, and in Neutral for Teacher I and Pupils I as recorded by Observer II for the ten periods of classroom visits. Superscripts indicate frequency of negative behavior in individual categories.

DAYS CATEGORY	1	2	3	4	5	6	7	8	9	1
Content				TEACHER I						
Structure	20	-	2	5	9	-	11^1	22^1	-	1
Techniques	17	-	12	8	14	-	6	6	-	1
Deductive	8	-	20	15	8	-	11	25^1	-	1
Inductive	0	-	9	9	6	-	17	1	-	
Mathematical	16	-	20	23	18	-	21	12	-	1
Other	0	-	0	3	6	-	6	2	-	
Process										
Analyzing	8	-	1	4	8	-	14	10	-	
Synthesizing	14	-	21	12	6	-	7	29	-	
Specializing	16	-	23	27	30	-	33	18	-	3
Generalizing	5	-	7	9	3	-	11	3	-	
Relevant	18	-	11	11	14	-	8	10	-	1
Attitude										
Curiosity	1	-	0	3	3	-	3	4	-	
Independence	45	-	33	47	48	-	55	48	-	4
Receptivity	15	-	30	13	10	-	15	18	-	1
Neutral	3	-	0	2	0	-	1	1	-	
TOTAL TEACHER	64	-	63	65	61	-	74	71	-	6
Content				PUPILS I						
Structure	9^1	-	3^1	0	3	-	3	11^2	-	
Techniques	0	-	1	2	0^1	-	2	0	-	
Deductive	2^1	-	8^2	13^1	4^1	-	3^1	6	-	
Inductive	0	-	0	1	2	-	7	0	-	
Mathematical	13	-	18	8^2	11^2	-	1^1	2	-	1
Other	0	-	0	0	1	-	0	0	-	
Process										
Analyzing	2	-	2	2	2	-	1	3	-	
Synthesizing	17	-	17	10	10	-	4	8	-	10
Specializing	3	-	9	7	12	-	8	9	-	15
Generalizing	4	-	3	6	1	-	4	1	-	
Relevant	0	-	2	2	0	-	1	0	-	
Attitude										
Curiosity	1	-	0	4	2	-	0	0	-	0
Independence	21	-	28	19	21	-	18	20	-	24
Receptivity	4	-	5	4	2	-	0	1	-	
Neutral	0	-	0	0	0	-	0	0	-	0
TOTAL PUPILS	26	-	33	27	25	-	18	21	-	29
TOTAL T & P	90	-	96	92	86	-	92	92	-	92

Table 147 **Frequency of behaviors in each category of Content, of Process and of Attitude, and in Neutral for Teacher II and Pupils II as recorded by Observer I for the ten periods of classroom visits. Superscripts indicate frequency of negative behavior in individual categories.**

DAYS CATEGORY	1	2	3	4	5	6	7	8	9	10
				TEACHER II						
Content										
Structure	13	8	8	12	3	6	14	23	22	23
Techniques	11	6	9	7	6	6	4	5	5	7
Deductive	16	7	17	8	0	13	16	25	19	11
Inductive	0	9	9	14	7	11	8	6	0	5
Mathematical	23	27	7	25	27	19	16	6	19	17
Other	0	0	0	1	9	3	6	1	2	0
Process										
Analyzing	7	11	7	6	2	10	7	8	16	14
Synthesizing	8	17	5	11	5	13	18	23	23	7
Specializing	23	17	20	32	28	24	23	19	12	31
Generalizing	11	5	7	8	5	6	9	7	5	2
Relevant	14	7	11	11	12	5	7	9	9	9
Attitude										
Curiosity	1	4	0	3	4	4	4	1	2	3
Independence	48	49	39	62	36	46	45	45	50	52
Receptivity	14	4	11	3	12	8	15	20	15	8
Neutral	1	2	6	1	0	5	2	3	0	3
OTAL TEACHER	64	59	56	69	52	63	66	69	67	66
				PUPILS II						
Content										
Structure	2^{1}	5^{1}	0	1	0	4^{1}	4^{2}	3^{1}	5^{2}	5
Techniques	2	0	0	0	0	3	2	1	1	3^{1}
Deductive	4^{1}	1	15^{4}	2^{5}	0	6	10^{1}	11^{2}	7	4^{1}
Inductive	0	3	0	0	2^{1}	2	5	0	0	0
Mathematical	16^{1}	16^{7}	14^{1}	10^{1}	2^{1}	10^{3}	2^{1}	2	9^{2}	10^{1}
Other	0	0	0	0	0	0	0	0	0	0
rocess										
Analyzing	1	3	1	0	0	5	3	1	3	4
Synthesizing	17	16	20	5	1	10	7	8	10	8
Specializing	4	11	9	14	2	6	10	9	10	8
Generalizing	1	0	4	0	2	4	6	1	0	0
Relevant	4	3	0	0	0	4	1	1	2	5
ttitude										
Curiosity	0	0	0	0	0	0	3	0	0	0
Independence	26	32	33	19	5	27	23	18	24	24
Receptivity	1	1	1	0	0	2	1	2	2	1
eutral	0	0	0	0	0	0	0	0	0	0
OTAL PUPILS	27	33	34	19	5	29	27	20	26	25
TOTAL T & P	91	92	90	88	57	92	93	89	93	91

Appendix A

Table 148 Frequency of behaviors in each category of Content, of Process and of Attitude, and in Neutral for Teacher II and Pupils II as recorded by Observer II for the ten periods of classroom visits. Superscripts indicate frequency of negative behavior in individual categories.

DAYS CATEGORY	1	2	3	4	5	6	7	8	9	10
Content				TEACHER II						
Structure	18	-	6	8	2	-	8^1	21	-	23
Techniques	15	-	10	10	6	-	5	4	-	9
Deductive	4	-	20	10	5	-	20	29	-	7
Inductive	0	-	7	19	4	-	11	3	-	1
Mathematical	23	-	10^1	17	15	-	12	7	-	23
Other	0	-	0	2	19	-	3	2	-	0
Process										
Analyzing	5	-	5	4	2	-	7	4	-	17
Synthesizing	10	-	10	11	5	-	16	19	-	9
Specializing	17	-	20	35	30	-	19	21	-	26
Generalizing	10	-	7	6	7	-	12	12	-	1
Relevant	18	-	12	10	7	-	6	10	-	10
Attitude										
Curiosity	0	-	2	2	8	-	3	1	-	6
Independence	41	-	37	51	28	-	35	43	-	46
Receptivity	19	-	15	13	15	-	22	22	-	11
Neutral	2	-	5	1	0	-	4	3	-	3
TOTAL TEACHER	62	-	59	67	51	-	64	69	-	66
Content				PUPILS II						
Structure	4^2	-	1	2	0	-	4^2	3^2	-	3^1
Techniques	2	-	1	1	0	-	0	2	-	3^1
Deductive	2	-	14	2^4	0	-	11^1	7^2	-	4^1
Inductive	0	-	2	0	1	-	4^1	0	-	0
Mathematical	19	-	11	11	2^1	-	4	5	-	12
Other	0	-	0	0	2	-	1^1	0	-	0
Process										
Analyzing	1	-	1	2	0	-	5	4	-	4
Synthesizing	20	-	20	5	2	-	7	6	-	13
Specializing	3	-	3	12	3	-	11	7	-	3
Generalizing	1	-	4	1	1	-	6	2	-	0
Relevant	4	-	1	0	0	-	0	2	-	5
Attitude										
Curiosity	0	-	0	0	0	-	4	0	-	1
Independence	27	-	29	18	6	-	25	18	-	23
Receptivity	2	-	0	2	0	-	0	3	-	1
Neutral	0	-	1	1	0	-	0	0	-	0
TOTAL PUPILS	29	-	30	21	6	-	29	21	-	25
TOTAL T & P	91	-	89	88	57	-	93	90	-	91

Table 149 Frequency of behaviors in each category of Content, of Process and of Attitude, and in Neutral for Teacher III and Pupils III as recorded by Observer I for the ten periods of classroom visits. Superscripts indicate frequency of negative behavior in individual categories.

DAYS CATEGORY	1	2	3	4	5	6	7	8	9	10
					TEACHER III					
Content										
Structure	6	11	16	5	13	7	15	4	30	8
Techniques	1	2	2	5	2	2	3	1	7	5
Deductive	17	34	39	21	60	7^1	22	54	19	5
Inductive	12	9	16	29	18	21	25	20	32	30^1
Mathematical	51	37^1	6	31	13	47	16	10	3	38
Other	1	1	13	0	0	2	0	5	1	2
Process										
Analyzing	9	14	13	8	28	16	17	13	19	6
Synthesizing	39	24	25	35	38	32	23	28	28	17
Specializing	22	21	14	25	22	23	13	31	29	47
Generalizing	15	30	25	17	15	10	23	21	12	12
Relevant	3	5	15	5	3	6	5	1	4	6
Attitude										
Curiosity	4	1	20	2	4	4	1	3	0	3
Independence	7	6	2	11	6	7	9	9	9	6
Receptivity	77	85^1	70	77^1	95^1	76	70	82	84	77^1
Neutral	1	0	1	0	0	1	13	2	0	0
TOTAL TEACHER	89	93	93	91	106	88	93	96	93	87
					PUPILS III					
Content										
Structure	0	0	0	0	0	1	0	0	1	0
Techniques	2	0	0	0	0	0	1	0	0	0
Deductive	1^2	0	0	1	0^1	0	2	0	0	1
Inductive	0	1	0	1	0	0	0	0	0^1	0
Mathematical	5	7	3	7	1	1^1	2	1	0	4
Other	0	0	0	0	0	0	0	0	0	0
Process										
Analyzing	2	0	0	1	0	0	1	0	1	0
Synthesizing	0	0	0	0	0	0	0	0	0	0
Specializing	5^1	8	3	8	1	3	2	1	1	5
Generalizing	0	0	0	0	0	0	1	0	0	0
Relevant	2	0	0	0	1	0	1	0	0	0
Attitude										
Curiosity	2	1	1	2	0	0	0	0	0	0
Independence	2	0^1	0	1	0	0	1	0	2	2^1
Receptivity	6	6	2	6	2	3	4	1	0	2
Neutral	0	0	0	0	0	0	0	0	0	0
TOTAL PUPILS	10	8	3	9	2	3	5	1	2	5
TOTAL T & P	99	101	96	100	108	91	98	97	95	92

Appendix A

Table 150 Frequency of behaviors in each category of Content, of Process and of Attitudes, and in Neutral for Teacher V and Pupils V as recorded by Observer I for the ten periods of classroom visits. Superscripts indicate frequency of negative behavior in individual categories.

DAYS CATEGORY	1	2	3	4	5	6	7	8	9	10
					TEACHER V					
Content										
Structure	4	19	9	7	9	22	3	10	8	22
Techniques	3	2	4	1	3	2^{1}	0	7	1	2
Deductive	8	55	43	9	69^{1}	7	48	32	45	9
Inductive	1	28	15	49	17	67	54	47	40	65
Mathematical	82	6	28	33	21	3	2	1	8	2
Other	0	0	2	3	0	0	0	3	3	7
Process										
Analyzing	12	19	5	9	22	16	12	6	11	8
Synthesizing	45	27	41	51	44	40	28	25	37	37
Specializing	29	35	16	21	27	26	46	36	44	47
Generalizing	7	27	29	16	11	17	21	32	12	13
Relevant	5	2	10	5	6	3	0	1	1	2
Attitude										
Curiosity	3	3	2	12	4	10	8	9	3	5
Independence	10	1	1	1	6	8	3	0	1	1
Receptivity	85	106	98	89	100	84	96	91	101	101
Neutral	1	0	0	0	0	1	0	7	1	0
TOTAL TEACHER	99	110	101	102	110	103	107	107	106	107
					PUPILS V					
Content										
Structure	1	0	0	0	0^{2}	0	0	0	0	0
Techniques	1	0	0	1	0	0	0	0	1	0
Deductive	1	6	1	0^{1}	0	1	1	0^{4}	1	0
Inductive	0	0	0	1	0	1	0	0	0	1
Mathematical	7	2	1	3	1	1	0	0	0	0
Other	0	0	0	0	0	0	0	0	0	0
Process										
Analyzing	2	1	0	0	1	1	0	0	0	0
Synthesizing	1	0	1	0	0	0	0	0	0	0
Specializing	5	6	1	3	2	2	1	2	2	1
Generalizing	0	1	0	1	0	0	0	2	0	0
Relevant	2	0	0	2	0	0	0	0	0	0
Attitude										
Curiosity	0	1	0	2	2	1	0	0	0	0
Independence	5^{1}	3	1	0	0	1	0	0	1	0
Receptivity	4	4	0^{1}	4	1	1	1	4	1	1
Neutral	0	0	0	0	0	0	0	0	0	0
TOTAL PUPILS	10	8	2	6	3	3	1	4	2	1
TOTAL T & P	109	118	103	108	113	106	108	111	108	108

Appendix A

Table. 151 Frequency of behaviors in each category of Content, of Process and of Attitudes, and in Neutral for Teacher VI and Pupils VI as recorded by Observer I for the ten periods of classroom visits. Superscripts indicate frequency of negative behavior in individual categories.

DAYS ATEGORY	1	2	3	4	5	6	7	8	9	10
ontent					TEACHER VI					
Structure	12	3	1	7	11	18^1	4	2	8	14
Techniques	8	16	21	9	17	6	14	28	18	23
Deductive	23	48	8	23	26	49	50	31	41	38
Inductive	25	10	38	0	0	13	14	22	0	0
Mathematical	10	0	30	43	41	3	0	5	23	32
Other	11	16	3	6	0	10	3	1	0	2
rocess										
Analyzing	6	7	4	8	12	11	1	10	15	17
Synthesizing	16	26	31	24	28	26	32	16	25	32
Specializing	41	25	33	40	32	36	23	30	23	37
Generalizing	11	6	11	3	7	14	12	11	5	5
Relevant	14	28	21	13	16	13	17	22	21	17
ttitude										
Curiosity	12	15	6	2	9	12	2	0	0	3
Independence	12	10	7	7	8	6	2	8	7^1	8
Receptivity	64^1	67^1	88	79	78	82	81	81	82	97^1
eutral	2	1	8	8	8	0	4	2	0	1
OTAL TEACHER	90	93	109	96	103	100	89	91	89	109
ontent					PUPILS VI					
Structure	0	0	0	0	0	1	0	1	0	0
Techniques	0	1	0	0	0	0	2	1	0	0
Deductive	1	0	0	0	2^1	0^1	1	1	1	1
Inductive	1	1	1	0	0	0	0	0	0	0
Mathematical	1	0	1	1	0	0	0	0	6	0
Other	0	0	0	0	0	0	0	0	0	0
rocess										
Analyzing	0	0	0	0	1	0	0	0	1	0
Synthesizing	1	1	0	0	0	1	0	1	1	0
Specializing	2	0	1	1	2	1	1	1	5	1
Generalizing	0	0	1	0	0	0	0	0	0	0
Relevant	0	1	0	0	0	0	2	1	0	0
ttitude										
Curiosity	0	0	0	0	1	0	0	0	0	0
Independence	2	2	1	1	2	1	1	1	4	1
Receptivity	1	0	1	0	0	1	2	2	3	0
eutral	0	0	0	0	0	0	0	0	0	0
OTAL PUPILS	3	2	2	1	3	2	3	3	7	1
TOTAL T & P	93	95	111	97	106	102	92	94	96	110

Appendix A

Table 152 Frequency of behaviors in each category of Content, of Process and of Attitudes, and in Neutral for Teacher VI and Pupils VI as recorded by Observer II for the ten periods of classroom visits. Superscripts indicate frequency of negative behavior in individual categories.

DAYS CATEGORY	1	2	3	4	5	6	7	8	9	10
Content				TEACHER VI						
Structure	-	3	0	7	-	-	8	-	-	15
Techniques	-	15	22	12	-	-	20	-	-	25
Deductive	-	51	10	23	-	-	40	-	-	37
Inductive	-	7	36	0	-	-	15	-	-	0
Mathematical	-	2	27	41	-	-	0	-	-	30
Other	-	15	4	5	-	-	3	-	-	3
Process										
Analyzing	-	3	6	16	-	-	4	-	-	15
Synthesizing	-	25	27	30	-	-	7	-	-	30
Specializing	-	33	35	27	-	-	43	-	-	36
Generalizing	-	4	10	3	-	-	12	-	-	11
Relevant	-	27	20	12	-	-	19	-	-	18
Attitude										
Curiosity	-	15	6	2	-	-	4	-	-	5
Independence	-	15	8	11	-	-	2	-	-	14
Receptivity	-	63	85	75	-	-	80	-	-	91
Neutral	-	2	9	8	-	-	3	-	-	0
TOTAL TEACHER	-	95	108	96	-	-	89	-	-	110
Content				PUPILS VI						
Structure	-	0	0	0^1	-	-	0	-	-	0
Techniques	-	0	0	0	-	-	2	-	-	0
Deductive	-	0	0	0	-	-	1	-	-	1
Inductive	-	0	2	0	-	-	0	-	-	0
Mathematical	-	1	1	0	-	-	0	-	-	0
Other	-	0	0	0	-	-	0	-	-	0
Process										
Analyzing	-	0	1	0	-	-	0	-	-	0
Synthesizing	-	0	0	0	-	-	0	-	-	0
Specializing	-	0	2	1	-	-	1	-	-	1
Generalizing	-	0	0	0	-	-	0	-	-	0
Relevant	-	1	0	0	-	-	2	-	-	0
Attitude										
Curiosity	-	0	0	0	-	-	0	-	-	0
Independence	-	1	2	1	-	-	0	-	-	1
Receptivity	-	0	1	0	-	-	3	-	-	0
Neutral	-	0	0	0	-	-	0	-	-	0
TOTAL PUPILS	-	1	3	1	-	-	3	-	-	1
TOTAL T & P	-	96	111	97	-	-	92	-	-	111

Appendix A

Table 153 Frequency of behaviors in each category of Content, of Process and of Attitudes, and in Neutral for Teacher VIII and Pupils VIII as recorded by Observer I for the ten periods of classroom visits. Superscripts indicate frequency of negative behavior in individual categories.

DAYS CATEGORY	1	2	3	4	5	6	7	8	9	10
Content					TEACHER VIII					
Structure	14	2	4	8	6	7	8	7	15^1	10
Techniques	17	5	17	16	19	20	28	14	17	22
Deductive	18	10	10	9	4	21	25	6	3	20
Inductive	0	21	2	0	0	1	13	0	14	13
Mathematical	29	17	34	47	46	7	15	20	25	6
Other	0^2	0	0	6	6	0	0	2	0	0
Process										
Analyzing	8	1	0	1	2	0	3	6	9	4
Synthesizing	15	13	14	16	29	6	11	9	11	6
Specializing	22	21	23	47	35	17	43	17	33	28
Generalizing	12	4	9	3	2	11	7	5	9	11
Relevant	21	16	21	19	13	22	25	12	13	22
Attitude										
Curiosity	0	0	0	0	0	0	2	0	0	6
Independence	39	34	43	53^1	33	11^1	57	21	25^1	12
Receptivity	41	21	24	32	48	44	30	28	47^1	53
Neutral	0	0	3	1	2	3	1	1	0	2
TOTAL TEACHER	80	55	70	87	83	59	90	50	75	73
Content					PUPILS VIII					
Structure	0	0	0	0	1	0	0	2	6	0
Techniques	0	1	0	1	2	0	1	1	0	0
Deductive	3	1	0	2^2	1	1^2	1	0	2	2^2
Inductive	0	2^1	0	0	0	0	1	0	0	3
Mathematical	7	11	6^2	4	6^1	4^1	3^1	2	11	1
Other	0	0	0	0	0	0	0	0	0	0
Process										
Analyzing	1	1	0	1	1	1	1	1	6	0
Synthesizing	2	8	4	1	1	2	1	1	5	0
Specializing	5	3^1	3	7	8	5	4	2	8	7
Generalizing	0	1	0	0	0	0	0	1	0	1
Relevant	2	2	0	0	0	0	1	0	0	0
Attitude										
Curiosity	0	0	0	0	0	0	0	1	3	0
Independence	7	10	4	4	6	4	3	3	16	6
Receptivity	3	6	3^1	5	5	4	4	1	0	2
Neutral	0	0	0	0	0	0	0	0	0	0
TOTAL PUPILS	10	16	8	9	11	8	7	5	19	8
TOTAL T & P	90	71	78	96	94	65	97	55	94	81

Appendix A

Table 154 Frequency of behaviors in each category of Content, of Process and of Attitudes, and in Neutral for Teacher VIII and Pupils VIII as recorded by Observer II for the ten periods of classroom visits. Superscripts indicate frequency of negative behavior in individual categories.

DAYS CATEGORY	1	2	3	4	5	6	7	8	9
Content					TEACHER VIII				
Structure	5^2	3	3	8	5	6^1	8	5	24^5
Techniques	20	4	11	20	21	18	24	8	17
Deductive	18	5	7	11	2	23^2	33	5	6^2
Inductive	0	20	4	9	9	9	16	0	3
Mathematical	28	23	41	39	48	6	8	16	15^2
Other	0^2	0	0	7	7	0	0	3	0
Process									
Analyzing	5	4	0	1	1	3	6	3	5
Synthesizing	13	13	20	17	26	4	9	9	11
Specializing	16	22	17	42	40	16	45	11	34
Generalizing	16^3	5	7	4	1	14	7	3	7^2
Relevant	22	11	22	21	15	19	22	11	15
Attitude									
Curiosity	0	1	0	0	0	0	1	1	5
Independence	27	29	42	42^1	32^2	12^2	53	16	21^4
Receptivity	48	25	24	42	49	42	35	20	44
Neutral	0	0	3	1	2	3	1	1	0
TOTAL TEACHER	75	55	69	86	85	59	90	38	74
Content					PUPILS VIII				
Structure	1	1	0	0	1^1	0	0	2	5^3
Techniques	2	1	0	1	2	0	1	0	0
Deductive	3	0	1	3^3	0	1^2	1	1	2
Inductive	0	2^1	0	0	0	0	1	0	0
Mathematical	8^1	11	4^4	3	4	4^1	4	3	9
Other	0	0	0	0	1	0	0	0	0
Process									
Analyzing	0	2	0	1	0	1	1	1	7
Synthesizing	5	8	5	2	2	0	3	0	4
Specializing	6	3	4	7	7	7	2	4	8
Generalizing	1	2	0	9	0	0	0	1	0
Relevant	3	1	0	0	0	0	1	0	0
Attitude									
Curiosity	0	0	1	0	0	0	0	1	3
Independence	9	8	1	7	5	5	4	5	14
Receptivity	6	8	7	3	4	3	3	0	2
Neutral	0	0	0	0	0	0	0	0	0
TOTAL PUPILS	15	16	9	10	9	8	7	6	19
TOTAL T & P	90	71	78	95	94	87	97	44	93

Table 155 Frequency of behaviors in each category of Content, of Process and of Attitudes, and in Neutral for Teacher IX and Pupils IX as recorded by Observer I for the ten periods of classroom visits. Superscripts indicate frequency of negative behavior in individual categories.

DAYS CATEGORY	1	2	3	4	5	6	7	8	9	10
Content				TEACHER IX						
Structure	1	7	4	11	12	15	7	11	17	18
Techniques	15	6	2	11	10	21	14	9	16	6
Deductive	1	17	7	3	12	4	12	16	7	36
Inductive	0	11	0	23	25	2	2	11	0	2
Mathematical	39	25	25	21	4	26	19	29	27	19
Other	0	0	7	0	0	0	9	4	0	0
Process										
Analyzing	3	1	1	4	9	5	2	11	10	7
Synthesizing	17	14	12	3	2	10	11	25	12	16
Specializing	22	34	21	43	24	38	31	20	36	36
Generalizing	1	8	7	11	16	3	0	13	5	14
Relevant	13	9	4	8	12	12	19	11	4	8
Attitude										
Curiosity	0	0	2	0	0	0	3	5	0	8
Independence	44	59	27	58	50	49	30	38	51	62
Receptivity	12	7	16	11	13	19	30	37	16	11
Neutral	0	0	0	4	0	0	1	0	0	0
TOTAL TEACHER	56	66	45	73	63	68	64	80	67	81
Content				PUPILS IX						
Structure	0	0	0	2^1	1^2	5^4	2	1^1	0	3
Techniques	1	1	0	0	0	11^1	2	0	5	0^1
Deductive	0	13	0	1	6^1	1	2	4^4	3	9^1
Inductive	0	0	0	0	4	0	0	0	0	0
Mathematical	18^1	11	7	2	10	10^2	10^7	3	7	1
Other	0	0	1^1	0	0	0	0	0	0	0
Process										
Analyzing	1	0	0	2	0	3^1	0	2	1	1
Synthesizing	11^1	11	6	0	2	10	7	1	5	4
Specializing	2^1	11	3	4	15	19	12	5	9	8
Generalizing	0	0	0	0	4	0	2	4	0	2
Relevant	4	3	0	0	3	1	2	1	0	0
Attitude										
Curiosity	0	0	0	0	2	0	0	1	0	0
Independence	16	12	6	5	17	24	17	8	9	7
Receptivity	4	13	3	1	5	10	6	4	6	8
Neutral	0	0	0	0	0	0	0	0	0	0
TOTAL PUPILS	20	25	9	6	24	34	23	13	15	15
TOTAL T & P	76	91	54	79	87	102	87	93	82	96

Appendix A

Table 156 **Frequency of behavior in each category of Content, of Process and of Attitudes, and in Neutral for Teacher IX and Pupils IX as recorded by Observer II for the ten periods of classroom visits. Superscripts indicate frequency of negative behavior in individual categories.**

DAYS CATEGORY	1	2	3	4	5	6	7	8	9	10
					TEACHER IX					
Content										
Structure	4	10	4	15	13^1	13	8	14	16	-
Techniques	15	12	2	23	14	23	14	9	9	-
Deductive	3	12	9	7	25	3	7	20	9	-
Inductive	0	9	0	17	7	1	2	5	0	-
Mathematical	37	19	20	8	3	25	17	26	33	-
Other	0	0	10	1	0	0	12	5	0	-
Process										
Analyzing	7	0	1	6	11	4	6	7	7	-
Synthesizing	19	18	11	9	1	17	8	15	14	-
Specializing	20	25	21	41	22	35	31	41	35	-
Generalizing	2	8	7	8	15	0	1	8	5	-
Relevant	11	11	5	7	14	9	14	8	6	-
Attitude										
Curiosity	0	1	2	5	1	0	5	3	0	-
Independence	40	38	20	57	44	38	24	31	37	-
Receptivity	18^1	23	23	9	18	27	31	45	30	-
Neutral	0	0	0	2	0	0	3	0	0	-
TOTAL TEACHER	59	62	45	73	63	65	63	79	67	-
					PUPILS IX					
Content										
Structure	0	1^1	0	2	4^1	4^7	2	3^3	2	-
Techniques	0	2	0	0	0	10^3	2	0	2	-
Deductive	0	10	0	1	4^1	1^3	3^1	2^2	2	-
Inductive	0	1	0	1	2	0	0	0	0	-
Mathematical	16^1	13^1	6^1	0	11	10	10^3	4	9	-
Other	0	0	1^1	0	0	0	3	0	0	-
Process										
Analyzing	1	0	0	2	0	4	2	1	1	-
Synthesizing	11^1	12^1	6	0	1	10	7	3	7	-
Specializing	2	11^1	2	2	16	18	10	5	6	-
Generalizing	0	2	0^1	0	5	0	2	5	0	-
Relevant	2	2	0	0	2	5	3	0	1	-
Attitude										
Curiosity	0	0	0	1	3	0	0	1	0	-
Independence	14	12	6	3	16	28	15	7	9	-
Receptivity	3	17	3	0	4	7^1	9	6	6	-
Neutral	0	0	0	0	0	0	0	0	0	-
TOTAL PUPILS	17	29	9	4	23	36	24	14	15	-
TOTAL T & P	76	91	54	77	86	101	87	93	82	-

Appendix A

Table 157 Frequency of behavior in each category of Content, of Process and of Attitudes, and in Neutral for Teacher X and Pupils X as recorded by Observer I for the ten periods of classroom visits. Superscripts indicate frequency of negative behavior in individual categories.

DAYS CATEGORY	1	2	3	4	5	6	7	8	9	10
					TEACHER X					
Content										
Structure	25^1	29	28	13	16	24	20	8	9	20
Techniques	9	9	9	12	6	7	2	5	0	6
Deductive	8	8	0	4	0	0	0	8	14	5
Inductive	1	24^1	0	0	2	0	0	12	30	1
Mathematical	30	14	26	1	23	40	50	21	3	38
Other	0	7	2	9	0	4	4	0	2	0
Process										
Analyzing	10	11	13	3	10	3	13	7	17	16
Synthesizing	23	25	17	1	12	20	28	10	11	25
Specializing	16	35	22	9	19	32	22	24	23	23
Generalizing	15	8	5	7	0	11	8	9	6	2
Relevant	9	12	7	19	6	9	5	4	0	4
ttitude										
Curiosity	4	5	0	6	3	1	1	0	0	0
Independence	11	8	20	6	8	12	8	17	15	29
Receptivity	59	79	45	27	36	62	67	37	43	41
eutral	7	0	2	6	3	2	0	13	1	5
OTAL TEACHER	81	92	67	45	50	77	76	67	59	75
					PUPILS X					
ontent										
Structure	2	0	4^1	1	3	3	4	0^1	0^2	2^2
Techniques	2	1	2	5	3	9	1	0	0	3
Deductive	2	1	1	5	0^1	0	0	2	0	0
Inductive	0	1	0	0	0	0	0	0	5	0
Mathematical	1	6	4^1	1	13	19	2^1	9^1	3	22
Other	0	0	0	2	0	0	0	0	0	0
rocess										
Analyzing	2	0	2	0	1	1	2	0	2	3
Synthesizing	0	3	5	0	7	10	0	8	1	13
Specializing	3	6	4	0	8	11	5	4	7	12
Generalizing	1	0	0	2	0	1	0	1	0	0
Relevant	1	0	2	12	4	8	1	0	0	1
ttitude										
Curiosity	2	0	0	0	1	1	1	0	0	0
Independence	3	7	8	2	8	18^2	4	9	6	28
Receptivity	2	2	5	11^1	11	9^1	3	4	4	1
eutral	2	0	0	0	0	0	1	0	0	0
OTAL PUPILS	9	9	13	14	20	31	9	13	10	29
TOTAL T & P	90	101	80	59	70	108	85	80	69	104

Appendix A

Table 158 Frequency of behavior in each category of Content, of Process and of Attitudes, and in Neutral for Teacher X and Pupils X as recorded by Observer II for the ten periods of classroom visits. Superscripts indicate frequency of negative behavior in individual categories.

DAYS CATEGORY	1	2	3	4	5	6	7	8	9	10
				TEACHER X						
Content										
Structure	26	30	20	-	-	-	-	-	10	22
Techniques	9^1	12	7	-	-	-	-	-	0	7
Deductive	9	8	0	-	-	-	-	-	13	3
Inductive	1	21	0	-	-	-	-	-	33	0
Mathematical	28	14	32	-	-	-	-	-	0	33
Other	0	7	2	-	-	-	-	-	2	1
Process										
Analyzing	9	9	14	-	-	-	-	-	19	14
Synthesizing	26	24	14	-	-	-	-	-	11	14
Specializing	18	35	20	-	-	-	-	-	20	31
Generalizing	12	13	7	-	-	-	-	-	8	1
Relevant	8	11	6	-	-	-	-	-	0	6
Attitude										
Curiosity	4	6	0	-	-	-	-	-	0	0
Independence	16	12	18	-	-	-	-	-	14	21
Receptivity	54	74	43	-	-	-	-	-	44	45
Neutral	7	0	2	-	-	-	-	-	1	5
TOTAL TEACHER	81	92	63	-	-	-	-	-	59	71
				PUPILS X						
Content										
Structure	2	0^1	2^1	-	-	-	-	-	5^3	4^2
Techniques	2	0^1	2	-	-	-	-	-	0	5
Deductive	2	1^1	0^1	-	-	-	-	-	0	0
Inductive	0	0	0	-	-	-	-	-	1	0
Mathematical	1	5	7^4	-	-	-	-	-	1	19^3
Other	0	0	0	-	-	-	-	-	0	0
Process										
Analyzing	1	0	2	-	-	-	-	-	4	3
Synthesizing	0	3	5	-	-	-	-	-	1	15
Specializing	3	2	7	-	-	-	-	-	4	13
Generalizing	1	3	1	-	-	-	-	-	1	1
Relevant	2	1	2	-	-	-	-	-	0	1
Attitude										
Curiosity	2	0	0	-	-	-	-	-	0	0
Independence	3	7	12	-	-	-	-	-	8	32
Receptivity	2	2	5	-	-	-	-	-	2	1
Neutral	2	0	0	-	-	-	-	-	0	0
TOTAL PUPILS	9	9	17	-	-	-	-	-	10	33
TOTAL T & P	90	101	80	-	-	-	-	-	69	104

Appendix A

Table 159 Frequency of behavior in each category of Content, of Process and of Attitudes, and in Neutral for Teacher XI and Pupils XI recorded by Observer I for the ten periods of classroom visits. Superscripts indicate frequency of negative behavior in individual categories.

DAYS ATEGORY	1	2	3	4	5	6	7	8	9	10
				TEACHER XI						
ontent										
Structure	18	4	3	3	27	8	5	17	11	10
Techniques	10	19	29	7	11	23	12	1	15	21
Deductive	38^1	0	1	21	10	7	13	3	13	3
Inductive	1	0	0	23	22	26	21	75	3	1
Mathematical	25	21	7	21	9	16	3	2	8	28
Other	0	5	9	6	2	3	1	0	0	1
rocess										
Analyzing	27	4	4	6	9	2	9	25	7	14
Synthesizing	19	10	2	13	6	7	7	20	7	9
Specializing	34	14	13	49	50	42	21	51	16	21
Generalizing	7	2	1	4	11	7	4	2	5	1
Relevant	5	19	29	8	5	25	14	0	15	19
ttitude										
Curiosity	3	0	10	6	2	4	1	7	1	1
Independence	32	28	14	34	43	48	15	67	30	19
Receptivity	56	21	25	41	36	31	39	24	19	44
eutral	0	5	0	0	6	8	6	1	8	2
OTAL TEACHER	91	54	49	81	87	91	61	99	58	66
				PUPILS XI						
ontent										
Structure	2	1^1	3	0^1	2^1	1	1	1	0	1
Techniques	0	0	1	0	1	0	2	0	1	0
Deductive	7^3	0^1	14	0^1	1	13	27	0	48	3^1
Inductive	0	0	0	6	6	1^1	3	8	0	0
Mathematical	6^1	41^1	40	8^1	0	4	12	0	0	16
Other	0	0	5	3	0	0	0	0	0	0
ocess										
Analyzing	7	2	9	2	1	3	1	1	2	2
Synthesizing	8	34	41	7	1	9	35	5	45	15
Specializing	6	9	12	8	9	6	7	3	1	4
Generalizing	0	0	0	0	0	2	0	0	0	0
Relevant	0	0	0	3	0	0	2	0	1	0
titude										
Curiosity	2	1	0	0	0	0	0	1	0	1
Independence	17	43	62	16	9	18	28	8	48	19
Receptivity	2	1	1	4	2	2	17	0	1	1
utral	0	0	0	0	0	0	0	0	0	0
TAL PUPILS	21	45	63	20	11	20	45	9	49	21
OTAL T & P	112	99	112	101	98	111	106	108	107	87

Appendix A

Table 160 **Frequency of behavior in each category of Content, of Process and of Attitudes, and in Neutral for Teacher XI and Pupils XI as recorded by Observer II for the ten periods of classroom visits. Superscripts indicate frequency of negative behavior in individual categories.**

DAYS	1	2	3	4	5	6	7	8	9	10
CATEGORY										
				TEACHER XI						
Content										
Structure	-	-	2	5	21	11	8	-	-	-
Techniques	-	-	31	11	15	16	10	-	-	-
Deductive	-	-	2	19	8	12	15	-	-	-
Inductive	-	-	0	21	18	19	18	-	-	-
Mathematical	-	-	7	18	10	18	2	-	-	-
Other	-	-	7	8	4	4	1	-	-	-
Process										
Analyzing	-	-	5	9	8	10	7	-	-	-
Synthesizing	-	-	2	9	6	7	11	-	-	-
Specializing	-	-	10	42	37	36	20	-	-	-
Generalizing	-	-	3	9	17	9	4	-	-	-
Relevant	-	-	29	13	8	18	12	-	-	-
Attitude										
Curiosity	-	-	7	8	2	2	1	-	-	-
Independence	-	-	13	33	38	33	12	-	-	-
Receptivity	-	-	29	41	36	45	41	-	-	-
Neutral	-	-	0	0	3	9	5	-	-	-
TOTAL TEACHER	-	-	49	82	79	89	59	-	-	-
				PUPILS XI						
Content										
Structure	-	-	2	0^2	6^1	1	4	-	-	-
Techniques	-	-	1	2	3	0	2	-	-	-
Deductive	-	-	14	0	1	13	27	-	-	-
Inductive	-	-	0	6^1	6^1	6	2	-	-	-
Mathematical	-	-	45	9	0	3	13	-	-	-
Other	-	-	1	3	0	0	0	-	-	-
Process										
Analyzing	-	-	12	2	2	2	2	-	-	-
Synthesizing	-	-	41	7	0	13	25	-	-	-
Specializing	-	-	10	8	12	7	19	-	-	-
Generalizing	-	-	0	2	3	1	0	-	-	-
Relevant	-	-	0	4	1	0	2	-	-	-
Attitude										
Curiosity	-	-	0	0	0	0	0	-	-	-
Independence	-	-	62	19	12	21	33	-	-	-
Receptivity	-	-	1	4	6	2	15	-	-	-
Neutral	-	-	0	0	0	0	1	-	-	-
TOTAL PUPILS	-	-	63	23	18	23	49	-	-	-
TOTAL T & P	-	-	112	105	97	112	108	-	-	-

Appendix A

Table 161 Frequency of behavior in each category of Content, of Process and of Attitudes, and in Neutral for Teacher XII and Pupils XII as recorded by Observer I for the ten periods of classroom visits. Superscripts indicate frequency of negative behavior in individual categories.

DAYS ATEGORY	1	2	3	4	5	6	7	8	9	10
ontent				TEACHER XII						
Structure	10	2	3	2	4	0	0	1	1	0
Techniques	10	2	7	17	2	3	7	6	10	8
Deductive	1	10	3	0	5	21	8	7	31	2
Inductive	1	29	34	0	25	25	1	5	16	0
Mathematical	53	9	3	37	0	16	13	12	15	14
Other	0	4	1	0	0	0	0	2	4	0
rocess										
Analyzing	7	0	4	3	4	7	1	4	2	1
Synthesizing	25	11	21	19	13	20	6	8	20	1
Specializing	31	21	16	19	8	30	13	13	39	11
Generalizing	1	15	3	0	7	4	2	2	4	2
Relevant	11	9	7	15	2	3	7	6	12	9
ttitude										
Curiosity	0	1	1	1	2	1	0	1	3	0
Independence	4	5	7	4	5	1	3	2	3	2
Receptivity	71	47^3	43	51	29	63	26	30	71	22
eutral	0	1	0	3	0	1	0	1	0	2
OTAL TEACHER	75	57	51	59	36	66	29	34	77	26
ontent				PUPILS XII						
Structure	2	1	1^1	0	0	0	0	1	1	0
Techniques	0	0	0	1	0	1	1	0	1	1
Deductive	0	1	0^1	0	0	0^1	2^2	1	2^1	0
Inductive	0	0	5	0	6	0	0	0	1	0
Mathematical	1	2	0	4	0	1	1	1	3	1
Other	0	1^1	0	0	0	0	0	0	1	0
rocess										
Analyzing	1	0	1	1	0	0	0	0	2	0
Synthesizing	0	0	1	0	0	0	0	0	0	0
Specializing	2	4	6	3	6	2	3	3	6	2
Generalizing	0	2	0	0	0	0	2	0	0	0
Relevant	0	0	0	1	0	1	1	0	2	0
ttitude										
Curiosity	0	3	2	1	0	0	0	2	3	0
Independence	2	3	6	4	5	1	3	0	3	0
Receptivity	1	0	0	0	1	2	3	1	4	2
eutral	0	1	0	0	0	1	0	0	0	0
OTAL PUPILS	3	7	8	5	6	4	6	3	10	2
TOTAL T & P	78	64	59	64	42	70	35	37	87	28

Appendix A

Table 162 Frequency of behavior in each category of Content, of Process and of Attitudes, and in Neutral for Teacher XII and Pupils XII as recorded by Observer II for the ten periods of classroom visits. Superscripts indicate frequency of negative behavior in individual categories.

DAYS CATEGORY	1	2	3	4	5	6	7	8	9	10
Content					TEACHER XII					
Structure	9^1	1	0	-	3^1	0	-	-	-	-
Techniques	9	6	7	-	15	7	-	-	-	-
Deductive	0	10^1	4	-	1	19^1	-	-	-	-
Inductive	1	23^2	38	-	0	18	-	-	-	-
Mathematical	55	12	2	-	36	19	-	-	-	-
Other	0	1	0	-	0	0	-	-	-	-
Process										
Analyzing	9	2	7	-	5	6	-	-	-	-
Synthesizing	21	10	12	-	18	17	-	-	-	-
Specializing	34	25	24	-	18	31	-	-	-	-
Generalizing	0	11	2	-	1	6	-	-	-	-
Relevant	11	8	6	-	14	4	-	-	-	-
Attitude										
Curiosity	0	0	1	-	0	0	-	-	-	-
Independence	6^1	4	3	-	3	3	-	-	-	-
Receptivity	68	52	47	-	53	61	-	-	-	-
Neutral	0	0	0	-	4	1	-	-	-	-
TOTAL TEACHER	75	56	51	-	60	65	-	-	-	-
Content					PUPILS XII					
Structure	2	1	1^1	-	1	0	-	-	-	-
Techniques	0	0	0	-	0	2	-	-	-	-
Deductive	0	3	0^1	-	0	0^1	-	-	-	-
Inductive	0	2	5	-	0	0	-	-	-	-
Mathematical	1	1	0	-	2^1	1	-	-	-	-
Other	0	0	0	-	0	0	-	-	-	-
Process										
Analyzing	0	2	3	-	1	0	-	-	-	
Synthesizing	0	0	0	-	0	0	-	-	-	
Specializing	3	3	5	-	3	2	-	-	-	
Generalizing	0	2	0	-	0	0	-	-	-	
Relevant	0	0	0	-	0	2	-	-	-	
Attitude										
Curiosity	0	2	0	-	0	0	-	-	-	
Independence	3	5	8	-	3	4	-	-	-	
Receptivity	0	0	0	-	1	0	-	-	-	
Neutral	0	1	0	-	0	0	-	-	-	-
TOTAL PUPILS	3	8	8	-	4	4	-	-	-	-
TOTAL T & P	78	64	59	-	64	69	-	-	-	-

Appendix A

Table 163 Frequency of behavior in each category of Content, of Process and of Attitudes, and in Neutral for Teacher XIII and Pupils XIII as recorded by Observer I for the ten periods of classroom visits. Superscripts indicate frequency of negative behavior in individual categories.

DAYS CATEGORY	1	2	3	4	5	6	7	8	9	10
				TEACHER XIII						
Content										
Structure	20	21	9	61	35	-	-	-	-	-
Techniques	11	3	7	1	6	-	-	-	-	-
Deductive	8	28	19	1	4	-	-	-	-	-
Inductive	36	11	18	14	4	-	-	-	-	-
Mathematical	5	13	11	1	39	-	-	-	-	-
Other	7	2	4	3	6	-	-	-	-	-
Process										
Analyzing	7	10	12	18	19	-	-	-	-	-
Synthesizing	16	26	12	7	22	-	-	-	-	-
Specializing	43	29	26	41	36	-	-	-	-	-
Generalizing	10	9	9	10	9	-	-	-	-	-
Relevant	11	4	8	5	8	-	-	-	-	-
ttitude										
Curiosity	18	0	3	10	2	-	-	-	-	-
Independence	23	46	39	32	53	-	-	-	-	-
Receptivity	46	32	26	39	39	-	-	-	-	-
eutral	2	1	1	1	1	-	-	-	-	-
OTAL TEACHER	89	79	69	82	95	-	-	-	-	-
				PUPILS XIII						
ontent										
Structure	2	14	2	15^2	4	-	-	-	-	-
Techniques	1	0	2	1	0	-	-	-	-	-
Deductive	1	6	2^1	3	0	-	-	-	-	-
Inductive	5	0	6	5	0	-	-	-	-	-
Mathematical	2	6	26	1	10	-	-	-	-	-
Other	0	1^1	2	0	1^1	-	-	-	-	-
rocess										
Analyzing	0	3	5	2	1	-	-	-	-	-
Synthesizing	3	11	25	6	4	-	-	-	-	-
Specializing	8	12	8	10	8	-	-	-	-	-
Generalizing	0	2	1	8	1	-	-	-	-	-
Relevant	0	0	2	1	2	-	-	-	-	-
ttitude										
Curiosity	2	1	1	0	1	-	-	-	-	-
Independence	6	21	36	22	11	-	-	-	-	-
Receptivity	3	6	4	5	4	-	-	-	-	-
eutral	0	0	0	0	0	-	-	-	-	-
OTAL PUPILS	11	28	41	27	16	-	-	-	-	-
TOTAL T & P	100	107	110	109	111	-	-	-	-	-

Appendix A

Table 164 Frequency of behavior in each category of Content, of Process and of Attitudes, and in Neutral for Teacher XIII and Pupils XIII as recorded by Observer II for the ten periods of classroom visits. Superscripts indicate frequency of negative behavior in individual categories.

DAYS CATEGORY	1	2	3	4	5	6	7	8	9	10
					TEACHER XIII					
Content										
Structure	22	21	7	54	26	6	17	13	12	11
Techniques	11	6	5	2^{1}	3	6	10	4	1	7
Deductive	9	27	16	2	3	0	0	21	16	8
Inductive	33	11	14	12	6	15	8	19	23	16
Mathematical	5	8	14	1	45	36	28	0	3	21
Other	7	3	4	3	7	5	4	10	5	11
Process										
Analyzing	10	13	16	19	19	26	9	10	9	23
Synthesizing	12	17	7	3	20	15	4	0	3	13
Specializing	47	31	25	32	35	20	34	43	40	26
Generalizing	10	9	4	16	11	1	10	10	8	8
Relevant	8	6	8	5	5	6	10	4	0	4
Attitude										
Curiosity	14	1	3	11	2	3	2	3	9	2
Independence	25	48	38	26	49	24	29	27	27	44
Receptivity	48	27	19	38	39	41	36	37	24	28
Neutral	2	1	0	1	0	3	0	0	1	1
TOTAL TEACHER	89	77	60	76	90	71	67	67	61	75
					PUPILS XIII					
Content										
Structure	3^{2}	14^{1}	2	18^{3}	9	0	7	6^{2}	7^{2}	4
Techniques	2	0	2	1	0	4	2	1	1	3
Deductive	0	6	4^{1}	3	1	0	0	19	24	5
Inductive	2^{1}	1	4	6	0	6	1	10	5	5
Mathematical	0^{1}	4^{2}	25	1	9	20^{1}	29^{2}	0	4	13
Other	0	1^{1}	1^{1}	1	2	4^{4}	1	1	3	7
Process										
Analyzing	1	1	10	4	2	13	10	12	7	11
Synthesizing	2	9	22	5	3	6	16	2	14	11
Specializing	8	14	5	15	13	16	12	21	21	12
Generalizing	0	3	1	8	3	0	2	4	4	3
Relevant	0	3	2	1	0	4	2	0	0	2
Attitude										
Curiosity	0	1	3	0	1	1	0	2	3	0
Independence	10	25	34	29	20	29	36	37	37	35
Receptivity	1	4	3	4	0	9	6	0	6	4
Neutral	0	0	0	0	0	0	0	0	0	0
TOTAL PUPILS	11	30	40	33	21	39	42	39	46	39
TOTAL T & P	100	107	100	109	111	110	108	106	107	114

Appendix A

Table 165 Frequency of behavior in each category of Content, of Process and of Attitudes, and in Neutral for Teacher XIV and Pupils XIV as recorded by Observer I for the ten periods of classroom visits. Superscripts indicate frequency of negative behavior in individual categories.

DAYS CATEGORY	1	2	3	4	5	6	7	8	9	10
					TEACHER XIV					
Content										
Structure	16	-	11	3	4	2	-	-	-	-
Techniques	18	-	11	6	5	14	-	-	-	-
Deductive	0	-	0	0	5	1	-	-	-	-
Inductive	1	-	0	2	0	0	-	-	-	-
Mathematical	25	-	12	8	42	36	-	-	-	-
Other	0	-	0	1	2	16	-	-	-	-
Process										
Analyzing	12	-	0	0	3	6	-	-	-	-
Synthesizing	18	-	8	7	30	10	-	-	-	-
Specializing	11	-	20	5	19	35	-	-	-	-
Generalizing	6	-	5	2	0	0	-	-	-	-
Relevant	13	-	6	6	7	18	-	-	-	-
Attitude										
Curiosity	0	-	1	0	2	0	-	-	-	-
Independence	6	-	6	2	8	6	-	-	-	-
Receptivity	54	-	32	18	49	63	-	-	-	-
Neutral	3	-	5	15	4	8	-	-	-	-
TOTAL TEACHER	63	-	44	35	63	77	-	-	-	-
					PUPILS XIV					
Content										
Structure	5	-	0	0	1	0	-	-	-	-
Techniques	3	-	0	1	2	4	-	-	-	-
Deductive	0	-	0	0	0	0	-	-	-	-
Inductive	0	-	0	0	0	0	-	-	-	-
Mathematical	5	-	1	1	16	12	-	-	-	-
Other	0	-	0	0	1	0	-	-	-	-
Process										
Analyzing	1	-	0	0	1	1	-	-	-	-
Synthesizing	3	-	1	1	10	3	-	-	-	-
Specializing	10	-	2	0	11	14	-	-	-	-
Generalizing	0	-	0	0	0	0	-	-	-	-
Relevant	3	-	0	1	5	3	-	-	-	-
Attitude										
Curiosity	0	-	0	0	2	0	-	-	-	-
Independence	10	-	3	2	18	13	-	-	-	-
Receptivity	7	-	0	0	7	8	-	-	-	-
eutral	0	-	1	0	0	0	-	-	-	-
OTAL PUPILS	17	-	4	2	27	21	-	-	-	-
TOTAL T & P	80	-	48	37	90	98	-	-	-	-

Appendix A

Table 166 Frequency of behavior in each category of Contents, of Process and of Attitudes, and in Neutral for Teacher XIV and Pupils XIV as recorded by Observer II for the ten periods of classroom visits. Superscripts indicate frequency of negative behavior in individual categories.

DAYS CATEGORY	1	2	3	4	5	6	7	8	9	10
					TEACHER XIV					
Content										
Structure	18	1	10^{8}	2	3^{1}	2	8^{1}	4	1^{3}	2^{7}
Techniques	16	9	11	10	7	20	33	13	14^{1}	16
Deductive	0	0	0	0	1	1	0	0	0	0
Inductive	0	0	0	1	0	0	1	10	0	0
Mathematical	26	37^{1}	8	10	42	30	38	34	31	34
Other	0	0	0	1	1	18	0	0	0	4
Process										
Analyzing	10	9	3	0	4	5	5	2	1	3
Synthesizing	20	15	6	7	22	9	40	16	21	31
Specializing	13	16	18	4	20	37	26	26	13	17
Generalizing	5	1	4	2	2	1	0	6	1	6
Relevant	12	7	6	11	7	19	11	11	14	6
Attitude										
Curiosity	0	0	1	0	0	0	0	0	0	0
Independence	10	4	6	6	9	4	6	4	1	10
Receptivity	50	44	30	18	46	67	63	57	49	53
Neutral	3	14	5	10	4	5	8	6	10	5
TOTAL TEACHER	63	62	42	34	59	76	77	67	60	68
					PUPILS XIV					
Content										
Structure	4^{2}	0^{1}	0	0	2^{2}	0^{2}	1^{2}	2^{3}	0^{2}	0
Techniques	3	4	1	0	4	4	7	2^{1}	2	14^{2}
Deductive	0	0	0	0	0	0	0	0	0	0
Inductive	0	0	0	0	0	0	0	0	0	0
Mathematical	7^{1}	4^{7}	1^{2}	1	16^{6}	9^{4}	5^{3}	8^{3}	10^{3}	4^{1}
Other	0	0	0	0	1	0^{1}	0	0	0	1
Process										
Analyzing	1	3	0	0	2	0	3	0	1	0
Synthesizing	6	0	1	1	9	2	1	6	1	1
Specializing	6	5	3	0	12	15	9	12	11	11
Generalizing	0	0	0	0	0	0	0	0	0	0
Relevant	4	8	0	0	8	3	5	1	4	10
Attitude										
Curiosity	0	0	0	0	3	0	0	0	0	1
Independence	12	10	4	1	23	14	13	10	12	7
Receptivity	5	7	0	0	5	6	8	9	5	14
Neutral	0	0	1	0	0	0	0	0	2	1
TOTAL PUPILS	17	17	5	1	31	20	21	19	19	23
TOTAL T & P	80	80	47	35	90	96	98	86	79	91

Appendix A

Table 167 Frequency of behavior in each category of Contents, of Process and of Attitudes, and in Neutral for Teacher XV and Pupils XV as recorded by Observer I for the ten periods of classroom visits. Superscripts indicate frequency of negative behavior in individual categories.

DAYS ATEGORY	1	2	3	4	5	6	7	8	9	10
Content					**TEACHER XV**					
Structure	-	-	5	-	6	1	-	1	-	1
Techniques	-	-	9	-	6	10	-	6	-	9
Deductive	-	-	1	-	0	0	-	19	-	0
Inductive	-	-	0	-	0	0	-	20	-	0
Mathematical	-	-	20	-	7	18	-	24	-	4
Other	-	-	2	-	0	5	-	0	-	9
rocess										
Analyzing	-	-	0	-	0	3	-	4	-	2
Synthesizing	-	-	4	-	5	5	-	20	-	1
Specializing	-	-	21	-	5	17	-	36	-	10
Generalizing	-	-	1	-	3	0	-	3	-	0
Relevant	-	-	11	-	6	9	-	7	-	10
ttitude										
Curiosity	-	-	0	-	0	0	-	0	-	1
Independence	-	-	12	-	11	5	-	19	-	13
Receptivity	-	-	25	-	8	29	-	51	-	9
eutral	-	-	5	-	8	5	-	1	-	5
OTAL TEACHER	-	-	42	-	27	39	-	71	-	28
ontent					**PUPILS XV**					
Structure	-	-	1	-	0	0	-	0	-	3
Techniques	-	-	2	-	1	1	-	3	-	1
Deductive	-	-	0	-	0	0	-	8	-	4
Inductive	-	-	0	-	0	0	-	1	-	0
Mathematical	-	-	39	-	53	5	-	13	-	37
Other	-	-	0	-	0	1	-	1	-	3
rocess										
Analyzing	-	-	0	-	0	0	-	2	-	0
Synthesizing	-	-	32	-	50	2	-	13	-	33
Specializing	-	-	7	-	3	4	-	5	-	14
Generalizing	-	-	0	-	0	0	-	2	-	0
Relevant	-	-	3	-	1	1	-	5	-	2
ttitude										
Curiosity	-	-	0	-	0	0	-	1	-	2
Independence	-	-	40	-	53	6	-	18	-	45
Receptivity	-	-	2	-	1	1	-	8	-	2
eutral	-	-	0	-	0	0	-	0	-	0
OTAL PUPILS	-	-	42	-	54	7	-	27	-	49
TOTAL T & P	-	-	84	-	81	46	-	98	-	77

Appendix A

Table 168 Frequency of behavior in each category of Contents, of Process and of Attitudes, and in Neutral for Teacher XV and Pupils XV as recorded by Observer II for the ten periods of classroom visits. Superscripts indicate frequency of negative behavior in individual categories.

DAYS CATEGORY	1	2	3	4	5	6	7	8	9	10
					TEACHER XV					
Content										
Structure	0	4	8	4	4	0	0	1	0	1
Techniques	18	12	8	20	10	11	7	11	6	8
Deductive	1	0	0	0	0	1	3	18	1	0
Inductive	5	11	0	2	0	0	12	11	0	0
Mathematical	12	12	18	8	5	19	5	28^1	6	5
Other	6	5	2	1	0	5	0	0	0	9
Process										
Analyzing	2	6	5	4	0	1	3	3	0	0
Synthesizing	5	7	2	5	1	4	6	20	2	2
Specializing	16	15	19	15	6	18	8	31	7	13
Generalizing	2	0	1	0	2	0	4	5	0	0
Relevant	18	16	9	11	10	13	6	11	4	8
Attitude										
Curiosity	4	0	0	1	0	0	0	0	0	1
Independence	27	29	11	12	5	9	9	17	5	9
Receptivity	12	15	25	22	14	27	18	53	8	13
Neutral	3	1	6	2	8	2	0	1	3	5
TOTAL TEACHER	46	45	42	37	27	38	27	71	16	28
					PUPILS XV					
Content										
Structure	1	4^1	2	2^1	0	0	0	0	0	2
Techniques	10^1	2	2	9	0	0	1	4	0	2
Deductive	3^1	0	0	0	0	0	2^2	6^1	1	1
Inductive	0	8	0	1	0	0	3	1	0	0
Mathematical	37^1	42	38	26^1	54	3^2	7^1	14	29^1	42
Other	3	5	0	0	0	2	0	1	0	1^2
Process										
Analyzing	5	10	2	2	2	1	1	1	2	0
Synthesizing	30	31	27	23	51	1	8	12	21	35
Specializing	10	17	10	9	1	5	3	6	8	13
Generalizing	1	2	0	1	0	0	3	2	0	0
Relevant	11	2	3	5	0	0	1	6	0	2
Attitude										
Curiosity	2	2	0	0	0	0	1	0	0	2
Independence	46	60	40	35	54	7	15	23	30	48
Receptivity	9	0	2	5	0	0	0	4	1	0
Neutral	1	0	0	0	0	0	0	0	0	0
TOTAL PUPILS	58	62	42	40	54	7	16	27	31	50
TOTAL T & P	104	107	84	77	81	45	43	98	47	78

Appendix A

Table 169 Frequency of behavior in each category of Contents, of Process and of Attitudes, and in Neutral for Teacher XVI and Pupils XVI as recorded by Observer I for the ten periods of classroom visits. Superscripts indicate frequency of negative behavior in individual categories.

DAYS CATEGORY	1	2	3	4	5	6	7	8	9	10
Content					**TEACHER XVI**					
Structure	-	-	5	-	2	0	1	2	-	-
Techniques	-	-	6	-	23	35	2	30	-	-
Deductive	-	-	1	-	5	0	1	13	-	-
Inductive	-	-	17	-	1	0	0	24	-	-
Mathematical	-	-	11	-	42	21	6	9	-	-
Other	-	-	0	-	0	0	0	1	-	-
Process										
Analyzing	-	-	5	-	2	0	0	0	-	-
Synthesizing	-	-	6	-	20	18	4	10	-	-
Specializing	-	-	22	-	32	24	5	52	-	-
Generalizing	-	-	0	-	0	2	0	8	-	-
Relevant	-	-	8	-	21	12	1	9	-	-
Attitude										
Curiosity	-	-	0	-	0	0	0	0	-	-
Independence	-	-	12	-	13	1	0	2	-	-
Receptivity	-	-	29	-	62	55	10	77	-	-
Neutral	-	-	0	-	1	0	0	0	-	-
TOTAL TEACHER	-	-	41	-	76	56	10	79	-	-
Content					**PUPILS XVI**					
Structure	-	-	0	-	0	1	0	1	-	-
Techniques	-	-	6	-	8	5	2	1	-	-
Deductive	-	-	0	-	0	0	1	0	-	-
Inductive	-	-	0	-	0	1	0	2	-	-
Mathematical	-	-	27	-	4	4	21	7	-	-
Other	-	-	0	-	0	0	0	0	-	-
Process										
Analyzing	-	-	0	-	2	2	0	1	-	-
Synthesizing	-	-	22	-	0	0	21	5	-	-
Specializing	-	-	5	-	7	7	2	4	-	-
Generalizing	-	-	0	-	0	0	0	0	-	-
Relevant	-	-	6	-	5	2	2	1	-	-
ttitude										
Curiosity	-	-	0	-	0	1	0	0	-	-
Independence	-	-	28	-	7	9	23	11	-	-
Receptivity	-	-	5	-	7	1	2	0	-	-
eutral	-	-	0	-	0	0	0	0	-	-
OTAL PUPILS	-	-	33	-	14	11	25	11	-	-
TOTAL T & P	-	-	74	-	90	67	35	90	-	-

Appendix A

Table 170 **Frequency of behavior in each category of Contents, of Process and of Attitudes, and in Neutral for Teacher XVI and Pupils XVI as recorded by Observer II for the ten periods of classroom visits. Superscripts indicate frequency of negative behavior in individual categories.**

DAYS CATEGORY	1	2	3	4	5	6	7	8	9	10
Content				TEACHER XVI						
Structure	5	0^6	3	1	1^1	0	1	1	1	0
Techniques	16	9	10	8	19	37	12	31	8	4
Deductive	4	8	0	2^1	4	2	2	11	0	2
Inductive	3	5	12	2	3	0	0	25^2	18	17
Mathematical	22	15	16	10^1	41	17	14	9	28	15
Other	8	0	0	0	0	0	0	0	0	1
Process										
Analyzing	4	4	3	2	5	0	1	1	1	6
Synthesizing	6	10	9	7	24	18	7	18	20	14
Specializing	26	16	18	6	25	17	15	39	23	14
Generalizing	5	4	2	2	0	4	0	9	6	1
Relevant	17	9	9	8	16	17	6	12	7	4
Attitude										
Curiosity	1	0	0	0	0	0	0	0	0	0
Independence	21	10	14	4	19	4	6	3	16	7
Receptivity	36	33	27	21	51	48^4	23	70^6	41	32
Neutral	2	2	0	5	1	0	0	0	0	0
TOTAL TEACHER	60	45	41	30	71	56	29	79	57	39
Content				PUPILS XVI						
Structure	5^1	1	0^1	0^2	2^1	2^1	0	0	0	0
Techniques	1	3	6	2	8	3^1	2	2	2	0
Deductive	1	0	0	0	1	0	2	0	0	0
Inductive	0	0	0	0	0^1	1	0	2	3	4
Mathematical	12^4	10^1	26	7^1	5	2^1	38^1	7	20	41
Other	2	0	0	0	0	0	0	0	0	0
Process										
Analyzing	1	1	0	2	2	1	0	3	3	1
Synthesizing	2	9	23	5	1	0	36	5	16	41
Specializing	20	1	4	3	10	9	5	2	6	3
Generalizing	2	0	0	0	0	0	0	0	0	0
Relevant	1	4	6	2	5	1	2	1	2	0
Attitude										
Curiosity	0	0	0	0	0	1	0	0	0	0
Independence	24	14	27	10	12	9	40	11	24	45
Receptivity	2	1	6	2	6	1	3	0	3	0
Neutral	1	0	0	1	0	0	0	0	0	0
TOTAL PUPILS	27	15	33	13	18	11	43	11	27	45
TOTAL T & P	87	60	74	43	89	67	72	90	84	84

Appendix A

Table 171 Frequency of behavior in each category of Contents, of Process and of Attitudes, and in Neutral for Teacher XVII and Pupils XVII as recorded by Observer I for the ten periods of classroom visits. Superscripts indicate frequency of negative behavior in individual categories.

DAYS CATEGORY	1	2	3	4	5	6	7	8	9	10
					TEACHER XVII					
Content										
Structure	5	1	3	6	0	4	10	7	13	6
Techniques	14	10	5	2	9	9	11	5	4	13
Deductive	17	27	6	10	25	13	24	28	13	7
Inductive	10	0	0	0	3	7	7	5	20	7
Mathematical	25	32	33	22	1	22	8	16	25	38
Other	2	0	0	1	3	0	0	0	0	2
Process										
Analyzing	3	1	4	4	8	5	6	12	7	8
Synthesizing	21	25	8	7	2	9	1	16	19	24
Specializing	14	21	20	23	13	23	31	21	22	20
Generalizing	11	7	2	5	3	8	8	5	23	8
Relevant	24	16	13	2	15	10	14	7	4	13
Attitude										
Curiosity	1	4	3	2	3	1	0	1	2	5
Independence	48	48	38	33	24	36	43	45	55	43
Receptivity	24	18	6	6	14	18	17	15	18	25
Neutral	1	0	16	8	3	2	0	0	0	0
TOTAL TEACHER	74	70	63	49	44	57	60	61	75	73
					PUPILS XVII					
Content										
Structure	1	0	0	1	0	2	3	2	1	0
Techniques	1	1	1	0	0	1	2	0	1	0
Deductive	2	6	1	2	35	2	23	15	5	1
Inductive	0	0	0	0	1	0	2	0	2	0
Mathematical	10	6	25	34	3	22	4	13	8	12
Other	0	0	0	0	2	0	0	0	0	0
Process										
Analyzing	0	0	0	1	0	0	4	1	1	0
Synthesizing	10	8	23^{1}	26	8	13	18	8	8	8
Specializing	2	5	6	11	29	15	11	19	8	6
Generalizing	3	0	0	0	7	1	1	2	0	0
Relevant	1	1	1	1	2	7	3	2	1	0
ttitude										
Curiosity	0	0	1	2	3	0	0	1	0	0
Independence	8	4	26	33	40	16	33	21	14	9
Receptivity	8	10	4	4	3	21	4	10	4	5
eutral	0	0	0	1	0	0	0	0	0	0
TOTAL PUPILS	16	14	31	40	46	37	37	32	18	14
TOTAL T & P	90	84	94	89	90	94	97	93	93	87

Appendix A

Table 172 Frequency of behavior in each category of Contents, of Process and of Attitudes, and in Neutral for Teacher XVII and Pupils XVII as recorded by Observer II for the ten periods of classroom visits. Superscripts indicate frequency of negative behavior in individual categories.

DAYS	1	2	3	4	5	6	7	8	9	10
CATEGORY										
					TEACHER XVII					
Content										
Structure	4	6	2	6	0	4	11	14	12	-
Techniques	13	9	10	3	12	11	12	9	6	-
Deductive	10	18	2	10	27	13	20	25	19	-
Inductive	11	0	0	0	1	4	2	0	22	-
Mathematical	32	34	31	19	1	24	11	12	20	-
Other	2	0	0	0	3	0	1	0	0	-
Process										
Analyzing	4	6	1	5	11	8	7	6	8	-
Synthesizing	15	19	9	6	2	8	2	10	18	-
Specializing	23	24	24	21	13	24	33	28	32	-
Generalizing	10	8	2	4	4	5	4	5	13	-
Relevant	20	10	9	2	14	11	11	11	8	-
Attitude										
Curiosity	1	6	3	1	0	1	1	0	4	-
Independence	44	40	32	28	25	37^1	38	41	54	-
Receptivity	27	21	10	9	18^1	17	18	19	21	-
Neutral	1	1	16	8	2	1	0	0	0	-
TOTAL TEACHER	73	68	61	46	46	57	57	60	79	-
					PUPILS XVII					
Content										
Structure	0	0	3	1^1	0^3	1^1	11	3^1	1	-
Techniques	1	2	0	1	0	3^1	2	1	0	-
Deductive	2^2	8	0	2	28^6	3	18^1	15	3	-
Inductive	0	0	0	0	5	0	2	0	2	-
Mathematical	13	5^1	27^3	35^2	0	26^2	3^2	12^1	7^1	-
Other	0	0	0	0	1^2	0	0	0	0	-
Process										
Analyzing	1	0	1	1	3	1	3	0	1	-
Synthesizing	10	9	22	26	3	16	16	15	9	-
Specializing	4	4^1	8	13	22	14	13	10	4	-
Generalizing	2	1	1	1	15	1	5	4	0	-
Relevant	1	1	1	1	1	5	1^1	4	0	-
Attitude										
Curiosity	0	0	2	1	5	0	0	0	0	-
Independence	14	6	21^1	36	35	19	32	27	7	-
Receptivity	4	10	9	5	3	18	7	6	7	-
Neutral	0	0	0	1	0	0	0	0	0	-
TOTAL PUPILS	18	16	33	43	43	37	39	33	14	-
TOTAL T & P	91	84	94	89	89	94	96	93	93	-

Appendix A

Table 173 Frequency of behavior in each category of Contents, of Process and of Attitudes, and in Neutral for Teacher XVIII and Pupils XVIII as recorded by Observer I for the ten periods of classroom visits. Superscripts indicate frequency of negative behavior in individual categories.

DAYS CATEGORY	1	2	3	4	5	6	7	8	9	10
					TEACHER XVIII					
Content										
Structure	2	2	7	3	9	11	4	9	16	
Techniques	2	10	16	15	25	3	5	30	20	
Deductive	7	21	22	22	11	0	2	0	1	
Inductive	1	7	0	23	0	0	35	0	0	
Mathematical	0	18	16	8	16	25	5	14	20	
Other	0	0	1	1	0	7	2	4	5	
Process										
Analyzing	4	5	4	1	6	10	8	2	16	
Synthesizing	0	5	14	9	8	15	5	1	7	
Specializing	5	33	27	43	24	17	29	24	18	
Generalizing	1	3	0	5	1	0	4	2	1	
Relevant	2	12	17	14	22	4	7	28	20	
Attitude										
Curiosity	0	0	1	0	1	0	1	0	0	
Independence	9	36	37	44	30	35	36	9	29	
Receptivity	3	22	24	28	30	11	16	48	33	
Neutral	1	1	6	1	2	4	8	13	4	
TOTAL TEACHER	13	59	68	73	63	50	61	70	66	
					PUPILS XVIII					
Content										
Structure	8	0	0	0	2	4	2	4	3	
Techniques	0	1	1	0	2	3	0	5	5	
Deductive	68	8	11	7	8	1	0	0	1	
Inductive	5	1	1	6	0	0	17	0	0	
Mathematical	0	23	3	4	12	21	4	4	9	
Other	0	0	1	0	0	1	0	0	4	
Process										
Analyzing	11	0	1	0	2	5	0	2	4	
Synthesizing	31	21	2	10	9	10	6	2	3	
Specializing	37	11	12	8	16	16	3	6	11	
Generalizing	2	0	1	1	0	0	4	0	2	
Relevant	0	1	1	0	2	3	0	5	5	
Attitude										
Curiosity	1	0	1	0	1	1	0	3	0	
Independence	80	30	15	18	26	27	23	6	19	
Receptivity	0	3	1	1	2	6	0	6	6	
Neutral	0	0	1	0	1	0	1	2	1	
TOTAL PUPILS	81	33	18	19	30	34	24	17	26	
TOTAL T & P	94	92	86	92	93	84	85	87	92	

Appendix A

Table 174. Frequency of behavior in each category of Contents, of Process and of Attitudes, and in Neutral for Teacher XVIII and Pupils XVIII as recorded by Observer II for the ten periods of classroom visits. Superscripts indicate frequency of negative behavior in individual categories.

DAYS CATEGORY	1	2	3	4	5	6	7	8	9
Content				TEACHER XVIII					
Structure	-	4	8	4	10	11	-	-	-
Techniques	-	12	15	17	25	4	-	-	-
Deductive	-	19	21	22	12	1	-	-	-
Inductive	-	6	0	22	0	0	-	-	-
Mathematical	-	18	13	3	13	22	-	-	-
Other	-	0	1	2	0	5	-	-	-
Process									
Analyzing	-	6	6	7	8	6	-	-	-
Synthesizing	-	6	12	6	7	14	-	-	-
Specializing	-	30	22	37	22	18	-	-	-
Generalizing	-	2	1	3	1	0	-	-	-
Relevant	-	15	17	17	22	5	-	-	-
Attitude									
Curiosity	-	0	1	0	1	0	-	-	-
Independence	-	38	33	42	21	27	-	-	-
Receptivity	-	21	24	28	38	16	-	-	-
Neutral	-	0	6	0	2	4	-	-	-
TOTAL TEACHER	-	59	64	70	62	47	-	-	-
Content				PUPILS XVIII					
Structure	-	0^{1}	0	0^{3}	2	3^{1}	-	-	-
Techniques	-	1	1	1	2^{1}	6	-	-	-
Deductive	-	8^{1}	11^{2}	8^{2}	6^{1}	0	-	-	-
Inductive	-	0	1	6	0	0	-	-	-
Mathematical	-	21^{1}	6	1	16^{2}	19^{1}	-	-	-
Other	-	0	1	0^{1}	0	6^{2}	-	-	-
Process									
Analyzing	-	1	2	3	4	2	-	-	-
Synthesizing	-	19	4	5	12	11	-	-	-
Specializing	-	11	12	11	11	19	-	-	-
Generalizing	-	0	3	2	0	0	-	-	-
Relevant	-	2	1	1	3	6	-	-	-
Attitude									
Curiosity	-	0	1	0	1	1	-	-	-
Independence	-	33	20	22	24	29	-	-	-
Receptivity	-	0	1	0	5	8	-	-	-
Neutral	-	0	1	0	1	0	-	-	-
TOTAL PUPILS	-	33	23	22	31	38	-	-	-
TOTAL T & P	-	92	87	92	93	85	-	-	-

Appendix A

Table 175 Cross classification of Pupil I behaviors in Content, Process and Attitude categories: work matrices and final matrix. Negative in Process and Attitude disregarded; in Content where noted by superscripts. Symbolically: $C_iP_jA_k$

i--1 to 6
j--1 to 5
k--1 to 3

	P_1A_1	P_2A_1	P_3A_1	P_4A_1	P_5A_1	P_1A_2	P_2A_2	P_3A_2	P_4A_2	P_5A_2	P_1A_3	P_2A_3	P_3A_3	P_4A_3	P_5A_3
#1							Observer I -- Periods 1, 3, 4, 5, 7, 8, 10								
C_1	0	1				2^1	3	8^3	4	0^1			3		
2			3			3	22^1	9^2	4^3	2			1		
3			2				36^2	30^1	1	2		2	0	0	
4			1						1				0	0	
5								0	0	4					4
6							3		2						
#2							Observer II -- Periods 1, 3, 4, 5, 7, 8, 10								
C_1			0			5	3^1	17^4	4				2^1	2	
2			0^1	2		4^1	21	3^2	6^1			1	3	0^1	
3			1			1	44^1	14^3	0^1	1		2	3		
4			1												
5								2		2					5
6			1			2	2	3	2				1		
Mean 1 & 2: #3							Joint Observers I & II								
C_1						3	3	12^3	4				2		
2			1			3	21	11^2	5^2	1			2		
3			1				40^1	22^2		1		2	1		
4			1												
5								1		3					4
6						1	2	1	2						
#4							Observer I -- Additional Periods 2, 6, 9								
C_1						3		0^1							
2						1	4^2	1^1	2					1	
3						1	12^2	3^2		2		1	1		
4															
5								1		3					
6							3	6							
#3 + 4							Summary final matrix								
C_1						6	3	12^4	4				2		
2			1			4	25^3	12^3	7^2	1			2	1	
3			1			1	52^3	25^4		3		3	2		
4			1												
5								2		6					4
6						1	5	7	2						

Appendix A

Table 176 Cross classification of Teacher II behaviors in Content, Process and Attitude categories: work matrices and final matrix. Negative in Process and Attitude disregarded; in Content where noted by superscripts. Symbolically: $C_iP_jA_k$ i--1 to

j--1 to

k--1 to

	P_1A_1	P_2A_1	P_3A_1	P_4A_1	P_5A_1	P_1A_2	P_2A_2	P_3A_2	P_4A_2	P_5A_2	P_1A_3	P_2A_3	P_3A_3	P_4A_3	P_5
#1							Observer I -- Periods 1, 3, 4, 5, 7, 8, 10								
C_1	1			3		21	4	33	8	2	4		8	7	5
2			1			15	22	18	14	4	2	10	4	1	2
3			1			3	31	65	4	7	1		6		1
4			3	1	3			5				1	3		1
5					2					28			3		16
6	1					3	8	23	9			1	2	2	
#2							Observer II -- Periods 1, 3, 4, 5, 7, 8, 10								
C_1	1			1	1	16	3	25	11	1	2		10	8	7
2		1	1			12	21	19	11	2	1	7	6	12	2
3	1	2	1		1	4	31	43	2	2		4	13	2	1
4			6	3	1	1		5					7		3
5					2			3		27		1	3		23
6						4	10	24	3		1		1	2	
Mean 1 & 2: #3							Joint Observers I & II								
C_1	1			2		18	3	29	9	1	3		9	7	6
2			1			13	21	18	12	3	1	8	5	6	2
3		1	1			3	31	54	3	4		2	9	1	2
4			4	2	2			5					5		2
5					2			1		27			3		19
6						3	9	23	6				1	2	
#4							Observer I -- Additional Periods 2, 6, 9								
C_1	2					11	2	7	1	1	7	2	1	1	
2				1		10	10	6	5		1	4	1	1	
3		1				5	31	21	1	3		2	1		
4			1	2	2										
5										11			1		5
6			1			1	1	13	4						
#3 + 4							Summary final matrix								
C_1	3			2		29	5	36	10	2	10	2	10	8	6
2			1	1		23	31	24	17	3	2	12	6	7	2
3		2	1	4		8	62	75	4	7		4	10	1	2
4			5		4			5					5		2
5					2			1		38			4		19
6			1			4	10	36	10				1	2	

Appendix A

Table 177 Cross classification of Pupil II behaviors in Content, Process and Attitude categories: work matrices and final matrix. Negative in Process and Sttitude disregarded; in Content where noted by superscripts. Symbolically: $C_iP_jA_k$

i--1 to 6
j--1 to 5
k--1 to 3

	P_1A_1	P_2A_1	P_3A_1	P_4A_1	P_5A_1	P_1A_2	P_2A_2	P_3A_2	P_4A_2	P_5A_2	P_1A_3	P_2A_3	P_3A_3	P_4A_3	P_5A_3
#1							Observer I -- Periods 1, 3, 4, 5, 7, 8, 10								
C_1			1			5	1^1	7^3						1	
2						4	21^4	139	8	0^1					
3							39	14^6		2			1		
4												1			
5								1		3^1					4
6				2		1		1	3						
#2							Observer II -- Periods 1, 3, 4, 5, 7, 8, 10								
C_1			1			8^1	2^1	4^4	2	0^1					
2						5	22^2	3^4	9	0^1					
3			1			1	45	12^1		1			$\overset{1}{3}$		1
4			0^1					3							
5					1			2		3^1					3
6	1					0^1	1	2	3						
Mean 1 & 2: #3							Joint Observers I & II								
C_1			1			6	1^1	5^3							
2						4	21^3	8^6	8	0^1					
3							42	13^3		1			2		
4								1							
5								1		3^1					3
6								1	3						
#4							Observer I -- Additional Periods 2, 6, 9								
C_1						4	1	7^3		1	1				
2						5	5	2	2						
3						1	26^4	4^6		1^2			2		1
4															
5										3					1
6								3	2						
#3 + 4							Summary final matrix								
C_1			1			10	2^1	12^6		1	1				
2						9	26^3	2	10	0^1					
3						1	68^4	17^9		2^2			4		1
4															
5								1		6^1					4
6								4	5						

Appendix A

Table 178 **Cross classification of Teachers III and V; Pupils III and V behaviors in Content, Process and Attitude categories: final matrices. Negative in Process and Attitude disregarded; in Content where noted by superscripts. Symbolically: $C_iP_jA_k$**

i--1 to 6
j--1 to 5
k--1 to 3

	P_1A_1	P_2A_1	P_3A_1	P_4A_1	P_5A_1	P_1A_2	P_2A_2	P_3A_2	P_4A_2	P_5A_2	P_1A_3	P_2A_3	P_3A_3	P_4A_3	P_5A_3
								Summary final matrix -- Teacher III							
C_1	5	1		7	2	3		2	3		46	5	13	27	1
2	1		1	2		1	1	6	4	1	51	84	31^1	90	5
3		1					6	8		2	13	126	86	2	5
4			2	2	12						1		2	6	
5										12			6		12
6		1	1	4		1	2	16			21	61	69	30	1
								Summary final matrix -- Pupils III							
C_1						1		2					1		
2	0^1					1		2	1		1		0^1		
3	1		2					1					24^1		1
4															
5															3
6			2										0		
								Summary final matrix -- Teacher V							
C_1	12		1	5		2		1			32	7	25	27	1
2	4		2	4			1	1			31^1	113	69	87	3
3		1		1		2	1	9		1	11	98	52	6	4
4			1	1	5				1		2		4	4	
5					1					4			3^1	3	14
6	1	1	13	6			1	7	1		22	152	138	39	2
								Summary final matrix -- Pulils V							
C_1			0^1			1					0^1				
2	2			0^1				4					3^2	1^2	
3			2			1	1	2		1			6		1
4															
5													1		2
6								1					2		

Appendix A

Table 179 Cross classification of Teacher VI behaviors in Content, Process and Attitude categories: work matrices and final matrix. Negative in Process and Attitude disregarded; in Content where noted by superscripts. Symbolically: $C_iP_jA_k$

i--1 to 6
j--1 to 5
k--1 to 3

	P_1A_1	P_2A_1	P_3A_1	P_4A_1	P_5A_1	P_1A_2	P_2A_2	P_3A_2	P_4A_2	P_5A_2	P_1A_3	P_2A_3	P_3A_3	P_4A_3	P_5A_3
#1 Observer I -- Periods 2, 3, 4, 7, 10															
C_1				2		2					5		18	2	
2						3	2	5	1		15	69	53	18	1
3							2	4		4	10	43	36	1	5
4			3	1	19								4		3
5					2			1		5			17	1	57
6			1			1	2	2			1	28	15	11	1
#2 Observer II -- Periods 2, 3, 4, 7, 10															
C_1				2	1	1	1	3			3	1	19	2	
2				2		4	4	9		1	15	52	55	19	
3						2	2	5		1	12	41	31		6
4			6	3	11						1		5	1	3
5					2	2		3		7		1	15	4	60
6		1		2	2	1		3		1	3	16	21	6	2
Mean 1 & 2: #3 Joint Observers I & II															
C_1				2		1		1			4		18	2	
2				1		3	3	4			15	60	54	18	
3						1	2	4		2	11	42	33		5
4			4	2	15								4		3
5					2	1		2		6			16	2	58
6				1	1	1	1	2			2	22	18	8	1
#4 Observer I--Additional Periods 1, 5, 6, 8, 9															
C_1	3		3	3		3		3			14	3	18	1	
2			1	3	1	6	2	9	2		15	57	44	28	2
3	1		3	1		1	2	4		1	3	26	35	1	4
4	1			2	9			1					4	1	
5								13		3					61
6			2		1	1		2			5	21	22	6	
#3 + 4 Summary final matrix															
C_1	3		3	5		4		4			18	3	36	3	
2			1	4	1	9	5	13	2		30	117	98	46	2
3	1		3	1		2	4	8		3	14	68	68	1	9
4	1		4	4	24			1					8		3
5					3	1		15		9			16	2	119
6			2	1	2	2	1	4			7	43	40	14	1

Appendix A

Table 180 Cross classification of Pupil VI behaviors in Content, Process and Attitude categories: work matrices and final matrix. Negative in Process and Attitude disregarded; in Content where noted by superscripts. Symbolically: $C_iP_jA_k$ i--1 to 6
j--1 to 5
k--1 to 3

	P_1A_1	P_2A_1	P_3A_1	P_4A_1	P_5A_1	P_1A_2	P_2A_2	P_3A_2	P_4A_2	P_5A_2	P_1A_3	P_2A_3	P_3A_3	P_4A_3	P_5A_3
# 1							Observer I -- Periods 2, 3, 4, 7, 10								
C_1															
2								2							
3								1					1		
4															
5								1		1					2
6							1		1						
# 2							Observer II -- Periods 2, 3, 4, 7, 10								
C_1								0^1							
2								1					1		
3										1			1		
4															
5															2
6								1							
Mean 1 & 2: #3							Joint Observers I & II								
C_1															
2								1							
3													1		
4															
5															2
6															
#4							Observer I--Additional Periods 1, 5, 6, 8, 9								
C_1							1	1							
2			1			0^1		2				1	1^1		
3						1		1				2	3		
4															
5															1
6								1							
#3 + 4							Summary final matrix								
C_1															
2			1			0^1		3				1	1^1		
3						1		1				2	4		
4															
5															3
6								1							

Appendix A

Table 181 Cross classification of Teacher VIII behaviors in Content, Process and Attitude categories: work matrices and final matrix. Negative in Process and Attitude disregarded; in Content where noted by superscripts. Symbolically: $C_iP_jA_k$

i--1 to 6
j--1 to 5
k--1 to 3

	P_1A_1	P_2A_1	P_3A_1	P_4A_1	P_5A_1	P_1A_2	P_2A_2	P_3A_2	P_4A_2	P_5A_2	P_1A_3	P_2A_3	P_3A_3	P_4A_3	P_5A_3
#1							Observer I -- Periods 1-9								
C_1						5	2	21^1		2	8	6	9	14	3
2			2			4	9	26	6	2	6	4	23	23	1
3						3	50	90	3	12	3	39	28	2	7
4							1	8					4	1	
5							3	9		32		1	8	1	98
6							5	19	3		1	4	6	8	5
# 2							Observer II -- Periods 1-9								
C_1	1	1	1			5	3	17^3	3	3	3	2	15^2	8^2	1
2	1		2	1		4	6	31	5	1	6	2	21^1	28^3	1
3			1			1	57	47	1	14	1	41	46	1	12
4						1		6	1				5	2	2
5							2	8		25			6	4	95
6						2	3	24	1		2	2	3	4	2
Mean 1 & 2: #3							Joint Observers I & II								
C_1						5	2	19^2	1	2	5	4	12^1	11^1	2
2			2			4	7	28	5	1	6	3	22	25^1	1
3						2	53	68	2	13	2	40^1	37	1	9
4								7					4	1	1
5							2	8		28			7	2	96
6						1	4	21	2		1	3	4	6	3
# 4							Observer I--Additional Periods 10								
C_1	1			1									5	3	
2			2	1				4			1		7	5	
3							1	1			1		2		1
4															
5										3			1		18
6			1				1	2			1	4	3	1	
#3 + 4							Summary final matrix								
C_1	1			1		5	2	19^2	1	2	5	4	17^1	14^1	2
2			4	1		4	7	32	5	1	7	3	29	30^1	1
3						2	54	69	2	13	3	40^1	39	1	10
4								7					4	1	1
5							2	8		31			8	2	114
6			1			1	5	23	2		2	7	7	7	3

Appendix A

Table 182 Cross classification of Pupil VIII behaviors in Content, Process and Attitude categories: work matrices and final matrix. Negative in Process and Attitude disregarded; in Content where noted by superscripts. Symbolically: $C_iP_jA_k$

i--1 to 6
j--1 to 5
k--1 to 3

	P_1A_1	P_2A_1	P_3A_1	P_4A_1	P_5A_1	P_1A_2	P_2A_2	P_3A_2	P_4A_2	P_5A_2	P_1A_3	P_2A_3	P_3A_3	P_4A_3	P_5A_3
# 1							Observer I -- Periods 1-9								
C_1	3			1		2	2						1		
2						2^1		5^1				0^1	3^1		1
3						3	13	18		2		7^1	11^3		0
4															
5								1		1		1	2		1
6						2			1						
# 2							Observer II -- Periods 1-9								
C_1	2		1	1		1^1	0^1	3^1						2	
2			1			2^1	1	6^2					2^2		
3						1	15	13^1		1^1	1	9^3	10^1		
4													1		
5								1		2			3		1
6						1^1			1		1				
Mean 1 & 2: #3							Joint Observers I & II								
C_1	2			1		1^1	1	2						1	
2						2^1		5^1					2^1		
3						2	14	15		1		8^2	10^2		
4															
5								1		1			2		1
6						1			1						
# 4							Observer I--Additional Periods 10								
C_1															
2									1				1^1		
3								1							
4															
5															
6								3							
# 3 + 4							Summary final matrix								
C_1	2			1		1^1	1	2						1	
2						2^1		5^1	1				3^2		
3						2	14	16		1		8^2	10^2		
4															
5								1		1			2		1
6						1		3	1						

Appendix A

Table 183 Cross classification of Teacher IX behaviors in Content, Process and Attitude categories: work matrices and final matrix. Negative in Process and Attitude disregarded; in Content where noted by superscripts. Symbolically: $C_iP_jA_k$

i--1 to 6
j--1 to 5
k--1 to 3

	P_1A_1	P_2A_1	P_3A_1	P_4A_1	P_5A_1	P_1A_2	P_2A_2	P_3A_2	P_4A_2	P_5A_2	P_1A_3	P_2A_3	P_3A_3	P_4A_3	P_5A_3
# 1 Observer I -- Periods 1-9															
C_1						19	3	29	5	1	0	4	11	12	1
2			2	1		9	7	30	6	2	2	6	3	8	3
3						9	48	96	3	9	4	19	22	3	2
4			1	1	2			1				2	6	4	3
5					1		4	27		36			5	1	30
6				2		3	6	35	16	2		7	1	2	
# 2 Observer II -- Periods 1-9															
C_1	1		1			15	6	31^1	4	2	4		26	7	
2	1		3	2		12	2	18	16	2	3	4	14	16	1
3						6	39	61	1	7	3	38	29		4
4			3		2		2	3				2	14	1	1
5			1			1	8	32		31		2	9	3	34
6	1			2		2	2	22	2	1		6	3		
Mean 1 & 2: # 3 Joint Observers I & II															
C_1						17	4	30	4	1	2	2	18	9	
2			2	1		10	4	24	11	2	2	5	8	12	2
3						7	43	78	2	8	3	28	25	1	3
4			2		2		1	2				2	8	2	2
5							6	29		33		1	7	2	32
6				2		2	4	28	9	1		6	2	1	
# 4 Observer I -- Additional Periods 10															
C_1		1				1	2	6	4		1	1	1	1	
2				1	2	4	4	16	4		1	1		2	
3							6	11				1			1
4															
5					3			1		2					
6									1					1	
# 3 + 4 Summary final matrix															
C_1		1				18	6	36	8	1	3	3	19	10	
2			2	2	2	14	8	40	15	2	3	6	8	14	2
3						7	49	89	2	8	3	29	25	1	4
4			2		2		1	2				2	8	2	2
5					3		6	30		35		1	7	2	32
6				2		2	4	28	10	1		6	2	2	

Appendix A

Table 184 Cross classification of Pupil IX behaviors in Content, Process and Attitude categories: work matrices and final matrix. Negative in Process and Attitude disregarded; in Content where noted by superscripts. Symbolically: $C_iP_jA_k$ i--1 to 6
j--1 to 5
k--1 to 3

	P_1A_1	P_2A_1	P_3A_1	P_4A_1	P_5A_1	P_1A_2	P_2A_2	P_3A_2	P_4A_2	P_5A_2	P_1A_3	P_2A_3	P_3A_3	P_4A_3	P_5A_3
# 1							Observer I - Periods 1-9								
C_1						3^1	3^1	3^5			1	1	0^1		
2		0^1		1		2	10	2^2	4^2				6	1	4
3			1			1	27^3	23^6		3		7	13		2
4													1^1		
5							1	6^1			1		7		5
6								2	2						
# 2							Observer II -- Periods 1-9								
C_1	1					31	2^5	3	4^1	0^1	1		3^4	1	
2				2		0^2	9	3^2	3^1		1		0^1	2	2
3			2				25^2	21^2		4	1	11^2	14		1
4						1		1		1			1		
5							1^1	3^1		3			5		3
6								4							
Mean 1 & 2: # 3							Joint Observers I & II								
C_1						3^1	2^3	3^2	2		1		1^2		
2				1		1^1	9	2^2	3^1				3	1	3
3			1				26^2	22^4		3		9^1	13		1
4													1^1		
5							1	4^1		1			6		4
6								3	1						
# 4							Observer I -- Additional Periods 10								
C_1								1				1	1		
2						1	1	3				1^1	1	2	
3								1							
4															
5													0^1		
6															
# 3 + 4							Summary final matrix								
C_1						3^1	2^3	4^2	2		1	1	2^2		
2				1		2^1	10	5^2	3^1			1^1	4	3	3
3			1				26^2	23^4		3		9^1	13		1
4													1^1		
5							1	4^1		1			6^1		4
6								3	1						

Appendix A

Table 185 Cross classification of Teacher X behaviors in Content, Process and Attitude categories: work matrices and final matrix. Negative in Process and Attitude disregarded; in Content where noted by superscripts. Symbolically: $C_iP_jA_k$

i--1 to 6
j--1 to 5
k--1 to 3

Observer I -- Periods 1, 2, 3, 9, 10

# 1	P_1A_1	P_2A_1	P_3A_1	P_4A_1	P_5A_1	P_1A_2	P_2A_2	P_3A_2	P_4A_2	P_5A_2	P_1A_3	P_2A_3	P_3A_3	P_4A_3	P_5A_3
C_1	2		1			14	3	6	2		28	5	31	20	
2	1							3	2		3	5	13	[illegible]	1
3					1	3	11	13			5	57	21		
4			2		2			2					1	1	3
5								1		14			5	2	11
6						4		5			7	20	17	4	

Observer II -- Periods 1, 2, 3, 9, 10

# 2	P_1A_1	P_2A_1	P_3A_1	P_4A_1	P_5A_1	P_1A_2	P_2A_2	P_3A_2	P_4A_2	P_5A_2	P_1A_3	P_2A_3	P_3A_3	P_4A_3	P_5A_3
C_1	1			1		10	2	6	4		25	6	35	17	1
2	1							2	3		6	7	10	4	
3					1	7	9	11			5	48	26		
4			2	2	1			2					3	1	1
5										16			6	2	12
6				1		2	3	3	1			14	18	5	

Joint Observers I & II

Mean 1 & 2: # 3	P_1A_1	P_2A_1	P_3A_1	P_4A_1	P_5A_1	P_1A_2	P_2A_2	P_3A_2	P_4A_2	P_5A_2	P_1A_3	P_2A_3	P_3A_3	P_4A_3	P_5A_3
C_1	1					12	2	6	3		26	5	33	18	
2	1							2	2		4	6	11	5	
3					1	5	10	12			5	52	23		
4			2	1	1			2					2	1	2
5										15			5	2	11
6						3	1	4			3	17	17	4	

Observer I -- Additional Periods 4, 5, 6, 7, 8

# 4	P_1A_1	P_2A_1	P_3A_1	P_4A_1	P_5A_1	P_1A_2	P_2A_2	P_3A_2	P_4A_2	P_5A_2	P_1A_3	P_2A_3	P_3A_3	P_4A_3	P_5A_3
C_1			3	2		6		9			15	7	21	19	
2							1		1	1	1	1	3	4	
3						2	8	10	1		12	54	44		4
4			1		4									5	7
5									1	3			3	1	24
6			1					7	1				5		

Summary final matrix

# 3 + 4	P_1A_1	P_2A_1	P_3A_1	P_4A_1	P_5A_1	P_1A_2	P_2A_2	P_3A_2	P_4A_2	P_5A_2	P_1A_3	P_2A_3	P_3A_3	P_4A_3	P_5A_3
C_1	1		3	2		18	2	15	3		41	12	54	37	
2	1						1	2	3		5	7	14	9	
3					1	7	18	22	1		17	106	67		4
4			3	1	5			2					2	6	9
5									1	18			8	3	35
6			1			3	1	11	1		3	17	22	4	

Appendix A

Table 186 Cross classification of Pupil X behaviors in Content, Process and Attitude categories: work matrices and final matrix. Negative in Process and Attitude disregarded; in Content where noted by superscripts. Symbolically: $C_iP_jA_k$

i--1 to 6
j--1 to 5
k--1 to 3

	P_1A_1	P_2A_1	P_3A_1	P_4A_1	P_5A_1	P_1A_2	P_2A_2	P_3A_2	P_4A_2	P_5A_2	P_1A_3	P_2A_3	P_3A_3	P_4A_3	P_5A_3
# 1 — Observer I -- Periods 1, 2, 3, 9, 10															
C_1						2^1	1^1	3^2				1	1^1		
2	1			1		1							1		
3						2	19	11^1					4		
4															
5								3		1			1		3
6						1		3			1		1		
# 2 — Observer II -- Periods 1, 2, 3, 9, 10															
C_1						3^2		7^3	1^1		1		0^1	1	
2	1			1		0^1			0^1					1	
3						2	20^2	9^3				2	0^2		
4															
5								4		1^1					4
6									1						
Mean 1 & 2: # 3 — Joint Observers I & II															
C_1						2^1		5^2					0^1		
2	1			1											
3						2	19^1	10^2				1	2^1		
4															
5								3		1					3
6						1		1			1				
# 4 — Observer I -- Additional Periods 4, 5, 6, 7, 8															
C_1	3							3^1	1				4		
2								0^1	2				1		4
3						1	22	7	1			3	8^1		2
4															2
5								1		0^2					13^2
6															
# 3 + 4 — Summary final matrix															
C_1	3					2^1		8^3	1				4^1		
2	1			1				0^1	2				1		4
3						3	41^1	17^2	1			4	10^2		2
4															2
5								4		1^2					16^2
6						1		1			1				

Appendix A

Table 187 Cross classification of Teacher XI behaviors in Content, Process and Attitude categories: work matrices and final matrix. Negative in Process and Attitude disregarded; in Content where noted by superscripts. Symbolically: $C_iP_jA_k$

i--1 to 6
j--1 to 5
k--1 to 3

	P_1A_1	P_2A_1	P_3A_1	P_4A_1	P_5A_1	P_1A_2	P_2A_2	P_3A_2	P_4A_2	P_5A_2	P_1A_3	P_2A_3	P_3A_3	P_4A_3	P_5A_3
# 1							Observer I -- Periods , 3, 4, 5, 6, 7.								
C_1						12		12	3		3		9	7	
2						2	5	11	3		5	3	15	8	1
3						2	5	14		1	1	8	25	1	
4			10	1	9			1		1					
5					1			2		25			9	1	44
6		1	2			2	10	41	3		3	3	26		
# 2							Observer II -- Periods , 3, 4, 5, 6, 7.								
C_1						9		11	6	1	6		9	5	
2						6	1	9	4		4	4	11	17	
3						5	5	10			2	11	21	1	
4			12	1	4	1			1	2			2		
5					1			2		16			7	2	55
6			2			5	6	27	2		1	8	22	3	
Mean 1 & 2: # 3							Joint Observers I & II								
C_1						10		11	4		4		9	6	
2						4	3	10	3		4	3	13	12	
3						3	5	12			1	9	23	1	
4			11	1	6					1			1		
5					1			2		20			8	1	49
6			2			3	8	34	2		2	5	24	1	
# 4							Observer I -- Additional Periods 1, 2, 8, 9, 10								
C_1	2		1			27		5	1		14		8	2	
2						8^1	5	8	9		7	9	9	2	
3	1					3	15	23			4	17	20		
4								4					1	1	
5										13			8		45
6	1	1	5	1		8	14	32	1		2	4	11		
# 3 + 4							Summary final matrix								
C_1	2		1			37		16	5		18		17	8	
2						12^1	8	18	12		11	12	22	14	
3	1					6	20	35			5	26	43	1	
4			11	1	6			4		1			2	1	
5					1			2		33			16	1	94
6	1	1	7	1		11	22	66	3		4	9	35	1	

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Table 188 Cross classification of Pupil XI behaviors in Content, Process and Attitude categories: work matrices and final matrix. Negative in Process and Attitude disregarded; in Content where noted by superscripts. Symbolically: $C_iP_jA_k$ i--1 to 6
j--1 to 5
k--1 to 3

	P_1A_1	P_2A_1	P_3A_1	P_4A_1	P_5A_1	P_1A_2	P_2A_2	P_3A_2	P_4A_2	P_5A_2	P_1A_3	P_2A_3	P_3A_3	P_4A_3	P_5A_3
#1 Observer I -- Periods 3, 4, 5, 6, 7															
C_1						3		3^2						1	
2						5	25	9^1				15	1	1	
3						2	50	10					1^1		
4						3		2		1					2
5								1		1			1		1
6						3	4	6^1					3		
# 2 Observer II -- Periods 3, 4, 5, 6, 7															
C_1						4^1	1	3	3^2				1	1	
2						2	28	10					15		
3						9	49	11				1			
4						1				1					2
5								2		1			2		3
6						3	7	8^1					2^1		
Mean 1 & 2: # 3 Joint Observers I & II															
C_1						3		3^1	1^1					1	
2						3	26	9				7	8		
3						5	49	10							
4						2		1		1					2
5								1		1			1		2
6						3	5	7^1					2		
# 4 Observer I -- Additional Periods 1, 2, 8, 9, 10															
C_1			0^1			3		1^2							
2	1					5^1	47	4^4			1				
3			1			2	54	5				0^1	1^1		
4															
5															1
6			1				5	2							
# 3 + 4 Summary final matrix															
C_1			0^1			6		43	1^1					1	
2	1					8^1	73	13^4			1	7	8		
3			1			7	103	15				0^1	1^1		
4						2		1		1					2
5								1		1			1		3
6			1			3	10	9^1					2		

Appendix A

Table 189 Cross classification of Teacher XII behaviors in Content, Process and Attitude categories: work matrices and final matrix. Negative in Process and Attitude disregarded; in Content where noted by superscripts. Symbolically: $C_iP_jA_k$

i--1 to 6
j--1 to 5
k--1 to 3

# 1 Observer I -- Periods 1, 2, 3, 5, 6

	P_1A_1	P_2A_1	P_3A_1	P_4A_1	P_5A_1	P_1A_2	P_2A_2	P_3A_2	P_4A_2	P_5A_2	P_1A_3	P_2A_3	P_3A_3	P_4A_3	P_5A_3
C_1	1					1			1		6	1	7	2	
2									1		5	14	10	9	1
3							1	2			3	32	41		2
4			1					1					2		1
5										7			2		15
6	2			1		2	3	2			3	39	39	16	3^3

# 2 Observer II -- Periods 1, 2, 3, 5, 6

	P_1A_1	P_2A_1	P_3A_1	P_4A_1	P_5A_1	P_1A_2	P_2A_2	P_3A_2	P_4A_2	P_5A_2	P_1A_3	P_2A_3	P_3A_3	P_4A_3	P_5A_3
C_1						1			1		4		6^2		
2						1			1		3	8^1	10	11^1	
3						2		1			6	53	58		4
4															1
5										7			8		29
6	1					2		2	1		9	16	43^1	4^1	2

Mean 1 & 2: #3 Joint Observers I & II

	P_1A_1	P_2A_1	P_3A_1	P_4A_1	P_5A_1	P_1A_2	P_2A_2	P_3A_2	P_4A_2	P_5A_2	P_1A_3	P_2A_3	P_3A_3	P_4A_3	P_5A_3
C_1						1			1		5		6^1	1	
2									1		4	11	10	10	
3						1		1			4	42	49		3
4													1		1
5										7			1		22
6	1					2	1	2			6	22	41	5	2^1

# 4 Observer I -- Additional Periods 4, 7, 8, 9, 10

	P_1A_1	P_2A_1	P_3A_1	P_4A_1	P_5A_1	P_1A_2	P_2A_2	P_3A_2	P_4A_2	P_5A_2	P_1A_3	P_2A_3	P_3A_3	P_4A_3	P_5A_3
C_1	1										1		2		
2				1				2	1		6	10	18	7	3
3								3			3	37	47		1
4			3										3		
5								1		7			2		38
6												7	14	1	

# 3 + 4 Summary final matrix

	P_1A_1	P_2A_1	P_3A_1	P_4A_1	P_5A_1	P_1A_2	P_2A_2	P_3A_2	P_4A_2	P_5A_2	P_1A_3	P_2A_3	P_3A_3	P_4A_3	P_5A_3
C_1	1					1			1		6		8^1	1	
2				1				2	2		10	21	28	17	3
3						1		4			7	79	96	1	4
4			3										4		1
5								1		14			3		60
6	1					2	1	2			6	29	55	6	2^1

Appendix A

Table 190 Cross classification of Pupil XII behaviors in Content, Process and Attitude categories: work matrices and final matrix. Negative in Process and Attitude disregarded; in Content where noted by superscripts. Symbolically: $C_iP_jA_k$ i--1 to 6
j--1 to 5
k--1 to 3

	P_1A_1	P_2A_1	P_3A_1	P_4A_1	P_5A_1	P_1A_2	P_2A_2	P_3A_2	P_4A_2	P_5A_2	P_1A_3	P_2A_3	P_3A_3	P_4A_3	P_5A_3
#1	Observer I -- Periods 1, 2, 3, 5, 6														
C_1			1			1		0^1	1				1		
2				1				0^2							
3								3					1		
4			1^1												
5															1
6			1			1	1	7					1		
#2	Observer II -- Periods 1, 2, 3, 5, 6														
C_1						2		2^1	1						
2				1		1		1^2							
3								4^1					1		
4															
5										2					
6	1					1		4	1						
Mean 1 & 2: #3	Joint Observers I & II														
C_1			1			1		1^1	1						
2				1				0^2							
3								3					1		
4															
5										1					
6						1		5							
#4	Observer I -- Additional Periods 4, 7, 8, 9, 10														
C_1	1		1					-							
2			2					2^2	1					0^1	
3	1							3					5		1
4								1							
5										1			1		2
6	1														
#3 + 4	Summary final matrix														
C_1	1		2			1		1^1	1						
2			2	1				2^4	1					0^1	
3	1							6					6		1
4								1							
5										2			1		2
6	1					1		5							

Appendix A

Table 191 Cross classification of Teacher XIII behaviors in Content, Process and Attitude categories: work matrices and final matrix. Negative in Process and Attitude disregarded; in Content where noted by superscripts. Symbolically: $C_iP_jA_k$ i--1 to 6
j--1 to 5
k--1 to 3

	P_1A_1	P_2A_1	P_3A_1	P_4A_1	P_5A_1	P_1A_2	P_2A_2	P_3A_2	P_4A_2	P_5A_2	P_1A_3	P_2A_3	P_3A_3	P_4A_3	P_5A_3
#1								Observer I -- Periods 1-5							
C_1	6			1		23	2	39	5	1	13	14	33	9	
2						8	9	8	7		3	11	5	9	
3		2	1			3	16	24			3	16	4		
4			3	2	9	1							1	3	3
5								5		5			3		15^1
6		1	8			5	5	24	3		2	7	17	8	3
#2								Observer II -- Periods 1-5							
C_1	7					20	3	35	8	1	14	2	29	11	
2						12	11	5	5		1	8	8	7	
3		1	2			9	9	21		1	4	17	9		
4			3	3	8	1					1		2	3	3
5				1				6		7			2		11
6			6			6	4	19	3		2	3^1	23	9	1
Mean 1 & 2: #3								Joint Observers I & II							
C_1	6					21	2	37	6	1	13	8	31	10	
2						10	10	6	6		2	9	6	8	
3		1	1			6	12	22			3	16	6		
4			3	2	8	1							1	3	3
5								5		6			2		13
6			7			5	4	21	3		2	5	20	8	2
#4								Observer I -- Additional Periods 6-10							
C_1			3			11		13	5		3		17	7	
2						7	3	15	3		2	1	13	1	
3			1			14	11	15	2		10	11	26		
4			6		1	4	1	3			5		9	2	2
5					4	2		1		6			4		11
6			4			12	5	15	3		7	3	18	14	
#3 + 4								Summary final matrix							
C_1	6		3			32	2	50	11	1	16	8	48	17	
2						17	13	21	9		4	10	19	9	
3		1	2			20	23	37			13	27	32		
4			9	2	9	5	1	3			5		10	5	5
5					4	2		6		12			6		24
6			11			17	9	36	6		9	8	38	22	2

Table 192 Cross classification of Pupil XIII behaviors in Content, Process and Attitude categories: work matrices and final matrix. Negative in Process and Attitude disregarded; in Content where noted by superscripts. Symbolically: $C_iP_jA_k$ i--1 to 6
j--1 to 5
k--1 to 3

	P_1A_1	P_2A_1	P_3A_1	P_4A_1	P_5A_1	P_1A_2	P_2A_2	P_3A_2	P_4A_2	P_5A_2	P_1A_3	P_2A_3	P_3A_3	P_4A_3	P_5A_3
#1							Observer I -- Periods 1-5								
C_1	2		1			4	8	9^1	8^1			1	3	1	
2						3	2	3^1	1				2	1	
3			1			1	24	13				1	5		
4						1	1	1					1^1		1
5															3
6			1				12	2					1		
#2							Observer II -- Periods 1-5								
C_1	1		1			4	4	21^6	11				3	1	
2	1					2^1	3	6	1					1	
3						3	24	10		0^1			1		1^1
4						2^1	1	2					0^1		
5								2							3
6	2					1	8	1^1	1						
Mean 1 & 2: #3							Joint Observers I & II								
C_1	1		1			4	6	15^3	9				3	1	
2						2	2	4	1				1	1	
3						2	24	11					3		
4						1	1	1					0^1		
5								1							3
6	1		1				10	1							
#4							Observer II -- Additional Periods 6-10								
C_1			1			5		11^4	6		0^1		1^1		
2						10	16	16	4				2		
3						18^1	30	10^2	1				5^3		
4			2			6^1	1	4^3					2		
5						2		2		1					6
6	1		2			7	1	9	2	1	1	1	2		
#3 + 4							Summary final matrix								
C_1	1		2			9	6	26^7	15		0^1		4^1	1	
2						12	18	20	5				3	1	
3						20^1	54	21^2	1				8^3		
4			2			7^1	2	5^3					2^1		
5						2		3		1					9
6	2		3			7	11	10	2	1	1	1	2		

Table 193 Cross classification of Teacher XIV behaviors in Content, Process and Attitude categories: work matrices and final matrix. Negative in Process and Attitude disregarded; in Content where noted by superscripts. Symbolically: $C_iP_jA_k$ i--1 to 6
j--1 to 5
k--1 to 3

	P A 1 1	P A 2 1	P A 3 1	P A 4 1	P A 5 1	P A 1 2	P A 2 2	P A 3 2	P A 4 2	P A 5 2	P A 1 3	P A 2 3	P A 3 3	P A 4 3	P A 5 3
#1								Observer I -- Periods, 1, 3, 4, 5, 6							
C_1			1			1		2			7		15^5	9^1	1
2							1					3	2		
3							9	7		1	7	58	36		5
4			1		1						4	1	11		1
5										7	2	1	8	2	34
6													2	1	
#2								Observer II -- Periods 1, 3, 4, 5, 6							
C_1			1					3			7	2	14^8	7^1	1
2												1		1	
3						1	7	6			9	53	39		1
4								1	1		3		12	1	2
5								1		15	2	1	7	2	36
6														1	
Mean 1 & 2: #3								Joint Observers I & II							
C_1			1					2			7	1	14^6	8^1	1
2												2			
3							8	6			8	55	37		3
4											3		11		1
5										11	2	1	7	2	35
6													1	1	
#4								Observer II -- Additional Periods 2, 7, 8, 9, 10							
C_1						2					1	1^3	8^4	3^4	
2															
3			1			4	5	7		1	12	101	43	1	1^1
4													4		
5							2	2		5	1	10	22	1^1	42
6													7	4	
#3 + 4								Summary final matrix							
C_1			1			2		2			8	2^3	22^{10}	11^5	1
2												2			
3			1			4	13	13		1	20	156	80	1	4^1
4											3		15		1
5							2	2		16	3	11	29	3^1	77
6													8	5	

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Table 194 Cross classification of Pupil XIV behaviors in Content, Process and Attitude categories: work matrices and final matrix. Negative in Process and Attitude disregarded; in Content where noted by superscripts. Symbolically: $C_iP_jA_k$

i--1 to 6
j--1 to 5
k--1 to 3

	P_1A_1	P_2A_1	P_3A_1	P_4A_1	P_5A_1	P_1A_2	P_2A_2	P_3A_2	P_4A_2	P_5A_2	P_1A_3	P_2A_3	P_3A_3	P_4A_3	P_5A_3
#1							Observer I -- Periods 1, 3, 4, 5, 6								
C_1	1					1		3^7					1^2		
2															
3						1	16^1	11^4				1	4^2		2^1
4			1										0^1		
5										2			1		7
6															
#2							Observer II -- Periods 1, 3, 4, 5, 6								
C_1	2					1	1	2^5					0^1		
2															
3							15	11^9		3^1		2	2^1		1^1
4			1										0^1		
5								3		2					7
6															
Mean 1 & 2: #3							Joint Observers I & II								
C_1	1					1		2^6					0^1		
2															
3							15	11^6		1		1	3^1		1^1
4			1										0^1		
5								1		2					7
6															
#4							Observer II -- Additional Periods 2, 7, 8, 9, 10								
C_1						1		2^5					0^2		1^-
2															
3						3	6	14^4		0^2		2	4^4		0^6
4													1		
5						2	1	4^2					4^1		18
6															
#3 + 4							Summary final matrix								
C_1	1					2		4^{11}					0^3		1^-
2															
3						3^1	21	25^{10}		1^2		3	7^5		1^7
4			1										1^1		
5						2	1	5^2		2			4^1		25
6															

Table 195 Cross classification of Teacher XV behaviors in Content, Process and Attitude categories: work matrices and final matrix. Negative in Process and Attitude disregarded; in Contnet where noted by superscripts. Symbolically: $C_iP_jA_k$

i--1 to 6
j--1 to 5
k--1 to 3

	P_1A_1	P_2A_1	P_3A_1	P_4A_1	P_5A_1	P_1A_2	P_2A_2	P_3A_2	P_4A_2	P_5A_2	P_1A_3	P_2A_3	P_3A_3	P_4A_3	P_5A_3
#1							Observer I -- Periods 3, 5, 6, 8, 10								
C_1								3	3		2		2	1	3
2								5	1		1	9	2	1	1
3						1	6	14		1	4	15	30		2
4			1					7		1		1	6		
5										15			5		20
6								3			1	4	11	1	
#2							Observer II -- Periods 3, 5, 6, 8, 10								
C_1								2	2				7	1	2
2							3	2	2		1	6	3	2	
3						3	3	9		2	4	12	39^1		3
4			1					4					11		
5							1			16			3		28
6							1	1			1	3	4	1	
Mean 1 & 2: #3							Joint Observers I & II								
C_1								2	2		1		4	1	2
2							1	3	1		1	7	2	1	
3						2	4	11		1	4	13	34		2
4			1					5					8		
5										15			4		24
6								2			1	3	7	1	
#4							Observer II -- Additional Periods 1, 2, 4, 7, 9								
C_1							1	4			2		1		
2							1		1				1	2	
3			1			3	8	13		2	3	5	7		1
4			1		2		1	3					3		2
5							2	3		21			11		26
6				1		7	1	8	1	1		5	5	1	
#3 + 4							Summary final matrix								
C_1							1	6	2		3		5	1	2
2							2	3	2		1	7	3	3	
3			1			5	12	24		3	7	18	31		3
4			2		2		1	8					11		2
5							2	3		36			15		50
6				1		7	1	10	1	1	1	8	12	2	

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Table 196 Cross classification of Pupil XV behaviors in Content, Process and Attitude categories: work matrices and final matrix. Negative in Process and Attitude disregarded; in Content where noted by superscripts. Symbolically: $C_iP_jA_k$ i--1 to 6 j--1 to 5 k--1 to 3

	P_1A_1	P_2A_1	P_3A_1	P_4A_1	P_5A_1	P_1A_2	P_2A_2	P_3A_2	P_4A_2	P_5A_2	P_1A_3	P_2A_3	P_3A_3	P_4A_3	P_5A
#1	Observer I -- Periods 3, 5, 6, 8, 10														
C_1			2					1		1					
2						1	4	3	1				1^1	1	1
3			1				125	18		2		1			
4						1		3^1					1		
5										1					7
6													1		
#2	Observer II -- Periods 3, 5, 6, 8, 10														
C_1			2					2							
2						1	2	2	1				0^1		1
3						4	124	21^2		2					
4						1		3^2							
5										4					4
6															
Mean 1 & 2: #3	Joint Observers I & II														
C_1			2					1							
2						1	3	2	1				0^1		1
3						2	124	19^1		2					
4						1		3^1							
5										2					5
6															
#4	Observer II -- Additional Periods 1, 2, 4, 7, 9														
C_1						2^1		5^1							
2						2		2^1	1^2					1	
3					1	10^1	110^1	17^1		1			2		0^1
4	1		2				1	4							
5								5	1	6			1		9^1
6			1			3	1	5	2						
#3 + 4	Summary final matrix														
C_1			2			2^1		6^1							
2						3	3	4^1	2^1				0^1	1	1
3					1	12^1	234^1	36^2		3			2		0^1
4	1		2			1	1	7^1							
5								5	1	8			1		14^1
6			1			3	1	5	2						

Appendix A

Table 197 Cross classification of Teacher XVI behaviors in Content, Process and Attitude categories: work matrices and final matrix. Negative in Process and Attitude disregarded; in Content where noted by superscripts. Symbolically: $C_iP_jA_k$ i--1 to 6
j--1 to 5
k--1 to 3

	P_1A_1	P_2A_1	P_3A_1	P_4A_1	P_5A_1	P_1A_2	P_2A_2	P_3A_2	P_4A_2	P_5A_2	P_1A_3	P_2A_3	P_3A_3	P_4A_3	P_5A_3
#1							Observer I -- Periods 3, 5, 6, 7, 8								
C_1											3		6^2		1
2													9^1	2^5	3
3						1	8	9				38	31		2
4													1		
5										3		3	44^3	3	39^2
6						3	1	3				8	26		1
#2							Observer II -- Periods 3, 5, 6, 7, 8								
C_1								2			0^1		3	2	1
2						1		3			2^1		7	3^5	1
3						1	19	12				36	29		
4															
5							1	1		6	1	12	29^4	3	5^1
6						1	1	3		1	2	7	24^2	2	
Mean 1 & 2: #3							Joint Observers I & II								
C_1								1			1		4^1	1	1
2								1			1		8	2^5	2
3						1	13	10				37	30		1
4															
5												7	36^3	3	45^1
6						2	1	3		4	1	7	25	1	
#4							Observer II -- Additional Periods 1, 2, 4, 9, 10								
C_1						1		1			1^1		2^6	2	
2						2		2	2		1	2	4	2^2	1
3	1					1	22	12			2	26^1	24	1	
4								1			1		5	1	1
5										10			3		31^1
6								3	1		6	5	22	6^1	1
#3 + 4							Summary final matrix								
C_1						1		2			2^1		6^7	3	1
2						2		3	2		2	2	12	4^7	2
3	1					2	35	22			2	63^1	54	1	1
4								1			1		5	1	1
5										14		7	39^3	3	76^2
6						2	1	6	1		7	12	47	7^1	1

Appendix A

Table 198 Cross classification of Pupil XVI behaviors in Content, Process and Attitude categories: work matrices and final matrix. Negative in Process and Attitude disregarded; in Content where noted by superscripts. Symbolically: $C_iP_jA_k$ i--1 to 6
j--1 to 5
k--1 to 3

	P_1A_1	P_2A_1	P_3A_1	P_4A_1	P_5A_1	P_1A_2	P_2A_2	P_3A_2	P_4A_2	P_5A_2	P_1A_3	P_2A_3	P_3A_3	P_4A_3	P_5A_3
#1 Observer I -- Periods 3, 5, 6, 7, 8															
C_1	1					0^1		1					0^1		
2								1					0^1		
3						2	48	13							
4															
5						1		4		4			2		11
6								2		1					
#2 Observer II -- Periods 3, 5, 6, 7, 8															
C_1	1					1		2^3							
2								3							
3						2	64	10^2				1	1		
4															
5						2		5^1		1					13
6								2		1			0^1		
Mean 1 & 2: #3 Joint Observers I & II															
C_1	1							1^1							
2								2							
3						2	56	11^1							
4															
5						1		4		2			1		12
6								2		1					
#4 Observer II -- Additional Periods 1, 2, 4, 9, 10															
C_1						0^1		6^2					0^1		
2									1						
3						3	72^1	12^4					3		0^1
4								1	1						
5										5					3
6						3^1		4							
#3 + 4 Summary final matrix															
C_1	1					0^1		7^3					0^1		
2								2	1						
3						5	128^1	23^5					3		0^1
4								1	1						
5						1		4		7			1		15
6						3^1		6		1					

Appendix A

Table 199 Cross classification of Teacher XVII behaviors in Content, Process and Attitude categories: work matrices and final matrix. Negative in Process and Attitude disregarded; in Content where noted by superscripts. Symbolically: $C_iP_jA_k$

i--1 to 6
j--1 to 5
k--1 to 3

#1 Observer I -- Periods 1-9

	P_1A_1	P_2A_1	P_3A_1	P_4A_1	P_5A_1	P_1A_2	P_2A_2	P_3A_2	P_4A_2	P_5A_2	P_1A_3	P_2A_3	P_3A_3	P_4A_3	P_5A_3
C_1	1					9	2	15	6		4		4	6	2
2	2	2	3	2		24	25	45	16	4		3	20	12	5
3			2	1		6	59	74	4	12		7	8	2	9
4			1		3										2
5							2	1		33					33
6						2	6	11	12	2	2	2	4	11	

#2 Observer II -- Periods 1-9

	P_1A_1	P_2A_1	P_3A_1	P_4A_1	P_5A_1	P_1A_2	P_2A_2	P_3A_2	P_4A_2	P_5A_2	P_1A_3	P_2A_3	P_3A_3	P_4A_3	P_5A_3
C_1						11	6	21	6		2		7	4	2
2		2	2	3	1	25	8	51	12^1	4	3	5	16	9^1	1
3	2	2	4	1		9	50	62	1	6		10	24	2	11
4								1		1			2		2
5								9		29	1		8	3	35
6						1	6	12	6	2	2			6	2

Mean 1 & 2: #3 Joint Observers I & II

	P_1A_1	P_2A_1	P_3A_1	P_4A_1	P_5A_1	P_1A_2	P_2A_2	P_3A_2	P_4A_2	P_5A_2	P_1A_3	P_2A_3	P_3A_3	P_4A_3	P_5A_3
C_1						10	4	18	6		3		5	5	2
2	1	2	2	2		24	16	48	14	4	1	4	18	10	3
3	1	1	3	1		7	54	68	2	9		8	16	2	10
4					1								1		2
5							1	5		31			4	1	34
6						1	6	11	9	2	2	1	2	8	1

#4 Observer II -- Additional Periods: 10

	P_1A_1	P_2A_1	P_3A_1	P_4A_1	P_5A_1	P_1A_2	P_2A_2	P_3A_2	P_4A_2	P_5A_2	P_1A_3	P_2A_3	P_3A_3	P_4A_3	P_5A_3
C_1						1	1	2			1	1			
2				3				2						2	
3						4	12	10		1		8	3		
4				1									1		
5								1		3					9
6	1					1	2	1	2						

#3 + 4 Summary final matrix

	P_1A_1	P_2A_1	P_3A_1	P_4A_1	P_5A_1	P_1A_2	P_2A_2	P_3A_2	P_4A_2	P_5A_2	P_1A_3	P_2A_3	P_3A_3	P_4A_3	P_5A_3
C_1						11	5	20	6		4	1	5	5	2
2	1	2	2	5		24	16	50	14	4	1	4	18	12	3
3	1	1	3	1		11	66	78	2	10		16	19	2	10
4				1	1								2		2
5							1	6		34			4	1	43
6	1					2	8	12	11	2	2	1	2	8	1

Table 200 Cross classification of Pupil XVII behaviors in Content, Process and Attitude categories: work matrices and final matrix. Negative in Process and Attitude disregarded; in Content where noted by superscripts. Symbolically: $C_iP_jA_k$

i--1 to 6
j--1 to 5
k--1 to 3

	P_1A_1	P_2A_1	P_3A_1	P_4A_1	P_5A_1	P_1A_2	P_2A_2	P_3A_2	P_4A_2	P_5A_2	P_1A_3	P_2A_3	P_3A_3	P_4A_2	P_5A
#1							Observer I -- Periods 1-9								
C_1						0^1	3	4^1					2^2	1	
2	1		3	1		2	23^3	39^2	9	3	1	4	5^2	0^1	
3		2				1^1	69^4	13^2		1^-		10^1	25^6		5
4										2					
5							1			5^2					1
6							1	1	2			1			
#2							Observer II -- Periods 1-9								
C_1	1					1	3^1	5^3	1			1	7^1	1	
2	1		3^1	1		4	21	17^3	18^4			5	5^1	1	
3		1				2	74^2	20		0^2		13^1	14^6		4
4								1^2							
5								1^1		4		1	1		3
6							2	1	4		1	1	3		
Mean 1 & 2: #3							Joint Observers I & II								
C_1							3	4^2					4^1	1	
2	1		3	1		3	22^1	28^2	13^2	1		4	5^1		
3		1				1	71^3	17^1		0^1		11^1	19^6		4
4								0^1		1					
5										4^1					2
6							1	1	3			1	1		
#4							Observer II -- Additional Periods: 10								
C_1															
2							0^1					1			
3							5	3				1	3		
4															
5															
6															
#3 + 4							Summary final matrix								
C_1							3	4^2					4^1	1	
2	1		3	1		3	22^2	28^2	13^2	1		5	5^1		
3		1				1	76^3	20^1		0^1		12^1	22^6		4
4								0^1		1					
5										4^1					2
6							1	1	3			1	1		

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Table 201 Cross classification of Teacher XVIII behaviors in Content, Process and Attitude categories: work matrices and final matrix. Negative in Process and Attitude disregarded; in Content where noted by superscripts. Symbolically: $C_iP_jA_k$ i--1 to 6
j--1 to 5
k--1 to 3

	P_1A_1	P_2A_1	P_3A_1	P_4A_1	P_5A_1	P_1A_2	P_2A_2	P_3A_2	P_4A_2	P_5A_2	P_1A_3	P_2A_3	P_3A_3	P_4A_3	P_5A_3
#1	Observer I -- Periods 2, 3, 4, 5, 6														
C_1	1					6	2	9			5		6	1	2
2	1					5	8	45				4	12	1	
3						3	27	38			1	3	10		1
4						2		6							1
5								2		7			2		58
6						2	7	9	4				5	3	
#2	Observer II -- Periods 2, 3, 4, 5, 6														
C_1	1					9	1	9			3		10	1	3
2						10	10	38	3	1	1	2	8	1	1
3	1					2	23	24				4	15		
4						2		4					1		1
5								1		5			2		65
6						4	5	10					7	2	
Mean 1 & 2: #3	Joint Observers I & II														
C_1	1					7	1	9			4		8	1	2
2						7	9	41	1			3	10	1	
3						3	25	31				3	13		
4						2		5							1
5								1		6			2		61
6						3	6	9	2				6	2	
#4	Observer II -- Additional Periods: 1, 7, 8, 9														
C_1						6	2	3			9		10	3	
2						3		4	3				1	1	
3						5	5	11				3	15		
4					1	1		4			2				2
5								3		12					42
6						2	5	18	3		2		6		
#3 + 4	Summary final matrix														
C_1	1					13	3	12			13		18	4	2
2						10	9	45	4			3	11	2	
3						7	30	42				6	28		
4					1	3		9			2				3
5								4		18			2		103
6						5	11	27	5		2		12	2	

Appendix A

Table 202 Cross classification of Pupil XVIII behaviors in Content, Process and Attitude categories: work matrices and final matrix. Negative in Process and Attitude disregarded; in Content where noted by superscripts. Symbolically: $C_iP_jA_k$

i--1 to 6
j--1 to 5
k--1 to 3

	P_1A_1	P_2A_1	P_3A_1	P_4A_1	P_5A_1	P_1A_2	P_2A_2	P_3A_2	P_4A_2	P_5A_2	P_1A_3	P_2A_3	P_3A_3	P_4A_3	P_5A
#1							Observer I - Periods 2, 3, 4, 5, 6								
C_1	1					3	1	0^1					1^1		
2	1					1	12	15^3	2			1	3		
3			1			2	33	26^2					1		
4								2^1					0^1		
5								0^1		2					5
6							4^1	4							
#2							Observer II -- Periods 2, 3, 4, 5, 6								
C_1	1					2	1^2	1^3							
2	1					3	10	14^5	5	0^1					
3			1			1	35^1	22^2				1	2		1
4								4^3					3		
5								0^1		4					7
6						3	1	3							
Mean 1 & 2: #3							Joint Observers I & II								
C_1	1					2	1^1	0^2							
2	1					2	11	14^4	3				1		
3			1			1	34	24^2					1		
4								3^2					1		
5								0^1		3					6
6						1	2	3							
#4							Observer II -- Additional Periods: 1, 7, 8, 9								
C_1	3		1			4	0^1	7^1	2						
2						6	31	30	2						
3						3	6	4^1					4^1		
4						1		3^1							
5										3					7
6							4	14	4						
#3 + 4							Summary final matrix								
C_1	4		1			6	1^2	7^1	2						
2	1					8	42	44^4	5				1		
3			1			4	40	28^3					5^1		
4						1		6^3					1		
5								0^1		6					13
6						1	6	17	4						

Appendix A

Table 203 Representative triple frame cross classification in Content (C), Process (P), and Attitude (A), categories in classrooms type (RP) individually. Classroom scores. Negatives disregarded. Symbolically: $C_iP_jA_k$: i--1 to 6; j--1 to 5; k--1 to 3. Scores 5 only included except where mean 5. Such scores 5 starred.

	P_1A_1	P_2A_1	P_3A_1	P_4A_1	P_5A_1	P_1A_2	P_2A_2	P_3A_2	P_4A_2	P_5A_2	P_1A_3	P_2A_3	P_3A_3	P_4A_3	P_5A_3
								Classroom I							
C_1						44	10	57	10		4*		17	9	
2						31	90	50	31	6	2*	8	17	8	6
3						9	128	100		16	1*	12	20		
4			1*		17			2*					4*		
5								4*		63			1*		38
6						9	16	42	15				4*		
								Classroom XI							
C_1						42		22	9		17		16	9	
2						21	77	32	11		11	19	29	14	
3						13	116	38			4*	27	43		
4			4*		6			4*					1*		
5								3*		32			16		92
6			8			13	30	72	3			9	35	1*	
								Classroom XIII							
C_1						38	7	76	25		16	7	49	17	
2						27	29	40	13		4*	9	20	9	
3						38	71	57			12	26	37		
4			10		8			7					12		
5								8		12			6		31
6			12			23	19	44	7		9	8	37	20	

*In next twelve tables:

Content:		Process:		Attitude:	
1	Structure	1	Analyzing	1	Curiosity
2	Deductive	2	Synthesizing	2	Independence
3	Mathematical	3	Specializing	3	Receptivity
4	Other	4	Generalizing		
5	Techniques	5	Relevant		
6	Inductive				

Appendix A

Table 204 Representative triple frame cross classification in Content (C), Process (P), and Attitude (A), categories in classrooms type (rP) individually. Classroom scores. Negatives disregarded. Symbolically: $C_iP_jA_k$: i--1 to 6; j--1 to 5; k--1 to 3. Scores 5 only included except where mean 5. Such scores 5 starred.

	P_1A_1	P_2A_1	P_3A_1	P_4A_1	P_5A_1	P_1A_2	P_2A_2	P_3A_2	P_4A_2	P_5A_2	P_1A_3	P_2A_3	P_3A_3	P_4A_3	P_5A
								Classroom IX							
C_1						26	14	51	12				28	13	
2						19	21	56	23			9	14	20	6
3						9	91	137		13		46	46		6
4								2					13		
5							8	43		44			17		43
6						2*	5	37	13			7	2*	3*	
								Classroom XV							
C_1						4*	1*	18	3*				7	2*	
2						4*	7	12	8			9	7	7	
3						24	345	83		8	9	25	56		5
4								21					16		
5								10		59			21		84
6						13	3*	20	4*			10	17	3*	
								Classroom XVII							
C_1						12	9	29	7				12	7	
2			5	7		30	44	88	32			10	26	14	
3						14	160	107		12		33	52		15
4								2*					2*		
5								7		43			4*		50
6						2*	10	14	15			2*	3*	9	

Appendix A

Table 205 Representative triple frame cross classification in Content (C), Process (P), and Attitude (A), categories in classrooms type (rp) individually. Classroom scores. Negative disregarded. Symbolically: $C_iP_jA_k$: i--1 to 6; j--1 to 5; k--1 to 3. Scores 5 only included except where mean 5. Such scores 5 starred.

	P_1A_1	P_2A_1	P_3A_1	P_4A_1	P_5A_1	P_1A_2	P_2A_2	P_3A_2	P_4A_2	P_5A_2	P_1A_3	P_2A_3	P_3A_3	P_4A_3	P_5A_3
								Classroom VIII							
C_1						9		27			6		22	20	
2						9	10	48	7		9	4*	42	38	
3							83	105		17	3*	62	61		12
4								9					6		
5								10		40			6		140
6						2*		32			4*	5	9	9	
								Classroom XII							
C_1								3*			10		17	2*	
2								14	5		17	37	49	31	5
3								17			12	137	177		9
4			7										7		
5								2		28			7		108
6	5					5	6	12			10	59	96	19	
								Classroom XVI							
C_1								15			4*		19	3*	
2								5	4*		3*	3*	16	8	
3						9	218	68			3*	85	76		3*
4													7		
5								7		29		9	53		121
6						8		16			9	16	64	8	

Appendix A

Table 206 Representative triple frame cross classification in Content (C). Process (P), and Attitude (A), categories in classrooms type (Rp) individually. Classroom scores. Negative disregarded. Symbolically: $C_iP_jA_k$: i--1 to 6; j--1 to 5; k--1 to 3. Scores 5 only included except where mean 5. Such scores 5 starred.

	P_1A_1	P_2A_1	P_3A_1	P_4A_1	P_5A_1	P_1A_2	P_2A_2	P_3A_2	P_4A_2	P_5A_2	P_1A_3	P_2A_3	P_3A_3	P_4A_3	P_5A
								Classroom III							
C_1	5			7							47	5	14	28	
2								8	5		53	86	34	92	5
3							6	9			13	129	114	6	6
4					12										
5										12			6		15
6			3*					17			22	63	71	40	
								Classroom V							
C_1	11			5							29	6	23	24	
2	5							5			29	104	68	83	
3								10			10	90	53		5
4					5										
5										4*			5		15
6			12	5				7			20	139	129	36	
								Classroom VI							
C_1	3*			5		5		5			18		36	3*	
2						10	5	19			30	118	100	46	
3							6	10			14	68	72		9
4					24								8		
5								14		4*			16		122
6			2*					6			7	43	40	14	

Appendix A

Table 207 Representative triple frame cross classification in Content (C), Process (P), and Attitude (A), categories in classrooms type (RP) individually. Teacher scores. Negatives disregarded. Symbolically: $C_iP_jA_k$: i--1 to 6; j--1 to 5; k--1 to 3. Scores 5 only included except where mean 5. Such scores 5 starred.

	P_1A_1	P_2A_1	P_3A_1	P_4A_1	P_5A_1	P_1A_2	P_2A_2	P_3A_2	P_4A_2	P_5A_2	P_1A_3	P_2A_3	P_3A_3	P_4A_3	P_5A_3
Teacher I															
C_1						37	7	36	6		5		14	8	
2						27	57	42	21		2*	8	15	7	
3						8	61	79		13	1*	8	17		
4			1*		17								5		
5										55			1*		32
6						8	10	32	13				3	2*	
Teacher XI															
C_1						35		15	5		17		16	8	
2						12	8	17	11		11	11	21	13	
3						6	19	33			5	25	41		
4			11		6								2		
5										31			15		90
6			7			11	21	63	3*			9	33	1*	
Teacher XIII															
C_1						30		46	10		15		44	16	
2						16	12	19	8		4*	7	18	8	
3						19	21	34			12	25	30		
4			8		2*	5							9		
5								6		11			6		22
6			10			16	8	33	6		8	7	35	20	

Appendix A

Table 208 Representative triple frame cross classification in Content (C), Process (P), and Attitude (A), categories in classrooms type (r P) individually. Teacher scores. Negative disregarded. Symbolically: $C_iP_jA_k$: i--1 to 6; j--1 to 5; k--1 to 3. Scores 5 only included except where mean 5. Such scores 5 starred.

	P_1A_1	P_2A_1	P_3A_1	P_4A_1	P_5A_1	P_1A_2	P_2A_2	P_3A_2	P_4A_2	P_5A_2	P_1A_3	P_2A_3	P_3A_3	P_4A_3	P_5A
Teacher IX															
C_1						21	7	43	9				23	12	
2						17	9	47	18			7	9	17	
3						8	58	105		9		34	30		5
4													9		
5								36		41			8		38
6							5	33	12			7	2*		
Teacher XV															
C_1								8	3*				7	1*	
2							3*	4*	3*			9	4*	4*	
3						7	16	31		4*	9	24	41		4*
4								10					14		
5								4*		47			20		65
6						9	1*	13	1*			10	16		
Teacher XVII															
C_1						12	5	22	7				5	5	
2						26	18	55	15			4*	20	13	
3						12	72	86		11		18	21		11
4													2*		
5								7		37			4*		47
6							9	13	12			1*	2*	9	

Appendix A

Table 209 Representative triple frame cross classification in Content (C), Process (P), and Attitude (A), categories in classrooms type (r_p) individually and mean. Teacher scores. Negative disregarded. Symbolically: $C_iP_jA_k$: i--1 to 6; j--1 to 5; k--1 to 3. Scores 5 only included except where mean 5. Such scores 5 starred.

	P_1A_1	P_2A_1	P_3A_1	P_4A_1	P_5A_1	P_1A_2	P_2A_2	P_3A_2	P_4A_2	P_5A_2	P_1A_3	P_2A_3	P_3A_3	P_4A_3	P_5A_3
Teacher VIII															
C_1								26			6		22	18	
2								39			9	4	35	38	
3							66	85		16	4*	50	48		12
4								9					5		
5								10		38			10		140
6								28			2*	9	9	9	
Teacher XII															
C_1											10		16	2*	
2								3*			17	37	49	30	
3								7			12	138	167		7
4			5										7		
5										24			5		105
6								3*			10	50	96	10	
Teacher XVI															
C_1								3*			4*		17	4*	
2								4*			3*	3*	16	12	
3							47	29			3*	85	72		1*
4													7		
5										19			56		104
6								8			9	16	63	11	

Appendix A

Table 210 Representative triple frame cross classification in Content (C), Process (.), and Attitude (A), categories in classrooms type (R_p) individually and mean. Teacher scores. Negative disregarded. Symbolically: $C_iP_jA_k$: i--1 to 6; j--1 to 5; k--1 to 3. Scores 5 only included except where mean 5. Such scores 5 starred.

	P_1A_1	P_2A_1	P_3A_1	P_4A_1	P_5A_1	P_1A_2	P_2A_2	P_3A_2	P_4A_2	P_5A_2	P_1A_3	P_2A_3	P_3A_3	P_4A_3	P_5A
Teacher III															
C_1	5			7							47		13	28	
2								6			52	86	33	92	
3								8			13	129	88		5
4					12										
5										12			6		12
6			1*					16			21	62	71	31	
Teacher V															
C_1	11			4*							29		23	25	
2								1*			29	104	63	80	
3								8			10	90	48		4
4					5										
5										4*			4*		13
6			12					6			20	140	127	36	
Teacher VI															
C_1	3*			5							18		36	3*	
2								13			30	117	98	46	
3								8			14	68	68		9
4					24										
5								15		9			16		119
6			2*					4*			7	43	40	14	

Appendix A

Table 211 Representative triple frame cross classification in Content (C), Process (P), and Attitude (A), categories in classrooms type (RP) individually. Pupils scores. Negatives disregarded. Symbolically: $C_iP_jA_k$: i--1 to 6; j--1 to 5; k--1 to 3. Scores 5 only included except where mean 5. Such scores 5 starred.

Pupils I

	P_1A_1	P_2A_1	P_3A_1	P_4A_1	P_5A_1	P_1A_2	P_2A_2	P_3A_2	P_4A_2	P_5A_2	P_1A_3	P_2A_3	P_3A_3	P_4A_3	P_5A_3
C_1						7		18	5						
2						5	32	17	10						
3						1*	64	33							
4															
5															5
6							6	8							

Pupils XI

	P_1A_1	P_2A_1	P_3A_1	P_4A_1	P_5A_1	P_1A_2	P_2A_2	P_3A_2	P_4A_2	P_5A_2	P_1A_3	P_2A_3	P_3A_3	P_4A_3	P_5A_3
C_1						6		7	2*						
2						9	70	16							
3						7	98	14							
4															
5															3*
6							10	10							

Pupils XIII

	P_1A_1	P_2A_1	P_3A_1	P_4A_1	P_5A_1	P_1A_2	P_2A_2	P_3A_2	P_4A_2	P_5A_2	P_1A_3	P_2A_3	P_3A_3	P_4A_3	P_5A_3
C_1						8		31	14						
2						11	17	19	5						
3						19	50	21					10		
4															
5															8
6							10	9							

Appendix A

Table 212 Representative triple frame cross classification in Content (C), Process (P), and Attitude (A), categories in classrooms type (rP) individually. Pupils scores. Negatives disregarded. Symbolically: $C_iP_jA_k$: i--1 to 6; j--1 to 5; k--1 to 3. Scores 5 only included except where mean 5. Such scores 5 starred.

	P_1A_1	P_2A_1	P_3A_1	P_4A_1	P_5A_1	P_1A_2	P_2A_2	P_3A_2	P_4A_2	P_5A_2	P_1A_3	P_2A_3	P_3A_3	P_4A_3	P_5A
							Pupils IX								
C_1								7							
2							12	8	5						
3							33	32				12	15		
4															
5										1*					5
6															
							Pupils XV								
C_1								9							
2							4*	7	5						
3						14	308	50					3*		
4															
5										10					20
6															
							Pupils XVII								
C_1								7							
2							26	33	16						
3						1*	87	23				14	31		
4															
5										5					2
6															

Appendix A

Table 213 Representative triple frame cross classification in Content (C), Process (P), and Attitude (A), categories in classrooms type (rp) individually. Pupils scores. Negatives disregarded. Symbolically: $C_iP_jA_k$: i--1 to 6; j--1 to 5; k--1 to 3. Scores 5 only included except where mean 5. Such scores 5 starred.

	P_1A_1	P_2A_1	P_3A_1	P_4A_1	P_5A_1	P_1A_2	P_2A_2	P_3A_2	P_4A_2	P_5A_2	P_1A_3	P_2A_3	P_3A_3	P_4A_3	P_5A_3
								Pupils VIII							
C_1								2*							
2								7							
3							17	20					15		
4															
5															1*
6								4*							
								Pupils XII							
C_1								3*							
2								10							
3								10					10		
4															
5															3*
6								9							
								Pupils XVI							
C_1								13							
2								3*							
3							172	37					4*		
4															
5															20
6								8							

Appendix A

Table 214 Representative triple frame cross classification in Content (C). Process (P), and Attitude (A), categories in classrooms type (R_p) individually. Pupils scores. Negatives disregarded. Symbolically: $C_iP_jA_k$: i--1 to 6; j--1 to 5; k--1 to 3. Scores 5 only included except where mean 5. Such scores 5 starred.

	P_1A_1	P_2A_1	P_3A_1	P_4A_1	P_5A_1	P_1A_2	P_2A_2	P_3A_2	P_4A_2	P_5A_2	P_1A_3	P_2A_3	P_3A_3	P_4A_3	P_5A
Pupils III															
C_1															
2															
3													26		
4															
5															
6															
Pupils V															
C_1															
2															
3													5		
4															
5															
6															
Pupils VI															
C_1															
2															
3													3*		
4															
5															
6															

Appendix A

Table 215 Representative classroom individual frame scores (T & P) of the twelve classrooms of the four types: high rigor, high participation (RP); low rigor, high participation (rP); low rigor, low participation (rp); high rigor, low participation (Rp), in each category of Content, Process and Attitude and in Neutral. Superscripts indicate negative behaviors.

TYPE		RP			rP			rp			Rp	
CLASS	I	XI	XIII	IX	XV	XVII	VIII	XII	XVI	III	V	VI
TEGORY												
ntent												
ructure	161^{8}	115^{5}	244^{9}	148^{13}	43^{3}	83^{5}	109^{7}	46^{4}	33^{20}	120	104^{2}	83^{2}
chniques	109^{1}	148	65^{1}	162^{4}	179	108^{2}	216	151	243^{1}	35	26	170
ductive	246^{12}	215^{6}	153^{1}	188^{8}	53^{5}	270^{10}	174^{10}	160^{11}	53^{4}	289^{4}	298^{6}	342^{2}
ductive	90	181^{1}	187^{1}	75	78^{1}	[illegible]	80^{1}	224^{2}	128^{3}	218	354	123
thematical	301^{12}	259^{2}	248^{4}	356^{9}	533^{7}	397^{15}	356^{9}	363^{1}	474^{10}	288^{2}	184	194
her	37	34	73^{5}	30^{1}	53^{2}	10^{1}	20^{2}	18^{1}	15	26	16^{1}	52
ocess												
alyzing	109	137	201	77	62	77	57	76	52	152	115	96
nthesizing	270	283	185	219	391	280	191	242	350	295	345	248
ecializing	346	353	435	464	301	363	407	435	369	292	322	348
neralizing	90	53	104	103	29	102	100	68	43	185	173	88
levant	162	140	66	131	174	145	229	160	170	59	36	188
itude												
riosity	33	37	57	30	18	32	21	30	3	49	60	64
dependence	770	567	577	650	642	663	458	113	415	84	40	100
ceptivity	174	362	357	314	297	272	505	838	566	849	891	804
tral	23	34	9	6	43	33	16	19	16	18	9	32
al cher and il	1,000	1,000	1,000	1,000	1,000	1,000	1,000	1,000	1,000	1,000	1,000	1,000

Appendix A

Table 216 Representative teacher and Pupils components of individual frame scores of the twelve classrooms of the four types: high rigor, high participation (RP); low rigor, high participation (rP); low rigor, low participation (rp); high rigor, low participation (Rp) in each category of Content, Process, Attitude and in Neutral. Superscripts indicate negative behaviors.

TYPE		RP			rP			rp			Rp	
CLASS	I	XI	XIII	IX	XV	XVII	VIII	XII	XVI	III	V	VI
CATEGORY												
						TEACHERS						
Content												
Structure	126^2	101	183	128^1	29	66	97^5	35^2	21^{11}	118	103	82^1
Techniques	95	141	51	141	141	99	208	142	204	31	23	166
Deductive	187	106^1	96	146	32	176	157^3	148^2	49^3	284^1	288^1	335
Inductive	70	157	150	70	60	58	73	206^2	115^1	216	351	120
Mathematical	197^3	134	147	261	153^1	244	291^2	337	249^1	257^1	170	185
Other	36	28	54	28	37	9	19	16	12	26	16	52
Process												
Analyzing	92	107	138	64	31	67	40	64	35	147	110	94
Synthesizing	159	95	98	148	75	135	158	240	168	295	343	244
Specializing	255	284	312	362	195	247	341	374	286	253	299	332
Generalizing	68	49	79	87	17	78	95	61	40	184	169	87
Relevant	144	133	54	114	134	125	223	151	137	54	32	184
Attitude												
Curiosity	25	33	47	25	8	24	15	14	2	43	54	63
Independence	540	304	316	509	180	437	382	61	130	74	29	84
Receptivity	153	331	318	241	264	191	460	815	534	816	870	794
Neutral	21	33	9	6	42	32	16	17	13	18	9	32
Total Teacher	740	701	690	781	494	684	873	906	679	951	962	973
						PUPILS						
Content												
Structure	35^6	14^5	61^9	20^{12}	14^3	17^5	12^2	11^2	12^9	2	1^2	1
Techniques	15^1	7	14^1	21^4	39	9^2	8	9	39^1	3^1	3	4
Deductive	59^{10}	109^5	57^1	42^8	21^5	94^{10}	17^7	12^9	4^1	5^3	10^5	7^2
Inductive	20	24^1	37^1	5	18^1	8	7^1	18	13^2	2	3	3
Mathematical	104^9	125^2	101^4	95^9	380^6	153^{15}	65^7	26^1	225^9	32^1	14	9
Other	1	6	19^5	2^1	16^2	1^1	1	2^1	3	0	0	0
Process												
Analyzing	17	30	63	13	31	10	16	12	17	5	5	2
Synthesizing	111	188	87	71	316	145	33	2	182	0	2	4
Specializing	91	69	123	102	106	116	67	61	83	39	23	16
Generalizing	22	4	25	16	12	24	5	7	3	1	4	1
Relevant	18	7	12	17	40	20	6	9	33	4	4	4
Attitude												
Curiosity	8	4	10	5	10	8	6	16	1	6	6	1
Independence	230	263	251	141	462	226	76	52	285	10	11	16
Receptivity	21	31	39	73	33	81	45	25	32	33	21	10
Neutral	1	1	0	0	1	1	0	3	3	0	0	0
Total Pupils	260	298	310	219	505	315	127	94	321	49	38	27
Total Teacher and Pupil	1,000	1,000	1,000	1,000	1,000	1,000	1,000	1,000	1,000	1,000	1,000	1,000

Appendix A

Table 217 Simple differences of cumulative proportions in each category of Content, of Process and of Attitude, and in Neutral for Teacher II and Pupils II. All decimals omitted. All differences in absolute value. Dash occurs where proportions = 0.

DAYS TEGORY	1-2	2-3	3-4	4-5	5-6	6-7	7-8	8-9	9-10
				TEACHER II					
ntent									
tructure	042*	016	000	009	005	004	018*	014*	013*
echniques	039*	001	002	000	007	007	005	004	001
eductive	017	037*	007	010	005	012	022*	005	005
nductive	049	013	031**	000	005	001	007	011	005
athematical	020	058*	006	020	006	013	020	000	002
her	-	-	004	033***	001	002	003	001	004
ocess									
alyzing	027*	009	006	006	007	001	001	011	008
nthesizing	043*	019	000	004	004	009	013	012	007
ecializing	018	007	042*	035*	004	008	008	017	008
neralizing	030	002	001	003	004	008	002	005	007
levant	050*	001	002	006	014	008	001	001	000
itude									
riosity	020**	005	002	012	001	001	003	002	002
dependence	022	028	038*	006	005	014	002	002	005
ceptivity	069*	011	008	017	008	011	013	001	004
tral	003	014	005	004	005	001	000	003	000
				PUPILS II					
tent									
ructure	011	013	003	004	003	003	000	003	001
chniques	011	002	001	001	005*	000	000	000	002
ductive	011	046***	011	008	003	009	005	001	002
ductive	016	001	004	002	002	005	003	002	001
thematical	009	014	012	017	006	015	011	001	001
her	-	-	-	002	000	000	000	000	000
cess									
alyzing	011	004	001	003	008	003	000	001	002
nthesizing	014	011	035*	019	007	009	007	002	001
ecializing	041**	004	018	007	004	005	000	002	002
neralizing	006	013***	003	002	005	006	001	004	002
levant	006	010	007	003	005	003	000	000	003
itude									
riosity	-	-	-	-	-	006	001	001	001
dependence	029	008	028	028	004	003	009	002	001
ceptivity	002	003	000	001	002	001	002	001	000
tral	-	002	001	001	000	000	001	000	000

* p = 0.34
** p = 0.05
*** p = 0.01

Appendix A

Table 218 Simple differences of cumulative proportions in each category of Content, of Process and of Attitude, and in Neutral for Teacher III and Pupils III. All decimals omitted. All differences in absolute value. Dash occurs where proportions = 0.

DAYS CATEGORY	1-2	2-3	3-4	4-5	5-6	6-7	7-8	8-9	9-10
Content				TEACHER III					
Structure	023	026 *	015	006	004	008	008	023**	003
Techniques	005	002	008*	001	001	001	001	005	003
Deductive	080**	049*	022	060**	040*	012	033*	013	023*
Inductive	017	020	042**	000	010	011	002	016*	012
Mathematical	079*	122**	000	041*	037*	021*	023*	025*	016*
Other	000	040***	012	008	001	005	004	002	000
Process									
Analyzing	023	006	009	032**	005	003	002	005	008
Synthesizing	082*	018	015	010	005	014	002	002	011
Specializing	009	022	015	000	007	012	015	009	027*
Generalizing	071*	011	015	017	014	006	003	008	005
Relevant	010	037	007	009	001	002	006	001	001
Attitude									
Curiosity	015	059***	016	007	002	007	003	001	001
Independence	007	014	015*	002	002	004	003	002	000
Receptivity	029	028	002	025 *	004	015	006	009	004
Neutral	005	002	002	001	001	018***	000	003	002
Content				PUPILS III					
Structure	-	-	-	-	002	001	000	001	000
Techniques	010	003	002	001	001	001	000	001	000
Deductive	005	002	002	001	001	003*	001	000	000
Inductive	005	002	002	001	001	000	000	001	000
Mathematical	008	009	005	009	006	003	003	004	002
Other	-	-	-	-	-	-	-	-	-
Process									
Analyzing	010	003	001	002	001	001	001	001	001
Synthesizing	-	-	-	-	-	-	-	-	-
Specializing	008	012	006	012	002	004	005	003	002
Generalizing	-	-	-	-	-	001	000	000	000
Relevant	010	003	002	001	001	001	001	000	001
Attitude									
Curiosity	005	002	002	003	002	001	001	001	001
Independence	005	005	000	002	001	000	001	002	002
Receptivity	002	012	003	006	002	000	004	004	001
Neutral	—	—	—	—	—	—	—	—	—

* p = 0.34
** p = 0.05
*** p = 0.01

Appendix A

Table 219 Simple differences of cumulative proportions in each category of Content, of Process and of Attitude, and in Neutral for Teacher V and Pupils V. All decimals omitted. All differences in absolute value. Dash occurs where proportions = 0.

DAYS TEGORY	1-2	2-3	3-4	4-5	5-6	6-7	7-8	8-9	9-10
				TEACHER V					
ntent									
tructure	064***	004	008	002	020*	011	001	003	011*
echniques	006	005	004	001	001	003	005	002	000
eductive	205***	043*	058**	053**	041**	025*	002	013	023*
nductive	119***	005	080***	013	069***	033	016	005	028*
athematical	364***	036*	011	032*	046**	034**	028*	014	017*
her	-	006	005*	002	001	001	002	002	005*
ocess									
nalyzing	027	028*	006	019*	004	002	009	001	004
nthesizing	096**	025	033*	002	000	016	017	001	000
ecializing	016	040*	011	001	002	028	008	015	014
neralizing	086***	041*	010	018	000	005	015*	008	006
levant	015	021*	002	001	004	006	004	003	002
itude									
riosity	002	002	022	003	009	003	003	003	001
dependence	044	012	006	004	007	002	005	002	003
ceptivity	061*	035*	011	003	013	006	005	008	007
tral	005	001	001	000	001	000	007***	000	001
				PUPILS V					
tent									
ructure	005	001	001	000	000	001	000	000	000
chniques	005	001	002	001	001	000	001	001	000
ductive	022*	007	006	003	001	001	002	001	002
ductive	-	-	002	000	001	000	001	000	001
thematical	024*	010	000	005	002	003	003	000	003
her	-	-	-	-	-	-	-	-	-
cess									
alyzing	005	004	002	000	001	001	001	000	001
nthesizing	005	002	001	001	001	000	001	000	000
ecializing	002	012	002	003	002	003	001	002	004
neralizing	004	001	002	001	001	000	002	000	001
levant	009	003	003	002	001	001	000	000	001
itude									
riosity	004	001	004	002	000	001	001	000	002
dependence	015	010	007	005	001	003	001	001	003
ceptivity	002	008	003	005	002	002	002	001	004
tral	-	-	-	-	-	-	-	-	-

* p = 0.34
** p = 0.05
*** p = 0.01

Table 220 Simple differences of cumulative proportions in each category of Content, of Process and of Attitude, and in Neutral for Teacher VI and Pupils VI. All decimals omitted. All differences in absolute value. Dash occurs where proportions = 0.

DAYS CATEGORY	1-2	2-3	3-4	4-5	5-6	6-7	7-8	8-9	9-10
					TEACHER VI				
Content									
Structure	049*	028*	006	009	019*	002	007	000	006
Techniques	039*	026*	011	005	014	007	021*	002	006
Deductive	137***	111***	008	004	037*	027*	005	006	001
Inductive	090*	058**	057**	037*	002	003	013	019*	015
Mathematical	048*	073***	074***	038*	036*	027*	013	006	012
Other	022	039*	011	019*	005	005	007	007	004
Process									
Analyzing	006	005	017*	009	005	007	004	008	006
Synthesizing	048*	018	010	004	001	005	006	000	004
Specializing	071*	024	001	011	002	004	001	015	000
Generalizing	033	004	014	002	011	008	004	006	002
Relevant	077**	015	020	008	010	004	009	001	003
Attitude									
Curiosity	014	032*	022*	001	005	007	009	009	004
Independence	001	023	003	006	006	009	001	001	002
Receptivity	004	034	018	001	011	019*	021*	003	008
Neutral	003	022**	010	006	010	003	002	005	004
					PUPILS VI				
Content									
Structure	-	-	-	-	002	001	002**	001	000
Techniques	003	001	001	000	000	003***	000	000	000
Deductive	006	002	000	003	001	001	000	001	000
Inductive	003	002	002	002	001	001	000	001	000
Mathematical	003	000	000	002	001	001	000	006***	001
Other	-	-	-	-	-	-	-	-	-
Process									
Analyzing	-	002	001	002	001	000	000	001	000
Synthesizing	003	003	001	001	001	000	000	001	000
Specializing	010	001	001	002	001	000	000	004	000
Generalizing	-	002	001	000	000	000	000	000	000
Relevant	005	002	000	001	000	002	001	000	001
Attitude									
Curiosity	-	-	-	002	000	001	000	000	000
Independence	003	001	002	001	001	001	001	003	000
Receptivity	006	002	002	001	001	003	002	002	001
Neutral	-	-	-	-	-	-	-	-	-

* p = 0.34
** p = 0.05
*** p = 0.01

Table 221 Simple differences of cumulative proportions in each category of Content, of Process and of Attitude, and in Neutral for Teacher VIII and Pupils VIII. All decimals omitted. All differences in absolute value. Dash occurs where proportions = 0.

DAYS TEGORY	1-2	2-3	3-4	4-5	5-6	6-7	7-8	8-9	9-10
					TEACHER VIII				
ntent									
tructure	031	010	005	002	004	001	004	017*	003
echniques	063*	012	009	011	015	012	002	003	007
eductive	042	016	011	020*	029*	026*	004	015	010
nductive	127	029*	028*	015	007	017*	005	004	009
athematical	016	059*	025	025	042*	041*	003	015	024*
ther	-	-	019	011	004	004	002	003	002
ocess									
nalyzing	016	018	008	003	001	004	004	005	001
ynthesizing	012	016	003	024*	017	014	000	007	009
pecializing	041	001	060**	019	011	021	004	002	[illegible]
eneralizing	053*	009	024*	018*	017*	004	001	001	004
elevant	022	019	008	017	013	003	000	010	006
titude									
uriosity	003	001	001	002	000	002	000	003	007
ndependence	034	047*	016	026	033*	027*	005	019	026*
eceptivity	075*	036	001	029*	031*	018	004	009	021*
utral	-	013	001	002	004	001	000	002	001
ntent									
tructure	000	002	001	002	001	001	003*	007**	001
echniques	001	004	001	003	002	000	000	001	001
eductive	011	005	002	003	000	001	000	001	001
nductive	012	004	002	001	001	001	000	001	003*
thematical	032*	017	017	006	002	006	002	006	006
her	-	-	-	001	000	000	000	000	000
ocess									
alyzing	006	004	001	001	001	000	001	008**	002
nthesizing	032*	004	015	008	004	003	002	002	004
ecializing	005	002	006	004	004	006	001	004	002
neralizing	006	004	002	001	001	001	002	001	001
levant	003	008	005	002	001	000	000	001	001
titude									
riosity	-	002	001	000	000	000	001	004	000
dependence	017	029*	005	003	000	006	002	012*	001
ceptivity	021	004	009	004	002	004	003	005	003
tral	-	-	-	-	-	-	-	-	-

* p = 0.34
** p = 0.05
*** p = 0.01

Appendix A

Table 222 Simple differences of cumulative proportions in each category of Content, of Process and of Attitude, and in Neutral for Teacher IX and Pupils IX. All decimals omitted. All differences in absolute value. Dash occurs where proportions = 0.

DAYS CATEGORY	1-2	2-3	3-4	4-5	5-6	6-7	7-8	8-9	9-10
Content					TEACHER IX				
Structure	033*	002	026*	011	007	004	004	010	007
Techniques	055*	027	026*	001	015	001	009	000	010
Deductive	072***	012	012	026*	019*	000	013	003	030**
Inductive	060	015	056***	019*	022*	012	000	009	007
Mathematical	146**	014	049*	065**	002	008	007	011	009
Other	-	039	009	007	004	015**	002	004	004
Process									
Analyzing	034*	003	009	017*	003	001	007	005	000
Synthesizing	035	002	034*	035*	001	004	011	002	002
Specializing	024	021	056*	026	001	000	004	009	001
Generalizing	037**	018	012	021*	020*	012	005	002	007
Relevant	027	013	006	008	004	011	004	007	004
Attitude									
Curiosity	003	008**	006	003	003	006*	003	002	007*
Independence	015	027	058*	008	028*	036*	018	005	017
Receptivity	015	045*	025	004	007	022*	031*	003	017*
Neutral	-	-	010	002	002	003	001	001	001
Content					PUPILS IX				
Structure	003	001	006**	005	007*	000	000	001	002
Techniques	005	003	002	002	021***	001	003	002	003
Deductive	069	017	010	004	008	001	001	001	007
Inductive	003	001	001	007**	002	001	001	001	000
Mathematical	052*	014	039*	000	005	000	011	001	011
Other	-	005	002	000	001	002	000	001	000
Process									
Analyzing	007	001	005	002	006*	000	000	000	000
Synthesizing	015	009	035*	019*	003	000	009	000	004
Specializing	051**	007	010	025*	019*	002	008	001	003
Generalizing	006	001	002	011***	003	002	005	002	001
Relevant	007	008	007	003	002	001	003	002	002
Attitude									
Curiosity	-	-	002	006**	002	001	001	001	000
Independence	037	013	026*	015	025*	003	012	005	009
Receptivity	064***	013	024*	005	004	002	003	000	001
Neutral	-	-	-	-	-	-	-	-	-

* p = 0.34
** p = 0.05
*** p = 0.01

Appendix A

Table 223 Simple differences of cumulative proportions in each category of Content, of Process and of Attitude, and in Neutral for Teacher X and Pupils X. All decimals omitted. All differences in absolute value. Dash occurs where proportions = 0.

DAYS TEGORY	1-2	2-3	3-4	4-5	5-6	6-7	7-8	8-9	9-10
ntent				TEACHER X					
tructure	002	002	013	009	010	004	019*	009	004
echniques	001	002	018	006	011	011	004	008	003
eductive	008	025*	001	011	011	005	007	015*	003
nductive	115***	037*	016	008	014*	007	012	038***	011
athematical	097**	041*	227***	201***	028*	045**	006	026*	007
ther	037***	004	027**	010	002	001	005	001	004
ocess									
nalyzing	004	020	013	006	019*	008	002	015*	003
ynthesizing	015	019	040*	004	002	019*	010	004	001
pecializing	081*	004	021	004	009	000	005	005	002
eneralizing	022	014	001	020*	001	000	002	000	010
elevant	007	007	039**	009	010	009	007	010	006
titude									
uriosity	006	015	012*	001	008	003	005	002	004
ndependence	027	034*	010	006	006	006	010	006	012
eceptivity	068*	043*	035*	018	006	029*	019	002	024*
utral	041*	004	012*	000	006	005	015***	003	000
ntent				PUPILS X					
tructure	012	008*	000	005	001	003	003	001	000
echniques	009	004	012	002	011	004	005	003	001
eductive	006	003	013*	005	004	003	002	002	002
nductive	003***	001	000	001	000	000	000	004***	001
athematical	023**	010	005	026**	024**	010	004	005	015*
her	000	000	006***	001	001	001	000	000	001
ocess									
alyzing	009	005	002	000	000	002	002	003	002
nthesizing	016***	014*	006	014*	011*	007	007	003	011
ecializing	004	009	008	013*	011	001	001	002	007
neralizing	002	002	004	002	001	002	000	000	001
levant	007	005	033***	002	005	006	006	004	004
itude									
riosity	012	003	001	002	000	000	001	000	001
dependence	019	022*	047***	001	014	013	001	002	022*
ceptivity	001	012*	031***	016*	003	007	003	003	007
tral	012	003	001	001	001	001	001	000	000

* p = 0.34
** p = 0.05
*** p = 0.01

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Table 224 Simple differences of cumulative proportions in each category of Content, of Process and of Attitude, and in Neutral for Teacher XI and Pupils XI. All decimals omitted. All differences in absolute value. Dash occurs where proportions = 0.

DAYS CATEGORY	1-2	2-3	3-4	4-5	5-6	6-7	7-8	8-9	9-10
				TEACHER XI					
Content									
Structure	057	028*	009	033**	003	005	008	001	001
Techniques	048*	045*	024*	005	004	008	018*	001	009
Deductive	159***	058**	018	009	008	001	012	001	006
Inductive	004	002	051***	066***	014	007	070***	020	015
Mathematical	055	054*	007	014	001	019*	015*	005	017*
Other	024	016*	008	003	003	004	005	004	001
Process									
Analyzing	094**	037*	009	003	008	002	018*	004	005
Synthesizing	033	041*	002	007	005	000	013*	004	001
Specializing	077*	043*	064***	037*	011	015	024*	017*	004
Generalizing	020	009	007	019*	002	004	005	000	004
Relevant	069***	050**	014	016	011	004	018*	002	008
Attitude									
Curiosity	013	022***	008	004	002	004	004	004	001
Independence	002	056*	023	031*	014	024*	044***	004	008
Receptivity	135**	043*	018	006	001	004	016	018	016
Neutral	024	009	003	006*	010*	004	003	005	001
				PUPILS XI					
Content									
Structure	004	003	004	005	002	001	001	001	001
Techniaues	-	003	002	003	002	002	001	000	000
Deductive	030*	032**	016*	007	013*	029***	011*	042***	006
Inductive	-	-	014	009*	001	000	007*	004	002
Mathematical	169***	054*	048*	042**	028*	006	019*	015*	005
Other	-	009	005	003	002	001	001	001	000
Process									
Analyzing	020	017*	010	006	004	004	003	002	000
Synthesizing	128***	058*	046*	039*	013	018*	016	029**	002
Specializing	017	009	000	005	005	006	007	008	002
Generalizing	-	-	002	003*	001	001	000	001	000
Receptivity	-	-	008	000	002	002	001	000	000
Attitude									
Curiosity	004	005	002	001	001	001	001	001	001
Independence	132***	094***	052	040	020	003	025*	023	004
Receptivity	004	002	007*	004	001	019***	005	003	002
Neutral	–	–	–	–	–	001	000	000	000

* p = 0.34
** p = 0.05
*** p = 0.01

Appendix A

Table 225 Simple differences of cumulative proportions in each category of Content, of Process and of Attitude, and in Neutral for Teacher XII and Pupils XII. All decimals omitted. All differences in absolute value. Dash occurs where proportions = 0.

DAYS ATEGORY	1-2	2-3	3-4	4-5	5-6	6-7	7-8	8-9	9-10
ontent					TEACHER XII				
Structure	045*	015	007	002	011	003	002	004	002
Techniques	027	007	040*	003	013	005	002	004	008
Deductive	068***	004	017	000	042***	012	006	039**	004
Inductive	177***	123***	075**	001	013	018	008	006	011
Mathematical	238***	121***	059*	008	024	001	003	030*	009
Other	018	003	004	002	001	001	004	005	000
rocess									
Analyzing	040*	009	006	003	006	004	003	008	002
Synthesizing	059*	013	011	006	000	008	003	004	011
Specializing	026	015	019	016	018	001	001	014	001
Generalizing	089***	015	020*	003	002	001	001	003	001
Relevant	004	008	026*	001	019	006	002	001	009
ttitude									
Curiosity	004	008	026*	001	019	006	002	001	009
Independence	001	005	003	000	007	001	001	005	001
Receptivity	042*	026	006	007	015	007	001	000	002
utral	004	002	011***	004	000	002	001	002	003
ntent					PUPILS XII				
tructure	005	001	005	001	002	001	001	000	001
echniques	-	-	004	001	003	002	000	000	002
eductive	014	004	002	002	001	004	002	002	001
nductive	007	023***	007	005	005	002	001	002	001
athematical	005	006	013*	001	002	000	001	002	000
ther	004	002	000	000	001	000	000	002**	000
ocess									
nalyzing	005	006	000	001	003	001	001	002	001
ynthesizing	-	002	000	000	001	000	000	000	000
pecializing	010	015	002	005	006	003	002	001	001
eneralizing	014	004	002	002	001	004	000	002	000
elevant	-	-	004	001	003	002	000	002	000
titude									
uriosity	-	-	004	001	003	002	000	002	000
ndependence	014	021*	001	002	006	002	005	004	003
eceptivity	002	002	000	003	001	007*	001	005	003
utral	007	002	001	001	001	000	001	000	000

* p = 0.34

** p = 0.05

*** p = 0.01

Appendix A

Table 226 Simple differences of cumulative proportions in each category of Content, of Process and of Attitude, and in Neutral for Teacher XIII and Pupils XIII. All decimals omitted. All differences in absolute value. Dash occurs where proportions = 0.

DAYS CATEGORY	1-2	2-3	3-4	4-5	5-6	6-7	7-8	8-9	9-10
					TEACHER XIII				
Content									
Structure	007	043*	095***	004	035*	010	011	010	010
Techniques	035*	006	014	003	000	006	003	005	001
Deductive	089***	003	040*	021*	019*	013*	015*	006	003
Inductive	125**	023	020	028*	002	011	006	008	001
Mathematical	025*	015	021*	064***	034**	013	022*	015	004
Other	024	003	004	004	001	001	006	000	005
Process									
Analyzing	009	012	015	009	018*	011	005	006	007
Synthesizing	031	027*	025*	014	001	014*	015*	009	002
Specializing	088*	040*	003	001	024*	002	012	007	010
Generalizing	008	010	009	000	014*	002	002	001	001
Relevant	025	002	007	001	002	005	004	007	002
Attitude									
Curiosity	080**	017	008	011	005	006	002	004	004
Independence	103**	008	022	027*	023*	010	009	006	008
Receptivity	100**	053	010	005	007	001	001	013	008
Neutral	006	003	000	002	003	001	002	000	000
					PUPILS XIII				
Content									
Structure	055***	021*	024*	005	013*	000	001	000	003
Techniques	008	004	000	003	005*	001	001	000	001
Deductive	026***	001	000	006	004	003	020***	021***	001
Inductive	016	010*	005	007	005	003	008*	001	000
Mathematical	019*	072***	024*	002	018*	024**	015*	007	001
Other	005	003	001	001	005*	004	001	002	005*
Process									
Analyzing	007	020**	001	004	016**	007	008	001	004
Synthesizing	035**	055***	016	014	006	010	008	005	001
Specializing	021	013	007	000	009	001	011	009	002
Generalizing	012	001	016**	002	004	000	002	001	001
Relevant	007	004	000	003	005*	001	002	001	001
Attitude									
Curiosity	000	003	003	001	000	001	001	002	001
Independence	070**	062**	005	016	011	017*	015	011	006
Receptivity	014	000	002	004	008	003	006	002	000
Neutral	-	-	-	-	-	-	-	-	-

* p = 0.34
** p = 0.05
*** p = 0.01

Appendix A

Table 227 Simple differences of cumulative proportions in each category of Content, of Process and of Attitude, and in Neutral for Teacher XIV and Pupils XIV. All decimals omitted. All differences in absolute value. Dash occurs where proportions = 0.

DAYS ATEGORY	1-2	2-3	3-4	4-5	5-6	6-7	7-8	8-9	9-10
ontent					TEACHER XIV				
Structure	097**	025	010	024*	019*	001	005	007	005
Techniques	046*	015	007	033*	005	032*	005	001	000
Deductive	-	-	-	009	000	001	002	000	001
Inductive	003	001	006**	002	001	001	015***	002	002
Mathematical	078*	042*	015	035*	007	003	004	007	002
Other	-	-	004	004	037***	008	005	004	002
rocess									
Analyzing	010	022	015	014	004	004	006	005	002
Synthesizing	021	015	001	025	029*	039**	007	008	014
Specializing	028	052*	016	002	035*	004	007	008	006
eneralizing	027	012	001	012	008	007	006	002	005
Relevant	026	003	017	016	016	005	000	007	008
ttitude									
uriosity	-	005	001	002	001	001	001	000	000
Independence	024	012	004	001	009	001	005	006	005
eceptivity	038	011	017	019	021	025*	005	007	002
utral	070***	000	035*	026	011	018*	003	006	003
ntent					PUPILS XIV				
tructure	028*	006	003	001	004	001	002	002	001
echniques	007	008	003	000	002	006	002	001	014*
eductive	-	-	-	-	-	-	-	-	-
nductive	-	-	-	-	-	-	-	-	-
athematical	011	010	004	035**	006	008	002	006	005
ther	-	-	-	003	001	000	000	001	002**
ocess									
nalyzing	013*	006	003	001	003	003	002	000	002
ynthesizing	028*	042***	006	011	011	010	002	004	005
pecializing	017	102***	027	008	000	011	000	002	001
eneralizing	-	-	-	-	-	-	-	-	-
elevant	029*	016	007	010	007	000	006	001	008
titude									
uriosity	-	-	-	008	002	001	001	000	001
ndependence	004	013	012	033*	000	010	002	005	006
eceptivity	001	017	009	005	004	006	006	006	010
utral	-	005	001	001	001	000	000	002	001

* p = 0.34
** p = 0.05
*** p = 0.01

Table 228 Simple differences of cumulative proportions in each category of Content, of Process and of Attitude, and in Neutral for Teacher XV and Pupils XV. All decimals omitted. All differences in absolute value. Dash occurs where proportions = 0.

DAYS CATEGORY	1-2	2-3	3-4	4-5	5-6	6-7	7-8	8-9	9-10
				TEACHER XV					
Content									
Structure	019	017*	003	004	003	003	004	002	002
Techniques	031	011	026*	010	007	001	010	002	003
Deductive	005	000	001	001	001	005*	028***	001	004
Inductive	028*	022*	006	008	004	019**	016*	005	006
Mathematical	001	032*	009	011	025*	002	018*	003	011
Other	006	008	006	007	007	003	005	002	009*
Process									
Analyzing	019*	002	003	007	001	003	000	002	003
Synthesizing	009	006	003	003	004	007	022**	003	006
Specializing	007	026*	004	019	021*	000	025*	004	005
Generalizing	010	001	002	004	001	007*	003	001	002
Relevant	012	012	001	009	009	000	009	004	001
Attitude									
Curiosity	019	005	001	002	001	001	001	001	001
Independence	005	036*	015	021	003	001	001	006	004
Receptivity	013	048**	023*	011	039**	015	044**	008	014
Neutral	010	013*	001	012*	003	004	004	001	003
				PUPILS XV					
Content									
Structure	014*	002	001	004	002	001	003	001	002
Techniques	039*	010	015*	010	004	002	001	003	003
Deductive	015	004	002	001	001	003	010**	000	001
Inductive	038	011	003	004	002	004	002	001	002
Mathematical	018	024	012	049*	032*	019	038*	019	015
Other	009	011	005	004	001	001	002	001	001
Process									
Analyzing	023*	017	006	006	003	001	004	001	004
Synthesizing	001	018	002	057**	030*	012	029*	011	013
Specializing	032*	008	000	017	001	002	007	005	008
Generalizing	004	004	001	002	001	005*	001	001	001
Relevant	044*	008	002	009	003	002	003	003	002
Attitude									
Curiosity	000	005	003	002	001	001	001	001	002
Independence	060*	007	008	031*	035*	010	041*	014	016
Receptivity	044	006	006	007	002	003	005	001	002
Neutral	005	002	000	001	000	000	000	001	000

* p = 0.34
** p = 0.05
*** p = 0.01

Appendix A

Table 229 Simple differences of cumulative proportions in each category of Content, of Process and of Attitude, and in Neutral for Teacher XVI and Pupils XVI. All decimals omitted. All differences in absolute value. Dash occurs where proportions = 0.

DAYS CATEGORY	1-2	2-3	3-4	4-5	5-6	6-7	7-8	8-9	9-10
				TEACHER XVI					
Content									
Structure	023	007	003	005	006	002	001	001	003
Techniques	014	021	006	020	058***	010	018	018	020*
Deductive	036*	025*	002	001	006	003	013*	007	003
Inductive	020*	048**	009	018	012	009	034***	016*	011
Mathematical	001	023	000	060**	001	014	027*	011	009
Other	038*	018	006	007	004	003	001	002	000
rocess									
Analyzing	008	000	001	003	008	004	005	003	005
Synthesizing	040*	003	010	033*	019	011	000	010	000
Specializing	013	005	023	015	005	010	037*	004	015
Generalizing	004	016	000	011	002	006	010*	004	003
Relevant	018	018	005	011	007	015	007	010	011
ttitude									
Curiosity	004	002	001	001	001	000	000	000	001
Independence	030	012	017	001	023*	011	018*	008	006
Receptivity	055*	030	008	047*	048*	032*	052**	009	019
eutral	004	009	016*	006	004	004	003	002	002
				PUPILS XVI					
ontent									
Structure	016	014	004	003	000	003	002	001	002
Techniques	001	024***	001	014	002	004	004	003	004
Deductive	004	002	001	000	000	003	001	001	000
Inductive	-	-	-	-	002	000	003*	004*	004
Mathematical	012	069**	009	040*	020	055***	020*	007	033
Other	009	005	001	002	001	001	001	000	000
rocess									
Analyzing	003	005	006	002	001	003	001	003	001
Synthesizing	052***	077***	006	036*	017	059***	015	007	039***
Specializing	087*	028	007	003	002	005	011	002	006
Generalizing	009	005	001	002	001	001	001	000	000
Relevant	023**	016*	001	002	005	002	005	002	004
ttitude									
Curiosity	-	-	-	-	002	000	000	000	001
Idenpendence	017	037*	010	046*	016	048**	023*	005	031*
Receptivity	003	018*	002	008	005	000	007	000	004
eutral	004	002	003	002	001	001	001	000	000

* p = 0.34
** p = 0.05
*** p = 0.01

Appendix A

Table 230 Simple differences of cumulative proportions in each category of Content, of Process and of Attitude, and in Neutral for Teacher XVII and Pupil XVII. All decimals omitted. All differences in absolute value. Dash occurs where proportions = 0.

DAYS CATEGORY	1-2	2-3	3-4	4-5	5-6	6-7	7-8	8-9	9-10
				TEACHER XVII					
Content									
Structure	004	007	007	009	001	011*	008	009	000
Techniques	017	018	022*	005	002	003	003	006	006
Deductive	057*	057*	009	030*	005	009	015	003	010
Inductive	056*	021	010	001	005	002	001	022***	002
Mathematical	026	000	028	060**	002	023*	009	002	020*
Other	011	004	000	005	002	001	001	001	002
Process									
Analyzing	001	005	004	014*	002	002	005	002	003
Synthesizing	030	048*	027*	026*	007	016*	005	010	014
Specializing	031	000	003	019	006	016	003	005	002
Generalizing	013	029*	005	006	001	000	002	015*	001
Relevant	042	030	037*	006	005	001	004	008	003
Attitude									
Curiosity	023**	000	005	002	003	003	002	002	003
Independence	008	050*	031*	032	002	003	008	019*	006
Receptivity	024	061*	028*	003	003	001	002	003	010
Neutral	002	056***	006	008	008	009	006	004	004
				PUPILS XVII					
Content									
Structure	003	004	001	001	001	010**	001	001	002
Techniques	003	003	001	002	002	002	001	001	001
Deductive	030**	017	003	064***	012	020*	007	008	009
Inductive	-	-	-	007	001	002	001	001	000
Mathematical	030	063***	057***	040*	013	023*	004	009	002
Other	-	-	-	003	000	001	000	000	000
Process									
Analyzing	003	001	002	002	001	005*	001	000	001
Synthesizing	004	047**	034*	025*	001	002	005	008	005
Specializing	013	012	019*	042**	006	000	003	007	004
Generalizing	011	004	002	023***	004	000	000	003	003
Relevant	000	000	000	001	010**	001	001	003	002
Attitude									
Curiosity	-	006	002	008*	003	002	001	001	001
Independence	030	057***	059***	043**	012	015	001	017*	012
Receptivity	026	008	008	009	024**	005	000	003	002
Neutral	-	-	003	001	000	000	001	000	000

* p = 0.34
** p = 0.05
*** p = 0.01

Appendix A

Table 231 Simple differences of cumulative proportions in each category of Content, of Process and of Attitude, and in Neutral for Teacher XVIII and Pupils XVIII. All decimals omitted. All differences in absolute value. Dash occurs where proportions = 0.

DAYS	1-2	2-3	3-4	4-5	5-6	6-7	7-8	8-9	9-10
CATEGORY									
Content					TEACHER XVIII				
Structure	006	019*	002	012*	011	002	011	012*	
Techniques	049***	035*	017	030*	017	011	000	012	
Deductive	071**	033*	015	014	027*	017*	005	015*	
Inductive	029**	012	054***	016	011	049***	004	013	
Mathematical	097	021	015	011	025*	011	017*	009	
Other	-	004	003	002	011***	001	005	004	
Process									
Analyzing	008	002	002	005	006	004	001	013*	
Synthesizing	030	038**	003	002	016*	004	002	001	
Specializing	143***	028	053**	006	010	011	027*	013	
Generalizing	008	004	007	002	003	004	002	001	
Relevant	062***	036*	013	021*	016	007	038*	006	
ttitude									
Curiosity	-	004	001	001	000	001	000	001	
Independence	151***	050*	043*	013	006	012	001	003	
Receptivity	100***	046*	032*	032*	013	006	066***	008	
Neutral	003	020**	006	000	004	009*	019**	002	
ontent					PUPILS XVIII				
Strucutre	042*	014	007	000	003	000	005	000	
Techniques	005	002	000	003	007*	003	008*	004	
Deductive	314***	090**	060**	037*	034*	025*	006	018*	
Inductive	023	006	010*	007	004	024***	002	005	
Mathematical	118	021	017	014	022*	009	002	001	
Other	-	004	001	001	006***	001	000	004	
rocess									
Analyzing	055*	014	008	002	001	006	002	001	
Synthesizing	056*	076**	029*	009	006	011	002	014	
Specializing	136**	038*	029*	010	004	004	003	008	
Generalizing	010	004	000	003	002	005*	000	000	
Relevant	008	001	001	004	006*	002	007*	004	
ttitude									
Curiosity	006	002	002	002	000	001	005*	002	
Independence	252***	126***	064**	029*	007	014	004	019	
Receptivity	008	001	001	006*	011	003	008*	004	
eutral	-	004	001	001	000	001	003	000	

* p = 0.34
** p = 0.05
*** p = 0.01

Appendix A

Table 232 **Probability levels for similarity in comparison of triple and individual frame scores of subgroups taken in pairs (10 comparisons).**

		Class			Small group & T		Small group		Indiv. & T	
		T&P	T	P	T&P	P	T&P	P	T&P	P
Small Group & T	Triple	<.05	<.05	>.05						
	C	.75	.40	.60						
	P	.40	.60	.08						
	A	.001	.60	.001						
Small Group	Triple	>.05		>.05	<.05	.01				
	C	.40		.15	.25	.25				
	P	<.05		.001	.40	.40				
	A	.001		.20	.40	<.05				
Individual & T	Triple	>.05		>.05	.01	.01	.01			
	C	<.05		.001	.25	.08	.01	.001		
	P	.08		.001	.02	<.05	.02	.02		
	A	.001		.15	.15	.40	.40	.85		
Individual	Triple	>.05		>.05	.05	.01		.01	.01	.01
	C	.40		.001	.85	.90		.08	.60	.60
	P	<.05		.001	<.05	.15		<.05	.98	.98
	A	.01		<.05	.01	.85		.40	.08	.40

N.B.: For triples, H_0 rejected if $p > 0.05$
For individual frames C, P, A, H_0 rejected if $p < 0.05$.

Appendix A

Table 233 Probability levels for similarity in comparison of triple frame and individual frame scores across several subgroups (1) all five (2) the four other than the class (3) the three with teacher.

Frame Scores	Five subgroups		Four lab. subgroups	Three subgroups with teacher	
	T&P	P	P	T&P	P
Triple	<0.001	<0.001	<0.001	0.05	0.05
Individual					
Content	0.15	0.07	<0.05	0.01	0.001
Process	0.01	0.001	0.02	0.01	0.001
Attitude	0.001	0.15	0.60	0.001	0.001

For triple frame scores: H_0 rejected if $p > 0.05$

For individual frame scores: H_0 rejected if $p < 0.05$

Washington University

SYSTEMATIC OBSERVATION OF VERBAL INTERACTION AS

A METHOD OF COMPARING MATHEMATICS LESSONS

APPENDIX B: A SELF TRAINING MANUAL

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Prepared at Washington University, St. Louis
under Cooperative Research Project No. 816, of
the Office of Education, U. S. Department of
Health, Education and Welfare

June, 1961

Department of Mathematics

THE WRIGHT-PROCTOR INSTRUMENT FOR OBSERVATION OF VERBAL BEHAVIORS IN THE MATHEMATICS CLASSROOM FROM VIEWPOINTS OF: CONTENT, PROCESS AND ATTITUDE

MANUAL FOR THE OBSERVER

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Introduction

The Wright-Proctor Instrument is designed for direct observation of verbal behaviors of teachers and pupils in the mathematics classroom in terms of the mathematical content, the psychological process and the sociological attitude. This manual is written to facilitate self training in the use of the instrument of persons already well prepared in mathematics.

The rationale for the direct study of the classroom, for the choice of the three frames of reference and for the dependence on verbal behaviors is presented in the context of related research and literature. The categories defining each frame of reference, the methods for classification of verbal behaviors in these categories and the observational and training procedures constitute the design. Graphic and quantitative methods of treatment of data directed to collection, description and comparison are presented, with examples of tests for validity and reliability and a discussion of statistics appropriate to the instrument.

The present evidences for validity, reliability and for care in use of the instrument are followed by a listing of studies in which such a tool would be useful.

The text is supplemented with references and training materials in Appendixes 1 and 2, respectively.

N.B. This research instrument is now sufficiently refined for use by investigators not trained at this laboratory. It should therefore be noted that its first requirement is observers well prepared in mathematics and thoroughly practised in the observational procedures. Its direct use in the classroom also places a responsibility on the investigator for consideration of the schools, teachers and pupils in collecting data and reporting results. This careful selection and preparation of observers and the consideration shown is well rewarded by the richness of results and the enlightening classroom experiences.

A detailed report of any studies conducted with this instrument will be much appreciated for accumulation of normative data, still at a minimum.

GLOSSARY OF SYMBOLS

*Content:

Symbols used in Report**		Symbols used in Manual	
C 1	Structure	C 1	Structure
C 2	Deductive	C 2	Techniques
C 3	Mathematical	C 3	Deductive
C 4	Other	C 4	Inductive
C 5	Techniques	C 5	Statements
C 6	Inductive	C 6	Mathematical
		C 7	Other

Process:

P 1 Analyzing
P 2 Synthesizing
P 3 Specializing
P 4 Generalizing
P 5 Relevant

Attitude:

A 1 Curiosity
A 2 Independence
A 3 Receptivity

Neutral: N

V Verbal
T Teacher
P Pupils
B Boy
G Girl
C Classroom
O Observer

RP High rigor, high participation
rP Low rigor, high participation
rp Low rigor, low participation
Rp High rigor, low participation

---

* Symbols used for Content categories in the report are changed in the Manual because of the addition of a new category and more logical ordering of the entire set.

** Throughout this Manual the main body of the report is termed Project 816.

RATIONALE

MEASUREMENT OF CLASSROOM VARIABILITY

Systematic study of the classroom on the whole has emphasized indirect assessment of the classroom variables by measurement involving paper and pencil tests given in the pre and post lesson periods. Most promising of these is a consideration of pupil change during the experimental period. Despite the general level of refinement of this type of measure, and the additional improvements in scaling likely to develop from Guttman's concern with ordering of task complexity in correspondence with the real numbers (1959) or in experimental design in the use of residual scores as discussed by DuBois (1959), there remains the question of validity of indirect assessment in such studies as evaluation of experimental programs introducing additional topics or emphasizing more rigorous or more creative thinking. In use of achievement measures based on the work studied it would have to be assumed that topics and acceptable reasoning of the control groups which provide a practical common ground of testing materials constitute a valid sample for measurement of knowledge of the experimental groups. It might be feasible to turn away from testing achievement in the specific subject matter to measures of problem solving, critical thinking or other abilities, such as those analyzed by Guilford (1956) which, it might be inferred, would be affected by changes in mathematics instruction. A better indirect study of classroom effects would be in long term comparison of success in later academic work of experimental and control groups.

If our interest is more immediate than could be served by this long term study and directed particularly to classroom variability, then the value of the refined indirect measure of somewhat questionable validity should be compared with that of the less precise direct measure possessing high validity obtained by systematic observation in the classroom itself.

Considerable support for the use of carefully developed schemes for systematic observation has been accumulating over the past several decades from the pioneer work of Thomas (1933) and of Olson (1931) to the richer tools and techniques of Bales (1950), Cartwright and Zander (1953) and Guetzkow in social interaction generally and of Anderson (1939), Withall (1948) Ryans and Wandt (1952) in the classroom. Excellent discussions of the development of systematic observation may be found in Selltiz et al (1959), Festinger (1953), Lindzey (1954) and more specifically its use in the classroom in the Encyclopaedia of Educational Research (1960) and in Review of Educational Research (December, 1960).

While current investigations continue to develop new systems or to modify and refine earlier ones within broad social psychology frameworks (Trow, 1960; Flanders, 1960; Newell, Lewis and Withall, 1960), a variety of operational definitions of the teaching-learning situation are implicit in instruments now reaching publication level (Hughes and associates, 1959; Kowatrakul, 1959).

These operational definitions necessarily fall into a multiple-criterion predication in their approach to description, modification and prediction of teaching-learning phenomena. Of particular note is the attempt to study the classroom from observable behavior with a minimum of theoretical preconceptions is the work of B. O. Smith (1960), Mary Jane Aschner (1960) and associates, in directing their study of linguistic behavior of teacher and pupil in a classroom situation to a fundamental aspect of language, itself; the process or logical framework.

Also of high interest is Suchman's (1961) concern with the low incidence of pupil-initiated questions in most classroom discussion. This concern has taken form, not in systematic field observation of existing discussion, but in an active laboratory program for developing in children a method of inquiry for determining answers about intriguing natural phenomena from an informed individual (the investigator).

RATIONALE

MEASUREMENT OF MATHEMATICS CLASSROOM VARIABILITY
by
WRIGHT-PROCTOR OBSERVATIONAL INSTRUMENT

Our own experience has been that an operational definition may gradually make explicit a broad theoretical framework, which may then direct further resolution of the initial operational definition. Beginning in 1956 from the multiple-criterion operational approach to classification of language used in mathematical discoursé, our instrument developed through several levels of refinement of category definition and interpretation (Wright, 1957, 1959, 1960; Proctor, 1960). Modifications resulted from use of the instrument in the field interspersed with reconsideration of the categories in terms of a minimum number necessary yet sufficient to classify the language used by the teacher and the pupils in their thinking about mathematical subject matter. Because the key aspect of the classroom is the mastery of particular subject matter, it was with the belief that the important yet peripheral aspects of the classroom forming the background of the studies cited above would not be neglected but would be placed in proper perspective that we developed our categories and their definitions from the specifics of mathematical language in the classroom. Thus as in the case of Smith et al, because we were studying the teaching-learning situation as evidenced by language we concentrated on the essential aspects of language; however, we did not limit ourselves simply to the logical process.

It became apparent that language involving mathematical argument was based on content which had to be carried in the broad vehicle of psychological process and together these were effected in the broader framework of sociological attitude. The cognitive aspects, then, resulted from impetus of the general or particular environment of the speaker.

It was convenient to present these three aspects of the study of language schematically by three concentric circles: the innermost area, mathematical content; the middle area, psychological process; and the outer area, sociological attitude. Fig.

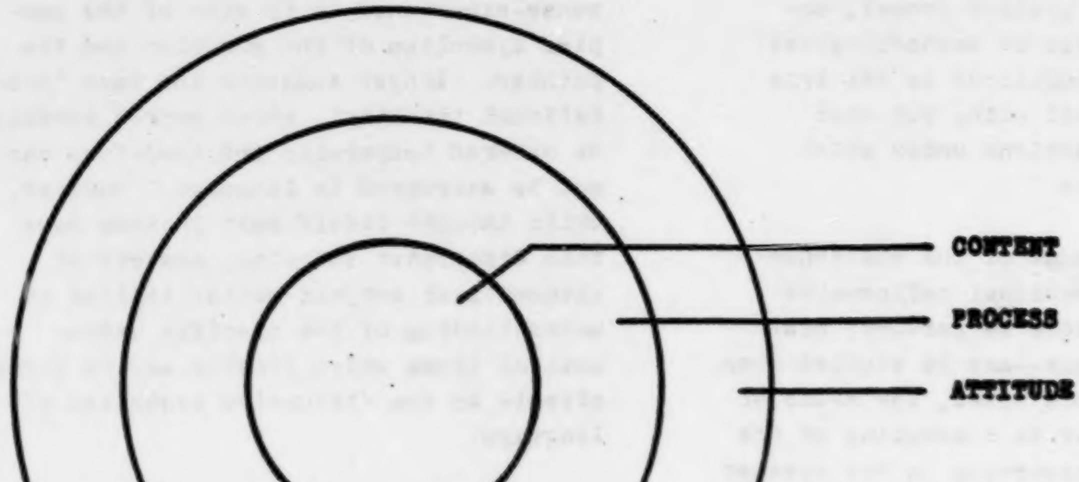
This schema implies that the environment has induced certain processes in the consideration of particular subject matter. This schema also is in marked agreement with Piaget's (1950) analysis of logical thinking where he points out the affective and cognitive aspects of thinking, his cognitive aspect here divided into process and content. This division of the cognitive aspect is in keeping with Cassirer's (1953, pp. 9, 345) consideration of thought as "thing-concepts" and "relation-concepts" with science presenting the things as distinct and certain, and psychology presenting the relations as a temporal process.

> "Psychology in this sense gives the approach to problems, which must seek their (complete) solution in logic and in their application to science."

The categories of process were selected at a functional level in consideration of subject matter rather than at the finest division of formal logic (Smith et al). That is, syllogistic, classificatory and relevant levels. It is also noted that the categories of the Process frame developed under the above considerations are in keeping with the broad division of the problem solving process reached empirically by Duncker (1945, p. 45).

The order of these concentric circles also may be commented on. The outer area, attitude, being larger than the process, permits inclusion of discursive thinking and other types of thought and the many specific attitudes that may be affective in different types of thinking. The center area, process, being larger than the content, implies application of the process in many subject areas. Viewing the circles from above brings out the importance of simultaneous consideration of the three areas for thorough assessment of the discussion of particular subject

TRIPLE VIEWPOINT OF THE WRIGHT-PROCTOR INSTRUMENT



MATHEMATICAL CONTENT

Structure, Techniques — **Fundamentals**
Deductive, Inductive, Statements — **Relations**
Mathematical, Other — **Applications**

PSYCHOLOGICAL PROCESS

Analyzing, Synthesizing — **Syllogistic**
Specializing, Generalizing — **Classificatory**
Relevant

SOCIOLOGICAL ATTITUDE

Curiosity
Independence
Receptivity

Figure 5

matter, yet implies that certain attitudes and certain processes may be pertinent only to a particular subject field. Kantor (1950, V. 1, p. 6) sums up this model as he asserts that

> "every particular interbehavior (of organisms and stimulus objects) is set in a special reference frame. As a result any specific type of logic, whether formal, empirical, symbolic or methodological is not only conditioned by the type of material dealt with, but also by general conditions under which the work is done."

While the language of the classroom--including its mathematical refinements and its approximations by gesture, hierogliphics, or drawings--may be studied from these three reference areas, the place of this verbal behavior as a sampling of the thinking actually occurring in the speaker should be questioned. Langer (1948, pp. 65, 70, 73, 79) points out that

> "language has a form which requires us to string out our ideas even though their objects rest one within the other"

and that this implies the limitation on language that only thoughts that can be arranged in such an order can be spoken at all. It seemed important at this stage to avoid the assumption that language is the only means of articulating thought and that everything which is not speakable thought is feeling. Such assumptions are surely inadmissible in the light of the mind's organized storing of the meaningless impressions of relatively simple sense-experience or in view of the complex symbolism of the musician and the painter. Langer suggests the term "presentational thinking", which cannot readily be ordered temporally and therefore cannot be expressed in language. However, while thought itself must include more than discursive thinking, mastery of mathematical subject matter implies an understanding of the specific mathematical ideas which finally may be demonstrable by the discursive symbolism of language.

From the general nature of these three frames of reference and their categories, it is interesting to consider that by modification and redefinition of the exact categories to meet the peculiarities of another subject area, comparison of categories common to two or more subject matter areas would be possible.

DESIGN

DEFINITION OF CATEGORIES: CONTENT

Mathematical systems consist of a set of undefined elements, a set of axioms or postulates, and the relations which are inherent in them and may be derived from them. Such systems find application in a physical situation if a correspondence exists between the structure and the applied field. A relationship is established rigorously by a sequence of steps, based on postulates and theorems of the system, moving from initial premises to a conclusion where each step is authorized by a rule of inference for which we have no practical doubt. The building of mathematical systems is the task of formal mathematical education and it may be done more or less rigorously, the degree dictated by the sophistication of the students. If the proof goes back to basic postulates and theorems, stated explicitly, a rigorous system has presumably been built and is now being developed. A more common practice in school mathematics is to develop unconnected sections of a system logically from accepted rules but without reference to the overall system and with little or no attention to the gaps in the system.

The categories of the Content frame have been selected to correspond to aspects of mathematical systems and functional classroom behavior. Classification of behavior into the Content categories answers the question, "What aspect of mathematics is being worked on?"

	Categories	Symbols	Former Symbols
Fundamentals	Structure	C 1	C 1**
	Techniques	C 2	C 5**
Relations	Deductive	C 3	C 2**
	Inductive	C 4	C 6**
	Statement	C 5	*
Applications	Mathematical	C 6	C 3**
	Other	C 7	C 4**

Other useful combinations of categories are:
Theory: structure + deductive + inductive
Practice in specific problems: inductive + mathematical

---

* Category was not used in Project 816: a result of study of reliability Chapter III

** Symbols used in earlier studies including Project 816. With the inclusion of the new category, a new numbering is begun in this Manual.

DESIGN

CLASSIFICATION OF VERBAL BEHAVIORS: IN THE FRAME CONTENT

Fundamentals — The body of mathematical knowledge at the command of the pupils; 'old' knowledge up to arbitrary cut off point such as last chapter or topic.

1. Structure

1.1 Fundamental elements, operations, postulates

1.2 Well established theory when understanding is apparent, e. g. definitions, suitable notation, theorems

1.3 Logical principles, e. g. consistency, inference, equivalence, proof

1.4 Strategies of problem solving, e. g. verification of facts, varying of conditions, testing hypotheses, inventing analogous problems, estimation of plausible answers, analysis of a method of problem solving

2. Techniques

2.1 Description and use of mechanical processes or rules where basic mathematical relation is not made apparent, e. g. mechanical use of tables

2.2 Reading of mathematical materials already developed, e. g. answers to homework problems, assignment of homework, first reading of a problem with no emphasis of specific conditions

Relations — The development and statement of 'new' relations

3. Deductive

3.1 Logical proof of new theory

4. Inductive

4.1 Use of specific examples selected to elicit new generalization or relation, e. g. problems used for this purpose usually begin quite simply and increase in technical complexity until pupils begin to look beyond the old method of for a new solution or for a general relationship

4.2 Use of graphs, diagrams to make a relation clear

4.3 *Intuitive approach to a relation, e. g. "What seems to be true?"*

5. Statement

5.1 Statements of new relations; may or may not be developed deductively or inductively; may be used in seeking method of problem solution in recent application, e. g. the statement may be right or wrong, may be pulled out for examination, and subsequently proved or disproved

5.2 Definitions, notation, terminology; mathematical conventions e. g. selection of means of describing empirical data such as means, mode, median, measures of dispersion, type of graphs in statistics

Applications — The use, place of the mathematical system in specific problems and in historical context

6. Mathematical

6.1 Solution of mathematical problems

7. Other

7.1 Brief statement of problem in other field before abstraction essentials

7.2 Examination of problems in terms of the concepts of the other field

7.3 References from mathematical history

7.4 Reference to new topics or different treatment to be met in later courses

7.5 Humor--when pertinent to mathematical activities

NEGATIVE behaviors are recorded when clearly false statement of mathematical content is made. If the observer is in doubt in the context, behavior is classed positive.

DESIGN

DEFINITION OF CATEGORIES: PROCESS

Logic is the tool of mathematical thinking and as such is the vehicle of the verbalized interaction occurring in the mathematics classroom where the teacher instructs and the pupils attend. Because mathematical thinking consists so largely of problem solving, both in building and in applying a system, the aspects of logic functional in problem-solving may usefully be identified to form the basis for classification of classroom verbal interaction.

Problems may be formulated in the course of classroom discussion or may merely be stated. They may be problems "to find" or problems "to prove"; in the former case the specific product is unknown; in the latter the generalization to be shown is given. When the specific product is unknown the problem is dissected and significant aspects such as the conditions and the goal selected. Relations among aspects of the problem are identified and within the context of the mathematical structure a plan for solution is adopted. Combining or synthesizing the problem facts and relations according to the theorems of the system yields the result. When a method of solution is familiar, the synthesizing becomes almost a mechanical process. Finally checking the correctness of the solution may be carried out by analyzing or confirming the connection of the result with the assumptions of the problem. When the generalization to be shown is given, a similar method of dissection, selection and synthesizing may be employed. Teachers seem partial to this approach for they already know an appropriate sequence of deductions. For the pupils this may be of limited usefulness--in achieving a solution to the particular problem. Particularly in early stages a process more productive in establishment of a broad strategy of problem-solving begins with recognition of goal and moving backwards from the goal towards the premises available. This offers what Duncker (1945) has called the 'organic' method of discovering proof. On the other hand at more sophisticated levels synthesizing may be the method of highly creative divergent thinking.

The syllogistic and classificatory categories of Process, then, all have the dynamic quality of a necessary sequence of classes and propositions to reach a desired goal.

	Category	Symbol
Syllogistic	Analyzing	P 1
	Synthesizing	P 2
Classificatory	Specializing	P 3
	Generalizing	P 4
Relevant	Relevant	P 5

DESIGN

CLASSIFICATION OF VERBAL BEHAVIORS: IN THE FRAME PROCESS

Syllogistic The syllogistic categories of analyzing and synthesizing require the logical operation of inference. Although synthesizing is often mechanical it may also be the method of highly creative divergent thinking.

1. Analyzing--from assumption of desired conclusion toward accepted principles
   1.1 Chain of backward implication--"is implied by"
   1.2 Less systematic moving backward from goal seeking connection with known premises to establish approach to proof
   1.3 Justification of a statement, e. g. Why? Because...; plausibility
   1.4 Moving backward over an argument to discover mistake or clarify meaning
2. Synthesizing--from accepted principles toward desired conclusion
   2.1 Chain of forward implication--"implies" , e. g. when moving forward from known premises to goal, synthesizing may be mechanical when method is a a familiar one; formal development or proof of theory or specific problem; reading entire proof already developed carefully step by step
   2.2 Consolidation of parts into a complete solution

Classificatory The classificatory categories of generalizing and specializing include the formulation of generalizations, their applications, and the less formal but very necessary heuristic process of problem dissection and focussing on goal.

3. Specializing--the use of significant attributes of a given set in an analogous set, or the application of a given set in a smaller included set
   3.1 Selection of significant parts of a problem--dissection, abstraction, e. g. verification of facts of problem; identification of necessary and sufficient conditions; identification of true and false statements
   3.2 Application of a generalization, e. g. substitution in a formula, use of theorem, definition
   3.3 Recognition of relation of corresponding sets, e. g. analogous problems
   3.4 Focussing on goal, e. g. recentering on goal at successive phases of solution
4. Generalizing--the recognition of significant attributes of a given set and the passing from the consideration of the given set to that of a larger inclusive set.
   4.1 Recognizing significant attributes and passing to a larger set, e. g. moving from particular examples to a common characteristic, a good guess, a hypothesis, the formulation of a problem, of a definition
   4.2 Statement of a formula, law, relation, definition to be proved or arising from development or to be examined for meaning

Relevant A more static category, the statement of relevant information occurs when mathematical information is presented but belongs to no apparent logical sequence.

5. Relevant
   5.1 Information about specific mathematics, e. g. problem read from book, reading of homework answers when no solution meaning given
   5.2 Information about more general aspects of mathematics, e. g. historical, biographical without logical analysis of the mathematical ideas that may thus be referred to

NEGATIVE behaviors are recorded when reasoning is invalid.

DESIGN

DEFINITION OF CATEGORIES: ATTITUDE

Mathematical behaviors of pupil in the classroom are passive or active largely as a direct result of the teacher's behavior because of the teacher's superior knowledge and his expected role. The art of teaching, Polya (1957) suggests, is in ensuring that the student shall do a "reasonable share of the work". In other terms DeCharms (1961) has viewed the interaction of teacher and pupils in a pedagogic situation in terms of a "contingency" model, where contingency indicates the dependency of the behavior of each person in a dyad on that of the other. Three possible forms of interaction in a dyad may be considered: non-contingent, asymmetrically contingent and symmetrically contingent. In a non-contingent interaction, the degenerate case, each person would follow a rigid predetermined course of behaviors unaffected by the other. In an asymmetrically contingent interaction one of the two person follows a rigid predetermined procedure while the second responds in behaviors depending on those of the first. In the third type, symmetric contingency, the behavior of each person is determined to some extent by the behavior of the other. It is this final type that B. O. Smith (1960) emphasizes in the two action steps in his detailing of reciprocal behaviors in a complete teaching cycle.

It seems important that both passive and active pupil behaviors or asymmetric and symmetric interaction occur in the course of mathematics teaching. While the learning of many skills depends on the pupil's desire and ability to imitate a teacher's successful activity and then to practice what he has observed, it is equally and often more valuable for the learner to move from this largely receptive role to greater independence. When the learner takes part in the development and in the application of the subject matter he is gaining skill in questioning and demonstrating himself, and may receive significant criticism and encouragement from the teacher and from the other pupils.

Spontaneous problem solving in general situations is usually motivated by the intrinsic interest or merely by the occurrence of the problem. In the classroom enthusiasm for mastery of subject matter beyond the passive or active behaviors discussed above may be stimulated by such means as intrinsic interest, explicit attention to the historical background, or by reference to use of a particular technique or system in the physical or social sciences.

The categories of Attitude answer the question, "How much initiative are the pupils asked to show, and how much do they demonstrate?"

Category	Symbol
Curiosity	A 1
Independence	A 2
Receptivity	A 3

DESIGN

CLASSIFICATION OF VERBAL BEHAVIORS: IN THE FRAME ATTITUDE

Teacher or Pupil The teacher demonstrates or encourages pupils behaviors in each category; the pupils demonstrate the behaviors in each category.

1. Curiosity -- fresh unusual material; a new direction
   - 1.1 Teacher statements relating present topic to other areas of mathematics or to other fields, or to more fundamental mathematical concepts or to historical context
   - 1.2 Teacher encouragement of unusual problem or new direction including positive support of pupil expression of unusual interest
   - 1.3 Pupils make statements as in 1.1
   - 1.4 Pupils ask questions about 1.1
2. Independence
   - 2.1 Teacher open questions or suggestions demanding pupil thinking beyond one carefully structured step, e. g. asking pupils to solve problems, asking pupils to discuss homework answers, asking pupil suggestion for relation apparent in a series of specific examples, requiring pupil development of proof of a relation, eliciting pupil criticism of his own work
   - 2.2 Turning of pupil-raised question back to same pupil or to the class
   - 2.3 Assignment of pupil topics for class demonstration including regular homework questions developed on blackboard by pupil
   - 2.4 Pupil statements moving problem solution forward more than one step during the interval
   - 2.5 Responsibility for development taken by pupil sometimes indicated by several steps forward or merely by one powerful step forward in a single interval
   - 2.6 Pupil initiates discussion by asking a question and noting specific aspects he has considered
3. Receptivity
   - 3.1 Teacher tells, states, solves problems
   - 3.2 Teacher asks rhetorical questions or questions limited to one-step often trivial or merely yes-no answers
   - 3.3 Teacher is responsive to signals that pupils understand, follow the discussion, are interested in the presentation
   - 3.4 Pupils respond appropriately when called on, but answer is limited to one relatively small step, e. g. I don't know; The square of 7 is 49; Yes; The answer to that homework question was x plus 2
   - 3.5 Pupils ask questions without indicating readiness to treat it themselves with teacher's assistance, e. g. How do you do this problem; I couldn't solve number 37

NEGATIVE behaviors are recorded when discouragement is apparent either in teacher or pupils.

INDEX OF INITIATIVE: Although each behavior can be classed in only one of the categories of Attitude, unlike the other two frames, the categories are related serially: Curiosity presupposes Independence and Independence presupposes Receptivity. This relationship of categories makes reasonable an Index of Initiative by a simple weighting of the positive behaviors in each category. This is discussed in Project 816 and in this Manual under Treatment of data-comparison.

DESIGN

CLASSIFICATION OF OTHER BEHAVIORS: NEUTRAL AND NON-VERBAL

Neutral

Verbal behaviors which concern non-mathematical matters are classified as Neutral. Examples of these are classroom organization behaviors, disciplinary comments, interruptions by school administration such as announcements over the public address system, inactivity or the occasional group bedlam.

Silent study

Mathematical study occurs in the classroom silently in several ways. 1. Short periods of silence may comprise a complete interval set aside for classification of class interaction. 2. Within the general discussion period, the teacher may direct that all the pupils individually, at their seats or with some at the blackboard, should develop a point for immediate use in class discussion. 3. Preceding or following the general class discussion the pupils may have a work period in which they may be doing assignments with individual pupils conferring with the teacher. 4. Tests of short duration--over the course, say, of ten minutes--may occur. Where tests require the entire class period no observation would be made.

These several types of silent study may be classified as S 1, 2, 3, 4 if it is found useful to differentiate them. Indeed in the cases of S 1 and S 2, the observer may be able to infer as well as in verbal discussion the behaviors being demonstrated by the pupils. If it is important in a given study, these two types, may in addition to the S classification, be noted in terms of the appropriate categories of content, process and attitude as if the behavior had been overt. e. g. S_1 (P 622)

DESIGN

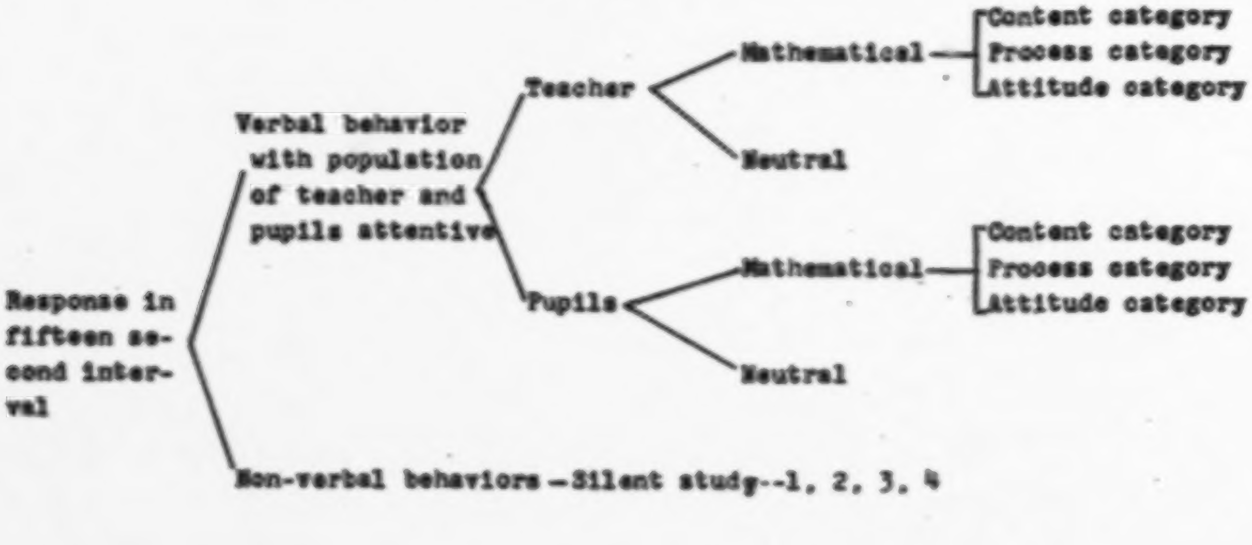
OBSERVATION PROCEDURES

Each verbal behavior occurring in the classroom, with the full population assumed to be attentive, is viewed in all three frames. If mathematical it is classified three ways under one and only one of the categories of each frame. If it is not mathematical, it is classified as Neutral. Non-verbal behaviors are classed as silent study with the possible qualifications as noted above. A schema of classification is given below.

Single behaviors The behavior observed is the verbal response of teacher or pupils plus any concurrent blackboard development, gestures, vocal inflections or facial expressions contributing to the meaning of the verbal interaction. The single behavior is obtained by time sampling: the response of the first speaker in a fifteen second interval. During each minute of observation two behaviors are classified. Of the four fifteen second intervals occurring in each minute, the first and third are used for observation and the second and fourth for classification. A stop watch or a watch with a sweep second hand is used to designate the intervals.

Period of observation The single lesson provides a natural unit for observation of the classroom. Despite the variation in the length of periods from one lesson to another, no fixed length of time for observation has proved feasible. Therefore all verbal interaction occurring while the teacher and pupils are working as a group is classified. The single lesson period is usually about fifty minutes long, resulting in classification of about one hundred behaviors. Stability studies indicate that a ten day sample gives representative values for each category. If the study is limited to the categories of Process or Attitude, or if an adequate number of phases of a small topic have been met, the sample may be derived from as few as six days. This is discussed in some detail in Project 816, Chapter IV.

Schema of classification of behaviors


Response in fifteen second interval
Verbal behavior with population of teacher and pupils attentive
Teacher
Mathematical
Content category
Process category
Attitude category
Neutral
Pupils
Mathematical
Content category
Process category
Attitude category
Neutral
Non-verbal behaviors —Silent study--1, 2, 3, 4

DESIGN

TRAINING PROCEDURES

I. Qualifications An essential qualification for the observer is a sound preparation in mathematics. Broadly, for observation in the elementary or the secondary school, this would be at the level of a mathematics major which includes courses in which the topics have been developed rigorously. In particular, courses such as Abstract Algebra, Non-Euclidean Geometry, Number Theory in which concepts fundamental to the topics of the school are desirable. In addition careful specific study of the topics and texts that will be met during observation should precede collection of data.

II. Observational methods This outlines a minimum training program. As much practice in classification as possible in a variety of classrooms is recommended following this training period before beginning a formal study. While the complexity of the Instrument demands a considerable period for training, this is well rewarded by the richness of the materials which then may be derived.

Preliminary steps

1. One of the following references should be read for a discussion of observational techniques in general:
   Festinger, L., and Katz, D., Research methods in the behavioral sciences. Dryden, New York: 1953.
   Lindzey, G., (ed) Handbook of social psychology. Addison-Wesley, Cambridge: 1954.
   Selltiz, M., Jahoda, M., Deutsch, M., Cook, S., Research methods in social relations. Holt, New York: 1959.
2. Careful study in this Manual of the rationale and the design should follow. Memorization of the categories will come with practice in using the instrument.
3. Before visiting a classroom, permission and cooperation should be secured from the principal of the school and the teacher to be observed. Explanation of the observational procedure and its goals should be made in general terms. We have found considerable interest in the results and have a practice of offering to forward any written results of our observations that will be of interest.
4. Materials needed for observation should be prepared. These are a stop watch or watch with large second hand, and daily recording sheets or cards (see collection of data, this Manual). Recall that materials sufficient for recording 100 behaviors per 50 minutes of observation are needed.
5. The training of observers can be done with an experienced observer and one or several novices depending on the opportunities for visitors in the cooperating training class. If self training is the procedure, it is recommended that this be done in pairs with the two novices observing at the same time during the first four days, with ample opportunity to discuss and resolve disagreements, together, and if possible with an experienced observer.

Actual classification

1. Day 1 in the classroom. Visit classroom noting the fifteen second

division. Consider responses in the broad terms of the Instrument. Expect to be confused!

2. Study the categories of Attitude closely, read and classify typescript # 1 in categories of Attitude only. Compare with given classification. Try to resolve difficulties.
3. Day 2 in the classroom. Practice attributing behaviors to teachers or pupils according to proper time intervals during first ten minutes. Then in addition, classify behaviors in the categories of Attitude only. Begin with short periods of five minutes, rest; then increase to ten minute periods by the end of the class.
4. At home study the categories of Content closely; review those of Attitude. Classify typescript # 2 in Content and Attitude frames. Compare with given classification. Try to resolve difficulties.
5. Day 3 in the classroom. Practice attributing behaviors to teachers or pupils, but in addition classify behaviors in the categories of both Attitude and Content. Begin with periods of ten minutes, rest; increase to one fifteen minute period by the end of the class.
6. At home study the categories of Process closely; review those of Content and of Attitude. Classify typescript # 3 in Content, Process and Attitude frames. Compare with given classification. Try to resolve difficulties.
7. Day 4 in the classroom. Classify behaviors in the categories of all three frames for fifteen minutes, rest; then classify for a period of thirty minutes.
8. Days 5 through 10 in the classroom. Classify behaviors in categories of all three frames during the entire period if at all possible.
9. Following day 10 in the classroom, at home classify typescript # 4 and test reliability of individual and triple frame scores with those of the given classification. Typescript # 5 is extra to use as you wish.

10. Following the training program, examination of the rest of the Manual in terms of treatment of data, validity, etc. will be of interest. In addition, the studies already completed with this instrument are listed in the References.

DESIGN

MARGINAL DIFFICULTIES

1. Speaker's purpose and group's interpretation
Conflict may occur when there is a difference between the speaker's purpose and the listeners' interpretation, as inferred by the observer. Early in the development it had been decided that the actual effect was the thing to consider, so the group's interpretation should be classified. The only exception to this occurs when the teacher develops relations inductively. For further discussion see # 8 below.

2. Two complete ideas in one observational interval
When two complete ideas occur in one fifteen second interval, classify the one which seems to be stronger in the context of the general discussion. e. g. Typescript # 1, behavior 8½.

3. Silent mathematical thinking
When behavior during a fifteen second interval consists of silence or of so few words that it cannot be classified, but when attention is directed to mathematical ideas, it should be classified as Silent study, type 1. The first two types of silent study discussed earlier bear comment. When silent study occurs and the observer may confidently infer the type of behavior, this may be classified as S 1 or 2 with the inferred classification into categories of the three frames in brackets. If desired in a particular study these behaviors can be handled as if overt. However, such decision should be clearly specified.

4. Negative behaviors
Because the pressure to classify each behavior requires concentration on the behavior rather than evaluation of its accuracy, validity or encouraging attitude, the observer may not identify negatives immediately. Negatives occur most often in Content and in pupil behaviors. Teachers we have observed rarely ventured into unfamiliar territory thus risking incorrect remarks, although the quality of their remarks may vary widely in terms of precision and rigor. The pupils negatives in Content occurred most frequently in classes where pupils participated in significant quantity. Many teachers of these high participant classes used the pupil errors as a constructive teaching device; they permitted pupils sufficient freedom to solve problems and develop theory and thus to make errors. Such freedom usually entailed pupil responsibility for the discovery and consideration of mistakes. Negatives in Process and Attitude are readily identified. In Process these have the quality of invalid thinking; in Attitude the quality of discouragement in the three categories. The decision to absorb the relatively small number of negatives into category totals as practiced in Project 816 was made with the recognition that they were handled constructively by the teachers. If negatives occur in numbers sufficiently large to be significant statistcally, then they probably are not being used constructively and decision should be made on their separation or absorption with this in mind.

5. Content: New Relations
Statement of a new relation may be made before it is developed, indeed independently of its development. When first stated it should be classified as C 5, statement. If the relation is being developed, statement of it should be classified according to the deductive or inductive treatment, C 3 or C 4.

6. Content: Old Relations

Statement of an old relations may occur in the development of new material or in problem solving. Note that the Instrument defines 'old' as the body of theory up to an arbitrarily chosen cut off point. In any given study an investigator should specify the cut off point. In project 816, texts or outlines were available for all classes observed and it was possible to specify cut off in this case as the end of the previous topic. If understanding of the fundamental principle of the 'old' relation is evident, the behavior is classified as Structure, C 1; if the relation is treated as a formal rule where no understanding is evident, it should be classified as Technique, C 2.

7. Content: Mathematical and Inductive

The observer recognizes the teacher's purpose in selection of the particular problem but does not infer that pupils are aware of the relation the teacher is trying to develop. Therefore when such behavior is attributed to the teacher it is classified as Inductive, C 4; when pupils work on problems the teacher has selected to elicit the relation, their behavior is generally classified as Mathematical C 6. In project 816 in the RP classes pupils were clearly aware of the strategy of inductive search for relationships and their handling of specific problems in the above context was reasonably classified as Inductive.

8. Content: Mathematical and Other

The first reading of word problems is classified as Other, C 7. If aspects of the problem are considered in terms of its particular field, the classification continues in Other. More frequently only a very superficial attention is given to the 'Other' aspects, hence the problem classification should quickly become Mathematical, C 6, or possibly Techniques, C 2.

9. Process: Specializing, and Analyzing or Synthesizing

Specific attributes of a problem may be selected (Specializing) and then used immediately in a necessary inference contributing to the solution during one interval (Analyzing or Synthesizing). When in doubt use the strong syllogistic category.

10. Process: Specializing and Generalizing

Restatement of earlier theory appropriate to the present problem may be made. Classification should depend on the context and a decision be based on the most apparent level of thinking. For example, a theorem, which is a generalization, may be quoted and then examined for its meaning. In that case it would be classified as Generalizing. On the other hand the same theorem may be quoted for use only in a specific aspect of a problem. Then it should be classified as Specializing.

11. Process: Specializing and Relevant

Mathematical information not caught up in the dynamic problem solving process belongs in Relevant. The difference between the two categories is merely an association of ideas in the case of Relevant, P 5, and the necessary logical bonds of problem facts and conditions in the case of Specializing, P 3. For example, the reading of homework, the reading of a problem preliminary to selection of significant facts and conditions, historical information given as a background for a topic or a problem, are all classified as Relevant, C 5, until a problem situation has been formulated and investigated. When the active problem solving process begins the behaviors belong in the dynamic categories of classificatory and syllogistic reasoning.

DESIGN

12. Attitude: Teacher independence and receptivity
Independence depends on the level of demand of the teacher's question. To be classified as Independence a question must be more than open, it must require more than one relatively simple step with the expectation that the pupil must justify his answer. Less demanding questions belong in Receptivity.

13. Attitude: Pupil Independence and receptivity
A short answer is usually classified as Receptivity. However if it requires careful thinking and if the climate of the class is such that any answer may have to be justified, then the short answer should be classified as Independence.

14. Classification clear in two of the three frames
If a behavior may be classified in two of the three frames readily, it may be 'forced' into a classification in the third frame.

DESIGN

EXCERPTS OF AN 8th GRADE CLASS IN ALGEBRA

Division of the verbal interaction into fifteen second intervals is indicated by the double space between the lines. Alternate intervals are numbered. In the 'live' situation only these numbered intervals are classified; here to exploit the material fully all are classified but the 'half' intervals are indented.

All behaviors are classified in the left hand column in the following way: Speaker, Content, Process, Attitude. Detailed explanation of the classification is made in the right hand column with decimal notation based on the definition of categories above.

(The class begins with the teacher asking pupils to read aloud their homework answers.)

T 253	**1)**	**T: On page 12 what are the factors of 6? Doug?** **P: 3 and 2.** **T: Of 10? Carol?** **P: 2 and 5.** **T: Of 21, Gary?**	**T 2.2, 5.1, 3.2**
P 253		**P: 3 and 7.** **T: Of abc, Tiggy?** **P: a, b, c.**	**P 2.2, 5.1, 3.4**
T 253	**2)**	**T: 8hk, Mark?** **P: 8, h, and k.** **T: All right, if we just wanted to talk about factors, we could say 8, h, and k. We're going to make an agreement now that whenever we talk about factors**	**T 2.2, 5.1, 3.2**
T 132		**of an expression we will talk about the prime factors. What do I mean by prime, Doug?** **P: The ones that are not divisible**	**T 1.2, 3.1, 2.1**
P 132	**3)**	**except by ones and by itself.** **T: OK a prime number is one that is not divisible by anything except itself and one; for example, what?** **P: 2.** **T: 2, what's another one?** **P: 3, 5.** **T: OK, that gives you an idea. So**	**P 1.2, 3.1, 2.5**
T 132		**if Mark said the factors of this are 8, h, and k, well, we will have to assume that h and k are prime. But what about 8. Is 8 a prime number, Ed?**	**T 1.2, 3.1, 2.1**

P: No.
T: No, so

T 632 4) how could you write prime factors of 8? T 6.1, 3.1, 2.1
P: 2, 2, 2.
T: OK, 8 is 2 x 2 x 2. So to write the prime factors we will want this. If you didn't do this on the assignment please mark it and do it in your corrections.

T 633 Give the prime factors. T 6.1, 3.1, 3.1
P: Could we say that...I had down, for example 8hk, and then I put down 2 with a little mark and 2. Should I write it out with the times?
T: This is all right. Actually they are just asking you to list what are the factors and this would be the best way of writing it.

P 253 5) P: I left out 6. Should I put down the answer? P 2.2, 5.1, 3.5
T: Just redo 4 and 6. Yes, Ross?
P: Well, I, uh, on the 8th, I had m over k and then I put 8 over 1/2, but how would you get 16 out of that?
T: Divided by 1/2? How do we divide fractions?

P 233 P: Oh, reverse them. P 2.1, 3.1, 3.4
T: What's the right word?
P: Invert.
T: Invert,

T 233 6) invert the divisor and multiply, right. Actually you should have this down here. Rennie? T 2.1, 3.1, 3.1

* * * * *

(Having checked the assignment, the pupils ask questions about difficulties.)

P 633 7) P: Well, on 10 I have 6, and then, well, you give the answer to be 0. P 6.1, 3.1, 3.4
T: Right. Tell me what you have.
P: Equals 6 times 0 times 2.
T: All right. Now multiply those for me. 6 times 0 is what?

P 133 P: Zero. P 1.1, 3.1, 3.4
T: 0 times 2 is?
P: Zero.
T: Zero. 0 times anything is?
P: Zero.
T: Gary?
P: Well, 8v on 14,
T: Um.
P: What was the answer to that?

P 653 8) I got 4, is that correct? P 6.1, 5.1, 3.5
T: No, that's the one we were just talking about here. Were you dividing 8 by 1/2? You have to invert and multiply to divide one fraction by another, to divide one number by a fraction.

* * * * *

(After most specific difficulties are handled, the teacher makes the assignment for the next day.)

T 252 9) T: Tomorrow's assignment will be No. 17. Read page 15, on page 16 do problems 1--25. That's it. Assignment 17, read page 15, T 2.2, 5.1, 2.3

T 752 on page 16 do problems 1--25. And also, it would be nice if we could know or find out a little bit more about the background of algebra. Is there anyone who would be interested in, for an T 7.3, 5.2, 2.1

T 752 10) extra credit report, looking up the history and giving us a report sometime next week on that? Someone who hasn't done a report in here before. I don't know who it has been. Ann, would you like to do that? T 7.3, 5.2, 2.1
P: How long would it take?
T: How long? Well it depends on how much material you find. Just

T 753 include whatever you think is important about the history and how algebra was involved, and so forth. If you, when you when you start looking for material, if you have any questions you can check with me about what you need to include. T 7.3, 5.2, 3.1

T 253 11) Have that up for about, oh, either Tuesday or Thursday. Is that, does that give you enough time? OK. Now from today's material do you all T 2.2, 5.2, 3.1

T 133 understand what we mean by factors and we will assume that when we say factors we can find prime factors. Is that part clear? Jean? T 1.2, 3.1, 3.2
P: I don't really...
T: I don't really?
P: I don't really understand it.
T: All right. Well

T 633 12) when you say prime factors that means that every factor we write down we want to mean a prime number. So if you were asking for the factors of 12, the prime factors of 12, well what's the first pair of factors you can think of? T 6.1, 3.2, 3.1

All right, 6 and 2, that's one set. Then we ask ourselves, are both of these prime numbers? Are they, Jean?
P: I don't think the 6 is.
T: The 6 isn't a prime number because it can be written

T 633 as what factors? 3 x 2. 3 x 2. So let's write that as 3 x 2 and then we also have this other times 2. T 6.1, 3.2, 3.1

T 633 13) Does 3 x 2 x 2 equal 12? T 6.1, 3.1, 3.2
P: Yes.
T: All right. Are each of these prime numbers? What's the definition of a prime number?

P N P: I don't know. P: N First pupil to speak; does not continue to participate.
T: Ross, can you remind her of what the definition of a prime number is?
P: It's something that is only divisible by itself and by one.
T: All right. A number that is divisible only by itself and one.
P: Oh.
T: 3 is a prime. Is 2 a prime?
P: Yeah.
T: Yes, and of course the other 2

T 623 14) is also, so here we have 3, 2 and 2. They are the prime factors of 12. T 6.1, 2.2, 3.1

* * * * *

(Moving into a discussion of new material, difficulties occur in an attempt to handle the 'traditional' presentation of the text in a more rigorous manner.)

P 543 15) P: May I ask a question about this? What is the definition of a term? P 5.2, 4.2, 3.5
T: It's hard to define.

T 543 I think your book gives a good description of it on page 15. When an algebraic expression is made up of certain parts connected by plus and minus signs, these parts with the signs immediately preceding them are called the terms of the expression, and that's just about all you can say about it. T 5.2, 4.2 , 3.1

T 132 16) Now let's see what we can do with terms. Say we have an expression, a binomial that has the terms 2x and 3x. One of our basic goals in algebra is to be able to write things in simple ways, T 1.4, 3.4, 3.1 Poorly expressed in use of term "simplest"

T 132 to take long complicated expressions and to try to write them the simplest way we can to make make our work easier. Can you think of any other way that we might be able to write T 1.4, 3.4, 2.1

this in a simpler way, John?

P $\bar{6}$32 17) P: 2x squared plus 3 — P $\bar{6}$.1, 3.1, 2.5 Bar over 6 indicates -ve.

T: Now, let's see. John thinks you can write it 2x squared plus 3. Anyone else have any ideas, Gary?

P: I'd say 5x.

T 432 — T: Gary thinks you can write it 5x. Any body else have any ideas? Mark? — T 4.1, 3.1, 2.2

P: 2, 2x, 2x plus 2...

P $\bar{6}$32 18) 2x plus 5. — P $\bar{6}$.1, 3.1, 2.5

T: 2x plus 5. Does anybody have any idea now about how we can find out if one of these is right or if any of them is right, or if they all seem to be right? Doug?

P: Well, uh, you say

P 432 — x and x are both the same in both situations and, uh, so you could say that both the first and the second terms are, have the same denominator--would you — P 4.1, 3.1, 2.5

P 432 19) say or? — P 4.1, 3.1, 2.5

T: Well, the same something.

P: Yeah, the same something. So you could just add the first part of the problem, uh, with both terms.

T 422 — T: Well you are using a reasoning sort of way and from your reasoning which one would you conclude? — T 4.1, 2.1, 2.2

P: I would say that the answer would be 5x.

T: Doug thinks it is 5x. Do you have, can any of you think of a way you can show it? Whether that expression is the same as this one or not?

P 112 20) P: Well, by making it a number. — P 1.4, 1.1, 2.5 (Checking is a strategy)

T: All right. Let's make letter x be a number. What number do you suggest?

P 633 — P: 3. — P 6.1, 3.1, 3.4

T: All right, 3. We don't, when we try this out, we don't use 2 because 2 has special properties, you know that 2 plus 2 is 4 and 2 x 2 is 4, so sometimes that can lead us astray, and one has even more special properties so we don't usually use 1, so 3 is a good choice. Let's evaluate this and find out what the value of the expression we started out with is. 2 x 3 plus 3 x 3 equals what?

P 633	21)	P: 6. T: 6 plus 9 equals 15. P: 15 T: Now let's evaluate these. (The possible) solutions) This would be 2 x 3 plus 3 which equals what?	P 6.1, 3.1, 3.4
P 633		P: 12, 24, 18 T: Plus 3, well that's not 15, so this one doesn't work. How about this one, 2 times 3 plus 5. P: That equals 11. T: ...plus 5 which equals 11. That doesn't work. Now this one which is 5 times 3 which equals 15, so it looks like this is at least	P 6.1, 3.1, 3.4
T 322	22)	the only possible one of the ones we have. Any body think of a way you can absolutely prove this is the right answer?...	T 3.1, 2.1, 2.1
T 322		using one of the principles we have discussed before? Ross? P: Well in the book, uh, it had a similar situation. You had to find out what the number was. T: I thing you're ahead of us reading those book answers; they're not proving it, they're just discussing it.	T 3.1, 2.1, 2.1
P N	23)	P: That was in the very first part. T: The first part, no. One of the principles we had, think of the basic principles that we've talked about, and using one of those we can prove this.	P N
T 112		Obviously it's not the commutative, because we're not switching anything. The associative law has to do with 3 numbers; by a process of elimination....	T 1.1, 1.2, 2.1
T 142	24)	rack your brains, and remember what the distributive said. What does the distributive principle say, Ross? P: Doesn't it say you can shift a few, well,	T 1.1, 4.2, 2.2
P 1̄33		you can put a few things in front like you T: You probably have the idea, but you're not saying it very well. Can you tell me what it is in symbols? P: The same number..what it is in symbols?	P 1̄.1, 3.1, 3.5
P 1̄33	25)	would it be 5? T: Let's use letters instead of numbers. P: OK, a plus b, uh, plus, no, times c is equal to	P.1̄.1, 3.1, 3.5
P 142		a times c plus b times c. T: (on board) $(a + b)c = ac + bc$	P 1.1, 4.2, 2.5

PROPOSED CLASSIFICATION FOR TYPESCRIPTS #1, #2

#1

1	T 553 T 751	21	T 632 T 423	41	T 632 T 132
2	T 333 T 133	22	T 432 P 632	42	P 632 T 632
3	T 133 P 133	23	P 632 T 432	43	T 622 T 632
4	T 253 T 253	24	T 442 S_1(P632)	44	T 622 S_1(P622)
5	T 233 T 132	25	S_1(P632) T 632	45	S_1(P622) S_1(P622)
6	T 132 T 132	26	P 633 T 143	46	T 622 T 112
7	T 133 T 132	27	T 632 T 132	47	T 112 T 112
8	T 112 T 232	28	P 132 T 132	48	T 112 P 112
9	T 133 T 333	29	T 632 T 643	49	T 632 T 633
10	T N T 322	30	T 643 T 133	50	T 633 T 112
11	T 322 T 332	31	T 143 T 151	51	T 622 T 622
12	T 332 T 132	32	T 632 T 443	52	T 633 T 632
13	T 133 T 432	33	T 443 T 433	53	T 443 T 633
14	T 432 T 143	34	T 433 T 653	54	T 632 P 642
15	T 751 T 333	35	T 432 T 432	55	P 632 P 632
16	T 342 P 342	36	T 632 P 632	56	T 612 T 622
17	T 151 T 133	37	T 443 T 442	57	T 622 P 632
18	T 453 T 143	38	T 632 T 633	58	T 632 T 632
19	T 432 T 432	39	T 132 T 632	59	P 632 T 632
20	T 432 P 632	40	P 112 P 322	60	T 623 P 632

#2

1	T 553 T 732	21	T 632 P 622	41	T 632 T 253
2	P 732 T 752	22	T 132 T 633	42	T 253 T 652
3	T 753 T 753	23	T 632 T 632	43	T 652 P 633
4	T 743 T 731	24	T 132 T 252	44	T 732 P 632
5	T 732 T 732	25	T 252 T 252	45	T 632 T 633
6	T 732 T 723	26	T 253 T 252	46	T 632 P 632
7	T 632 T 633	27	P 253 T 632	47	P 653 P 653
8	T 632 P 632	28	P 632 P 632	48	T 632 P 633
9	T 633 T 253	29	T 632 P 632	49	T 653 T 653
10	P 253 T 133	30	T 632 T 633	50	T 633 T 632
11	T 132 T 632	31	T 633 T 632	51	P 633 T 633
12	P 632 P 632	32	T 633 T 112	52	T 633 T 133
13	T 632 T 633	33	T 632 P 633	53	T 133 P 633
14	T 632 T 132	34	P 633 T 633	54	T 632 P 633
15	T 253 T 633	35	T 633 T 632	55	T 633 T 632
16	T 632 T N	36	T 632 T 632	56	T 632 P 633
17	T 633 T 632	37	T 632 P 632	57	T 632 T 232
18	T 633 T 133	38	T 132 T 132	58	T 632 T 632
19	T 632 T 112	39	P 132 P 632	59	T 632 P 233
20	P 142 T 632	40	P 632 T 632	60	T 233 T 133

PROPOSED CLASSIFICATION FOR TYPESCRIPTS #3, #4

#3

1	T N T N	21	T N T 333	41	P 112 T 332
2	T 253 T 253	22	T 332 T 332	42	T 332 T 353
3	T 632 T 432	23	T 312 P 312	43	T 322 T 323
4	P 332 P 633	24	P 312 T 143	44	T 322 P 322
5	T 751 T 333	25	T 112 T 342	45	T 132 T 333
6	T 332 P 333	26	T 342 P 342	46	T 333 T 133
7	T 333 T 332	27	T 342 T 332	47	T 152 T 333
8	P N T 332	28	T 333 T 332	48	T 332 T 332
9	T 332 T 332	29	P 333 T 322	49	P 332 T 332
10	T 332 T 313	30	T 322 T 322	50	T 332 T 332
11	T 413 T 313	31	T 112 T 112	51	T 153 T 153
12	T 322 P 133	32	T 312 T 313	52	T 322 P 322
13	T 322 T 332	33	T 323 T 113	53	T 322 T 322
14	T 312 T 312	34	T 113 T 333	54	P 333 P 322
15	T 753 T 333	35	T 322 T 322	55	P 322 S_1(P322)
16	T 343 T 332	36	T 332 T 332	56	P 322 P 322
17	P 332 P 122	37	P 132 S_1(P312)	57	T 322 T 332
18	P 112 P 112	38	T 312 P 332	58	T 333 T 332
19	T 332 T 332	39	T 313 T 312	59	P 332 P 333
20	T 322 P 322	40	T 113 T 112	60	P 333 T 332

#4

1	T 522 T 522	21	T 332 T 333	41	T 143 P 332
2	T 522 P 332	22	T 312 P 112	42	T 253 T 253
3	P 332 P 333	23	T 322 P 252	43	T 533 T 533
4	T 353 T 333	24	P 322 P 322	44	T 632 T 633
5	T 322 T 353	25	P 322 T 313	45	T 632 T 632
6	T 333 T 333	26	T 331 T 323	46	T 433 T 433
7	P 322 T 312	27	T 522 P 322	47	T 432 T 432
8	T 313 T 333	28	P 322 P 332	48	T 433 T 432
9	T 332 P 322	29	P 132 T 132	49	S_2(P432) P 432
10	P 322 T 333	30	P 133 T 123	50	T 432 P 432
11	T 143 T 323	31	T 123 T 322	51	T 433 T 432
12	T 333 T N	32	P 442 T 143	52	T 432 S_2(T433)
13	T 323 T 323	33	T 142 P 322	53	S_2(T432) T 432
14	P 332 P 332	34	P 112 P 322	54	T 432 T 432
15	P 332 T 333	35	P 312 T 112	55	P 633 P 632
16	T 333 T 323	36	P 322 P 322	56	P 632 T 432
17	T 322 P 332	37	P 322 T 322	57	T 433 P 633
18	P 333 T N	38	P 322 T 322	58	T 633 S_1(T433)
19	T 332 P 332	39	P 322 P 322	59	T 433 T 443
20	P 332 P 332	40	P 322 T 153	60	T 443 T 443

PROPOSED CLASSIFICATION FOR TYPESCRIPT #5

1	T N	T N
2	T 253	T 133
3	T 633	T 633
4	T 633	T 633
5	T 442	T 632
6	P 442	T 442
7	P 443	T 112
8	T 632	T 433
9	T 632	T 632
10	T 631	T 631
11	T 632	T 633
12	P 642	T 632
13	T 631	T 631
14	T 132	T 133
15	T 632	T 632
16	T 631	T 632
17	T 632	P 633
18	P 633	T 632
19	P 632	T 613
20	P 612	P 632
21	T 632	T 632
22	S_2(P632)	T 632
23	T 612	P 632
24	T 632	P 732
25	T 132	T 133
26	T 112	T 143
27	T 741	T 632
28	P 632	P 632
29	T 623	T 133
30	T 633	T 632
31	T 632	S_1(P632)
32	P 612	P 612
33	P 633	T 622
34	T 632	T 632
35	P 632	T 132
36	P 622	none
37	T 632	T 133
38	P 632	T 633
39	P 632	T 632
40	T 632	P 633
41	T 633	T 633
42	T 633	T 632
43	P 612	T 632
44	T 632	P 632
45	T 132	P 632
46	T 632	P 632
47	T 632	P 642
48	T 132	T 132
49	P 632	T 632
50	P 112	T 613
51	T 613	P 632
52	T 622	T 632
53	P 612	P 112
54	P 622	T 632
55	P 633	T 132
56	T 632	T 632
57	T 723	T 631
58	P 632	P 612
59	T 631	P 633
60	T 632	T 641

TREATMENT OF DATA

COLLECTION

The recording sheets occurring on the following pages have proved their use to us in many studies and may be readily and inexpensively duplicated. Pages 30 and 31 present the sheets used for daily recording of the individual behaviors in Content, Process and Attitude by category number, and summary of this detailed classification by individual and triple frame scores. Page 30 includes ample space for recording remarks needing later discussion and also details of blackboard development. The summary sheet may be used as a means of tallying triple frame scores followed by determination of individual frame scores by calculating appropriate totals. Page 32 is also used daily and assures the keeping of a broad description of classroom activities as well as important specifics of text, topic and assignment. We have found brief yet detailed account of the distribution of class time valuable, e. g.

> "Reading of answers by teacher to assignment of previous day; question period for difficulties. Teacher commentary on variety of quadratic equations for which solution methods found to date; introduction of $x^2 + 2x + 2 = 0$; introduction of name for new type of number--complex, of form $a + bi$. Teacher reading from text of operations with complex numbers. Students question value of arbitrary new numbers. Vague teacher reference to electrical engineering applications. Assignment of many problems of this type."

Pages 33 and 34 afford summaries of teacher and pupil characteristics. Many of these will be useful to control in particular studies. The former page may be rapidly completed by the teacher himself; the latter may require administrative permission and assistance in obtaining the necessary data for some of the calculations. Generally it is desirable to record individual student names by code to preserve anonymity.

Classification of classroom discourse, behavior by behavior, may be made directly on cards later used for machine computation. A card used in Project 816 is pictured on page 35 . The usefulness of such cards will depend on the purpose of the study and should be approached with some caution. In Project 816 two marked values were apparent in determination of the stability of accumulating samples and in determination of cross-classification matrixes of the two observers for the entire set of some 8,000 behaviors jointly classified. These were important aspects of validity for this study. Most other aspects could have been handled directly from duplicated recording sheets. While using the cards, we experienced some difficulty in continuing to record remarks of particular interest as well as to outline blackboard development. While this may have been related to time necessary for clear recording on the cards, it was felt more likely that it resulted from an inertia following completion of marking of the card. This difficulty was not experienced in the use of recording sheets. The value of the use of cards for maching computation from raw data, then, should be approached with due attention to the size and purposes of the study; when cards are used care should be directed to adequate recording of pertinent remarks and blackboard.

TREATMENT OF DATA

CODE ______

		TEACHER							PUPILS						
		C		P		A		N	C		P		A		N
		+	−	+	−	+	−	S	+	−	+	−	+	−	S
	1														
	2														
	3														
	4														
	5														
	6														
	7														
	8														
	9														
	10														
	11														
	12														
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	19														
	20														
	21														
	22														
	23														
	24														
	25														
	26														
	27														
	28														
	29														
	30														

TREATMENT OF DATA

TRIPLE FRAME SCORES IN CATEGORIES OF CONTENT, PROCESS AND ATTITUDE

CODE ________

	TEACHER						PUPILS					
	P_1A_1	P_2A_1	P_3A_1	P_4A_1	P_5A_1	Total	P_1A_1	P_2A_1	P_3A_1	P_4A_1	P_5A_1	Total
C_1												
C_2												
C_3												
C_4												
C_5												
C_6												
C_7												
Total												
	P_1A_2	P_2A_2	P_3A_2	P_4A_2	P_5A_2		P_1A_2	P_2A_2	P_3A_2	P_4A_2	P_5A_2	
C_1												
C_2												
C_3												
C_4												
C_5												
C_6												
C_7												
Total												
	P_1A_3	P_2A_3	P_3A_3	P_4A_3	P_5A_3		P_1A_3	P_2A_3	P_3A_3	P_4A_3	P_5A_3	
C_1												
C_2												
C_3												
C_4												
C_5												
C_6												
C_7												
Total												

INDIVIDUAL FRAME SCORES IN SINGLE CATEGORIES OF CONTENT, PROCESS, ATTITUDE

	C+	C-	P+	P-	A+	A-	N	S	C+	C-	P+	P-	A+	A-	N	S
1																
2																
3																
4																
5																
6																
7																
Total																

TREATMENT OF DATA

DAILY CLASSROOM ACTIVITIES

CODE ______
DATE ______
OBSERVERS ______

TIMING: DAY ______ BEGINNING ______ ENDING ______

SCHOOL PERIOD ______

GENERAL DISTRIBUTION OF CLASS TIME:

TEXT:

TOPICS:

ASSIGNMENTS:

OTHER REMARKS:

TREATMENT OF DATA

TEACHER CHARACTERISTICS

CODE ______

DATE ______

EXPERIENCE: TEACHING MATHEMATICS ______ less 5 yr ______ 5-10 yr ______ over 10 yr

OTHER RELATED ______________________________

____

____

FORMAL POSITIONS (chairman, etc.) ______________________________

____

FORMAL MATHEMATICS PREPARATION: UNIVERSITY COURSES ______________ APPROX. HRS

RELATED SEMINARS ______________ APPROX. HRS

REMARKS ______________________________

PRESENT TEACHING RESPONSIBILITIES: NUMBER OF MATHEMATICS CLASSES ______________

NUMBER OTHER CLASSES ______________

TOTAL NUMBER OF PUPILS ______________

BRIEF ESTIMATE OF CLASS OBSERVED:

TREATMENT OF DATA
PUPILS CHARACTERISTICS

CODE ______
DATE ______
OBSERVERS ______

NUMBER: OF BOYS ______ OF GIRLS ______

AGE: MEAN ______ S. D. ______

APTITUDE MEASURES: TYPE ______ MEAN ______ S. D. ______

______ ______ ______

______ ______ ______

MATHEMATICAL ACHIEVEMENT MEASURES: TYPE ______ MEAN ______ S. D. ______

______ ______ ______

______ ______ ______

MATHEMATICAL PREPARATION: ______

______

______

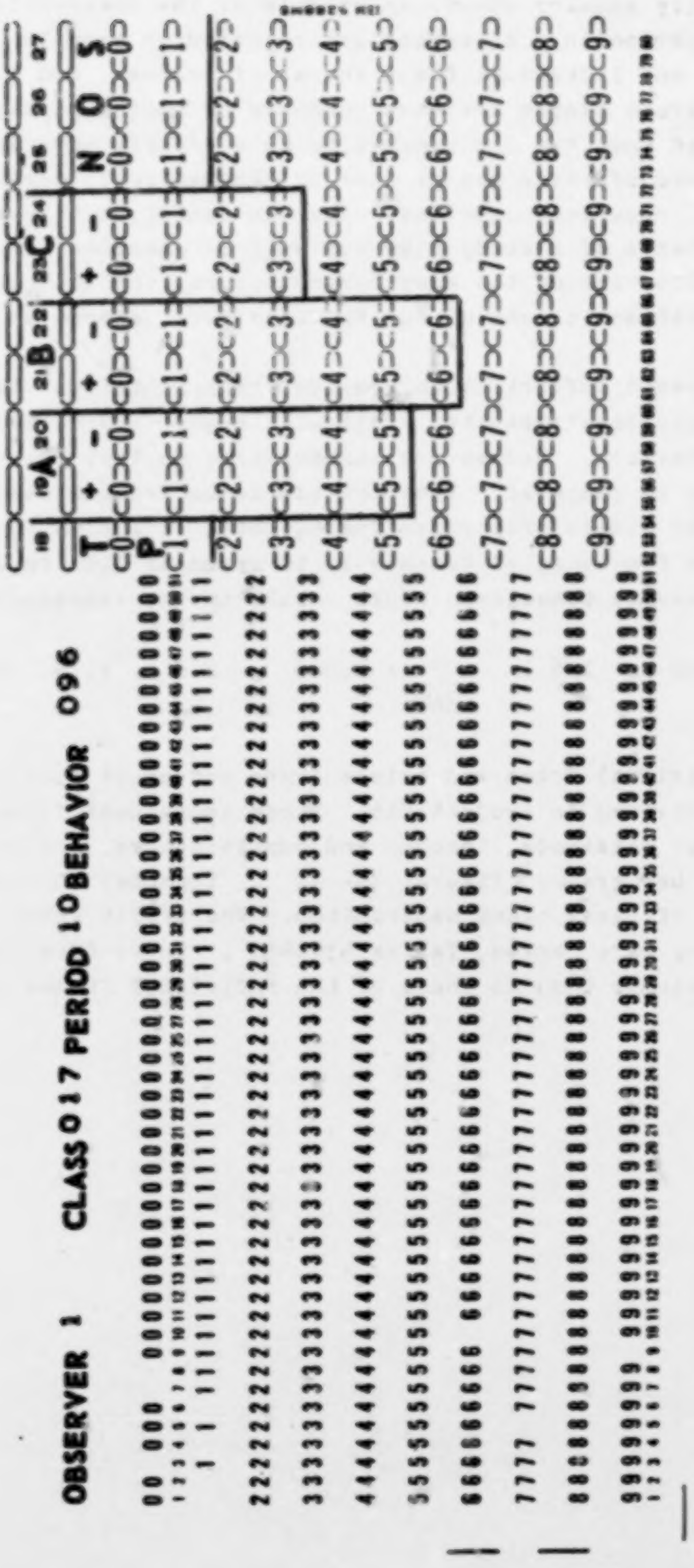
______

SCHOOL ECONOMIC STATUS: ______

COLLEGE EXPECTANCY: ______

CLASSROOM HABITS:

TREATMENT OF DATA


OBSERVER 1
CLASS 017 PERIOD 10 BEHAVIOR 096
T
P
A
B
C
+
-
N
O
S

Card used for machine computation, Project 816

TREATMENT OF DATA

DESCRIPTION

Using the daily summary sheet, an example of the observations made by one observer for one period in a classroom are recorded on page 37 . From the tallies both triple frame and individual frame scores of teacher and pupils behaviors may be derived. Classroom scores are then composed by adding teacher and pupils components. It is noted that the frequencies in this example were all positive; if negatives occur record of these may be made by superscripts, perhaps in colored ink. In our experience, negatives occur most often in Contet, but in such small quantities that for later aspects of a study they may well be absorbed into a single score. When joint classification of the same period occurs, the best scores should be obtained from the arithmetic mean of the two observers' scores in each cell.

The sample deemed sufficient to provide 'representative' behaviors is derived from ten such single days' results. This will number approximately 1,000 verbal and non-verbal behaviors. Scores are standardized so that the verbal behaviors of each classroom may be compared. Thus the raw score frequencies of verbal categories are standardized by simple proportion (using total verbal as n) to a base 1,000. Thus the raw score frequency of Teacher I, if given as 109^2 in Structure in a total of 868 classroom verbal behaviors, would result in the representative scores of:

$$\frac{109}{868} \times 1,000 = 126 \; ; \quad \frac{2}{868} \times 1,000 = 2 \; ; \quad \text{i. e. } 126^2$$

The mean individual frame and triple frame scores of four different types of classrooms were obtained in Project 816. These individual frame scores are recorded in Table 38 for classroom, teacher and pupils scores, and will be used as a basis for drawing bar graphs, Figures 39--42 . Together such tables and graphs provide the means of clear class description. The triple frame scores of the same types of classrooms were ranked, Tables 43--45 . These form the basis for class description even richer than do those of the individual frames except when viewed in entirety.

TREATMENT OF DATA

TRIPLE FRAME SCORES IN CATEGORIES OF CONTENT, PROCESS AND ATTITUDE

CODE XXI, 2, a

	TEACHER						PUPILS					
	P1A1	P2A1	P3A1	P4A1	P5A1	Total	P1A1	P2A1	P3A1	P4A1	P5A1	Total
C1												
C2												
C3												
C4												
C5												
C6												
C7												
Total												
	P1A2	P2A2	P3A2	P4A2	P5A2		P1A2	P2A2	P3A2	P4A2	P5A2	
C1												
C2	|		||			3						
C3		卌 ||	卌卌卌 卌 |||	|		31		卌 |	卌 |||			14
C4												
C5					卌	5					|	1
C6												
C7												
Total	1	7	25	1	5	39						15
	P1A3	P2A3	P3A3	P4A3	P5A3		P1A3	P2A3	P3A3	P4A3	P5A3	
C1												
C2			|||	|		4						
C3	|	||	卌 卌 |		|	15			||			2
C4												
C5					||	2						
C6												
C7												
Total	1	2	14	1	3	21			2			2

INDIVIDUAL FRAME SCORES IN SINGLE CATEGORIES OF CONTENT, PROCESS, ATTITUDE

	C+	C-	P+	P-	A+	A-	N	S	O	C+	C-	P+	P-	A+	A-	N	S	O
1	0		2		0					0		0		0				
2	7		9		39					0		6		15				
3	46		39		21					16		10		2				
4	0		2							0		0						
5	7		8							1		1						
6	0									0								
7	0									—								
Total	60		60		60					17		17		17				

Table 38 Mean individual frame scores for each type of classroom: high rigor, high participation (RP); low rigor, high participation (rP); low rigor, low participation (rp); high rigor, low participation (Rp) in each category of Content, Process and Attitude and in Neutral. Classroom scores, Teacher and Pupils components (C,T,P). Frequencies per 1,000 total behaviors.

Type	RP			rP			rp			Rp		
Class												
Component	C	T	P	C	T	P	C	T	P	C	T	P
CATEGORY												
Content												
Structure	181	138	43	98	74	24	73	57	16	104	102	2
Techniques	108	96	12	152	127	25	204	185	19	76	73	3
Deductive	211	130	81	178	118	60	137	121	16	314	303	11
Inductive	153	125	28	73	63	10	146	132	14	232	229	3
Mathematical	275	160	115	439	220	219	404	293	111	223	204	19
Other	50	40	10	32	24	8	19	16	3	31	31	0
Process												
Analyzing	149	112	37	72	54	18	62	46	16	121	117	4
Synthesizing	246	117	129	297	119	178	261	189	72	296	294	2
Specializing	378	284	94	376	268	108	404	334	70	321	295	26
Generalizing	82	65	17	78	61	17	70	65	5	149	147	2
Relevant	123	111	12	150	124	26	186	170	16	94	90	4
Attitude												
Curiosity	42	35	7	27	19	8	18	10	8	58	53	5
Independence	638	387	251	651	375	276	329	191	138	74	62	12
Receptivity	298	267	31	294	232	62	636	603	33	848	827	21
Neutral	22	21	1	28	27	1	17	15	2	20	20	0
Totals	1,000	710	290	1,000	653	347	1,000	819	181	1,000	962	38

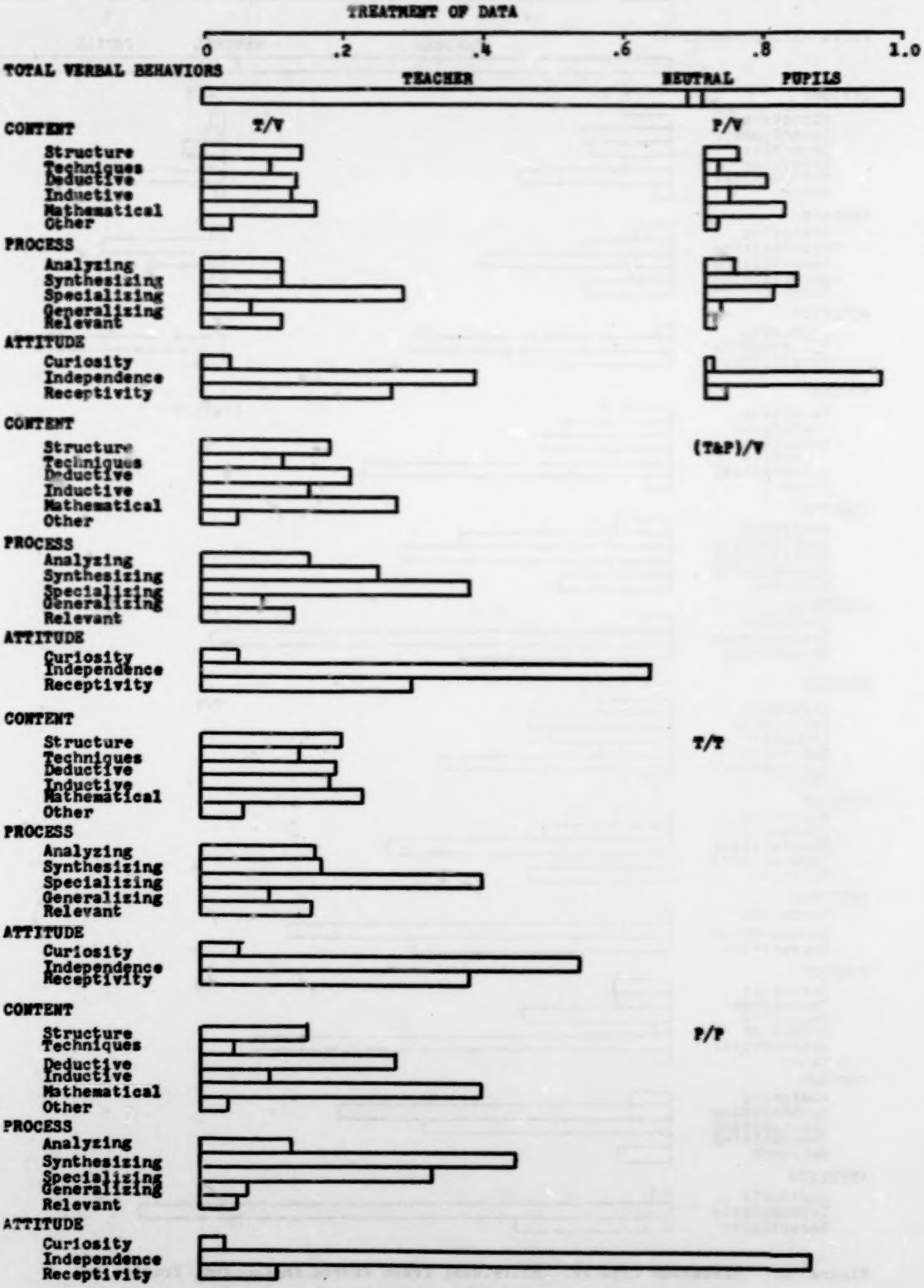


Figure 39 Classroom type RP. Individual frame scores in Content, Process and Attitude for classroom (T&P)/V, teacher T/V, pupil P/V; and teacher proportions T/T, pupil proportions P/P.

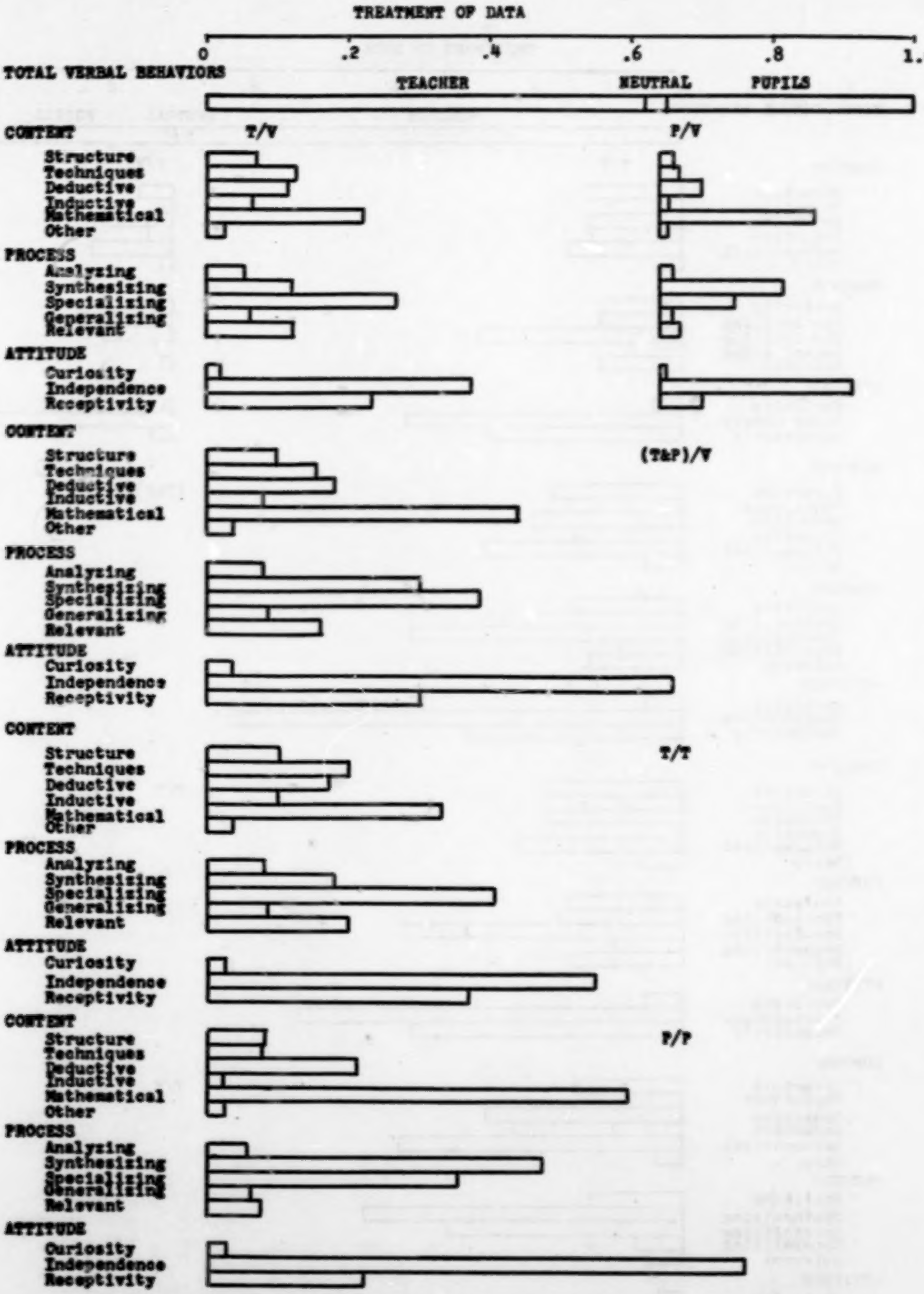


Figure 40 Classroom type rP. Individual frame scores in Content, Process and Attitude for classroom (T&P)/V, teacher T/V, pupil P/V; and teacher proportions T/T, pupil proportions P/P.

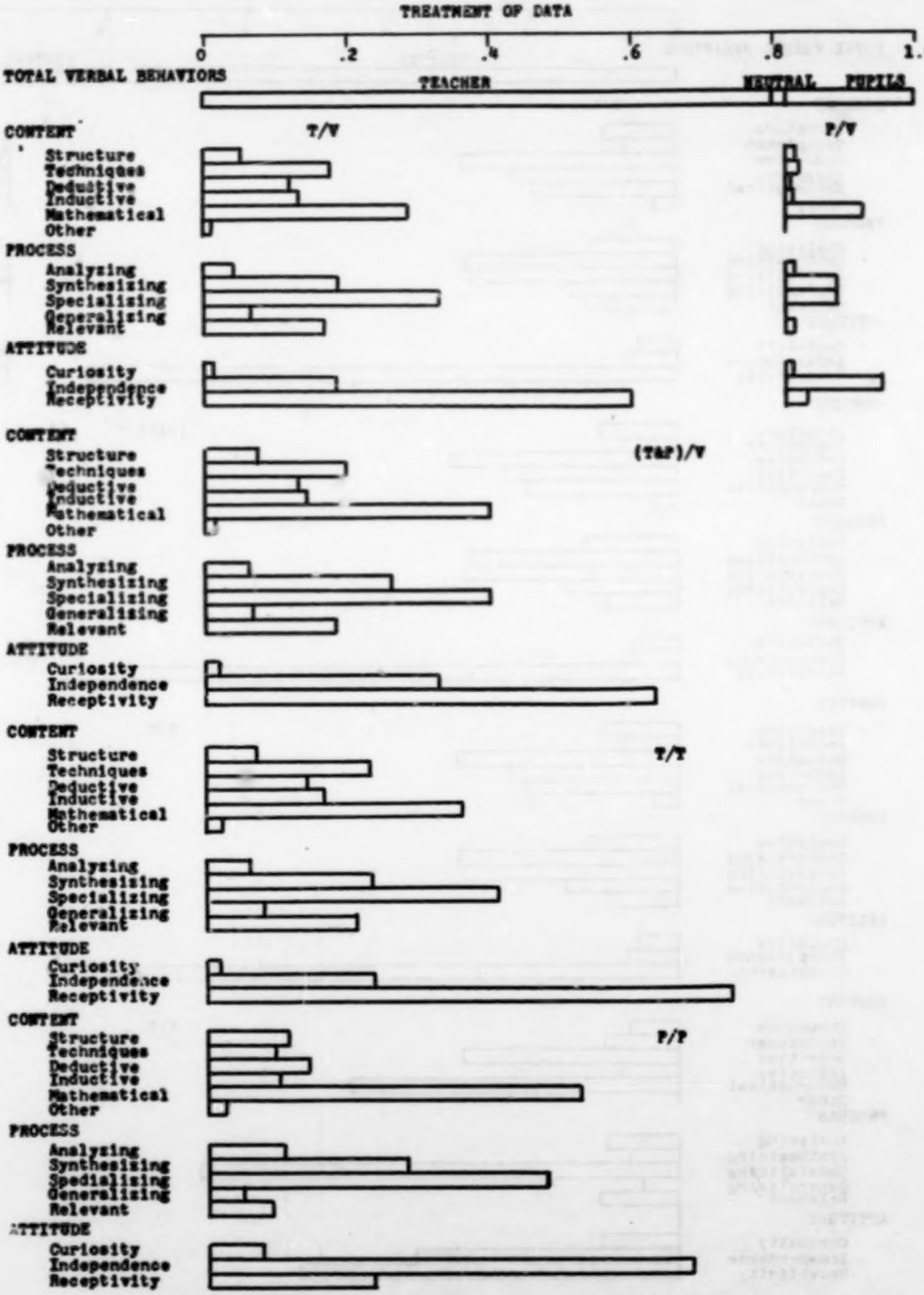


Figure 41 Classroom type rp. Individual frame scores in Content, Process and Attitude for classroom (T&P)/V, teacher T/V, pupil P/V; and teacher proportions T/T, pupil proportions P/P.

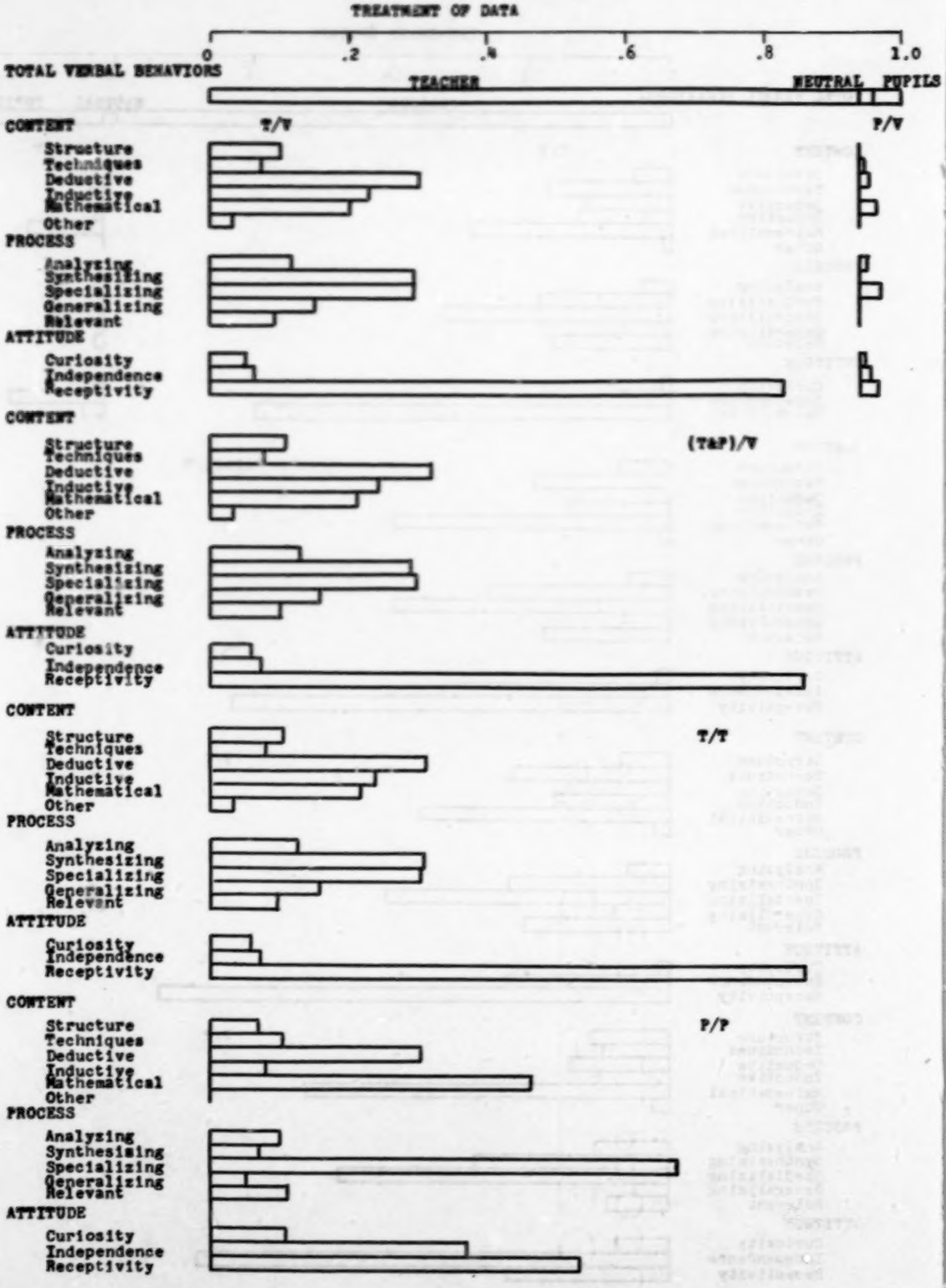


Figure 42 Classroom type Rp. Individual frame scores in Content, Process and Attitude for classroom (T&P)/V, teacher T/V, pupil P/V; and teacher proportions T/T, pupil proportions P/P.

Table 43 First 10 ranks of triple classifications of Classroom behaviors in Content, Process and Attitude in classrooms of high rigor, high participation (RP); low rigor, high participation (rP); low rigor, low participation (rp); and high rigor, low participation (Rp). Frequencies, number per 1,000.

	RP		rP		rp		Rp
105	Mathematical Synthesizing Independence	199	Mathematical Synthesizing Independence	123	Techniques Relevant Receptivity	103	Deductive Synthesizing Receptivity
65	Deductive Synthesizing Independence	109	Mathematical Specializing Independence	105	Mathematical Specializing Receptivity	96	Mathematical Synthesizing Receptivity
65	Mathematical Specializing Independence	59	Techniques Relevant Receptivity	100	Mathematical Synthesizing Independence	82	Inductive Synthesizing Receptivity
53	Inductive Specializing Independence	52	Deductive Specializing Independence	95	Mathematical Synthesizing Receptivity	80	Inductive Specializing Receptivity
53	Techniques Relevant Receptivity	51	Mathematical Specializing Receptivity	63	Mathematical Specializing Independence	74	Deductive Generalizing Receptivity
52	Structure Specializing Independence	49	Techniques Relevant Independence	56	Inductive Specializing Receptivity	67	Deductive Specializing Receptivity
41	Structure Analyzing Independence	35	Mathematical Synthesizing Receptivity	36	Deductive Specializing Receptivity	51	Techniques Relevant Receptivity
41	Deductive Specializing Independence	35	Structure Specializing Independence	32	Techniques Relevant Independence	37	Deductive Analyzing Receptivity
36	Techniques Relevant Independence	24	Inductive Specializing Independence	28	Inductive Synthesizing Receptivity	33	Mathematical Specializing Receptivity
33	Mathematical Specializing Receptivity	24	Deductive Synthesizing Independence	26	Inductive Generalizing Receptivity	31	Structure Analyzing Receptivity

Table 44 First 10 ranks of triple classifications of Teacher behaviors in Content, Process and Attitude in classrooms of high rigor, high participation (RP); low rigor, high participation (rP); low rigor, low participation (rp); and high rigor, low participation (Rp). Frequencies, number per 1,000.

	RP		rP		rp		Rp
49	Mathematical Specializing Independence	74	Mathematical Specializing Independence	116	Techniques Relevant Receptivity	102	Deductive Synthesizing Receptivity
48	Techniques Relevant Receptivity	50	Techniques Relevant Receptivity	96	Mathematical Specializing Receptivity	96	Mathematical Synthesizing Receptivity
43	Inductive Specializing Independence	49	Mathematical Synthesizing Independence	91	Mathematical Synthesizing Receptivity	82	Inductive Synthesizing Receptivity
34	Structure Analyzing Independence	42	Techniques Relevant Independence	56	Inductive Specializing Receptivity	79	Inductive Specializing Receptivity
34	Mathematical Synthesizing Independence	35	Deductive Specializing Independence	40	Mathematical Specializing Independence	73	Deductive Generalizing Receptivity
32	Structure Specializing Independence	31	Mathematical Specializing Receptivity	38	Mathematical Synthesizing Independence	68	Mathematical Specializing Receptivity
32	Techniques Relevant Independence	25	Mathematical Synthesizing Receptivity	33	Deductive Specializing Receptivity	65	Deductive Specializing Receptivity
29	Mathematical Specializing Receptivity	24	Structure Specializing Independence	27	Techniques Relevant Independence	48	Techniques Relevant Receptivity
26	Deductive Synthesizing Independence	20	Inductive Specializing Independence	27	Deductive Generalizing Receptivity	37	Deductive Analyzing Receptivity
26	Deductive Specializing Independence	16	Techniques Specializing Independence	25	Inductive Synthesizing Receptivity	31	Structure Analyzing Receptivity

Table 45 All ranks of triple classification (frequencies ≥ 5) of pupils behaviors in Content, Process and Attitude in classrooms of high rigor, high participation, (RP); low rigor, high participation (rP); low rigor, low participation (rp); and high rigor, low participation (Rp). Frequencies, number per 1,000.

	RP		rP		rp		Rp
71	Mathematical Synthesizing Independence	143	Mathematical Synthesizing Independence	63	Mathematical Synthesizing Independence	11	Mathematical Specializing Receptivity
40	Deductive Synthesizing Independence	35	Mathematical Specializing Independence	22	Mathematical Specializing Independence		
23	Mathematical Specializing Independence	16	Deductive Specializing Independence	10	Mathematical Specializing Receptivity		
19	Structure Specializing Independence	16	Mathematical Specializing Receptivity	8	Techniques Relevant Receptivity		
17	Deductive Specializing Independence	14	Deductive Synthesizing Independence	7	Deductive Specializing Independence		
9	Mathematical Analyzing Independence	9	Mathematical Synthesizing Receptivity	7	Inductive Specializing Independence		
9	Inductive Specializing Independence	9	Deductive Generalizing Independence	6	Structure Specializing Independence		
9	Inductive Synthesizing Independence	9	Techniques Relevant Receptivity				
8	Deductive Analyzing Independence	8	Structure Specializing Independence				
7	Structure Analyzing Independence	5	Techniques Relevant Independence				
7	Structure Generalizing Independence						
5	Deductive Generalizing Independence						
5	Techniques Relevant						

TREATMENT OF DATA

COMPARISON

The representative scores both individual frame and triple frame developed under the descriptive treatment of data serve also for careful comparison of two or more classrooms. In individual frame scores we have found that a ranking check of single category emphasis within each frame is generally too rough; but a careful chi-square test of all the single categories at a time within a frame is often too stringent. Wide variation in even one category may mask significant similarities in a major part of a frame. The most fruitful comparisons we have found have been by study of single categories or by single categories in meaningful combinations. Such combinations were noted earlier in the method of classification: i. Content: Theory--structure + deductive + inductive + statement; Relations--deductive + inductive + statements; Applications--mathematical + other; Skill in specific problems--mathematical + inductive. ii. Process: Syllogistic--analyzing + synthesizing; Classificatory--specializing + generalizing.

In the comparison of the four types of classrooms, three of each type, in Project 816, it was necessary to consider whether the four types were truly differentiated in the categories of the dependent variables (derived from the Wright-Proctor Instrument). Here analysis of variance of the 4 columns each of three values for within and between column variance was an appropriate statistic.

For comparison between pairs of types, chi-square assessment of scores in single categories was appropriate:

Category	Rp	RP	Totals
P_1	363	447	810
$V - P_1$	2637	2553	5190
Totals	3000	3000	6000

X^2 is 9.83, significant at $p < 0.01$.

Thus $RP > Rp$ in this category

Exactly analogous analyses were applied to the several categories in combination in comparing one classroom to another, or to T_I/T_V and P_I/P_V scores. All these comparisons may fruitfully be illustrated in bar graphs, Figures 48-50.

When one observes a sufficiently large population to consider normative values, mean population values and standard errors may be obtained and used for designinf profile graphs for rapid comparison of new classrooms or classroom types with population values. An example of this is given in the profile graph plotted from values of the twelve classrooms mentioned above. In addition to this graph, four profiles indicating the four classroom types are drawn (values in description section, Table 38) and may be used with appropriate caution and due attention to the types of classrooms forming the population. Figures 51-58.

In the triple frame scores, comparisons were made across several classrooms by the Kendall coefficient of rank correlation, W, or between two by Spearman, r_S. An example of the latter applied to triple frame scores appears under Observer Reliability. An example of the former is given in Table 59. One hazard, practically, is that of empty cells in the rankings of one or more observers. While an exact treatment has been described by Benard and Van Elteren, 1953), this is far beyond the level of precision occurring in other aspects of treatment of ranked data. One means of approaching this problem is to apply the W formula in several approximate ways which will provide values above and below the true value, thus permitting approximation. This was illustrated in Project 816 as follows:

i. Use of all triples with mean across the several sets $\geqslant 5$. Where cells in the individual rankings are empty, they should be tied for the final rank regardless of the multiplicity of the tie encountered as if no empty cells or very few.

ii. Elimination of some of the empty cells of (i) by limiting the selection of triples to those with mean $\geqslant 10$. Any remaining cell in any ranking would then be considered as the final rank as in (i).

iii. Only triples occurring in each of the four mean rankings to be used.

The W of (i) would be much inflated; of (ii) somewhat inflated; of (iii) somewhat depressed. A practical indication of the approximate level could then be obtained by examination of the three values.

A useful Index of Initiative may be derived from the categories of the Attitude frame. This is feasible because the three categories of this frame--receptivity, independence and curiosity--may be considered as increasingly indicative of the broad attitude of initiative characterizing the entire frame. A weighting which seems to take far account both of frequency and quality of behaviors and whether behaviors are attributed to teacher or to pupils follows:

	Teacher	Pupils
Curiosity	3	4
Independence	2	3
Receptivity	1	2
Neutral	0	0

For a classroom with representative scores of teacher--25, 540, 153, 21 and of pupil--8, 230, 21, 1, both in the order listed, the Index of Initiative would be

$$1/1{,}000(\ 3.25 + 2.540 + 1.153 + 0.21 + 4.8 + 3.20 + 2.21 + 0.1\ ) = 2.072.$$

References most useful to us in the statistical treatment have been Kendall, (1948), Siegel,(1960) and the tables of Fisher and Yates, (1957).

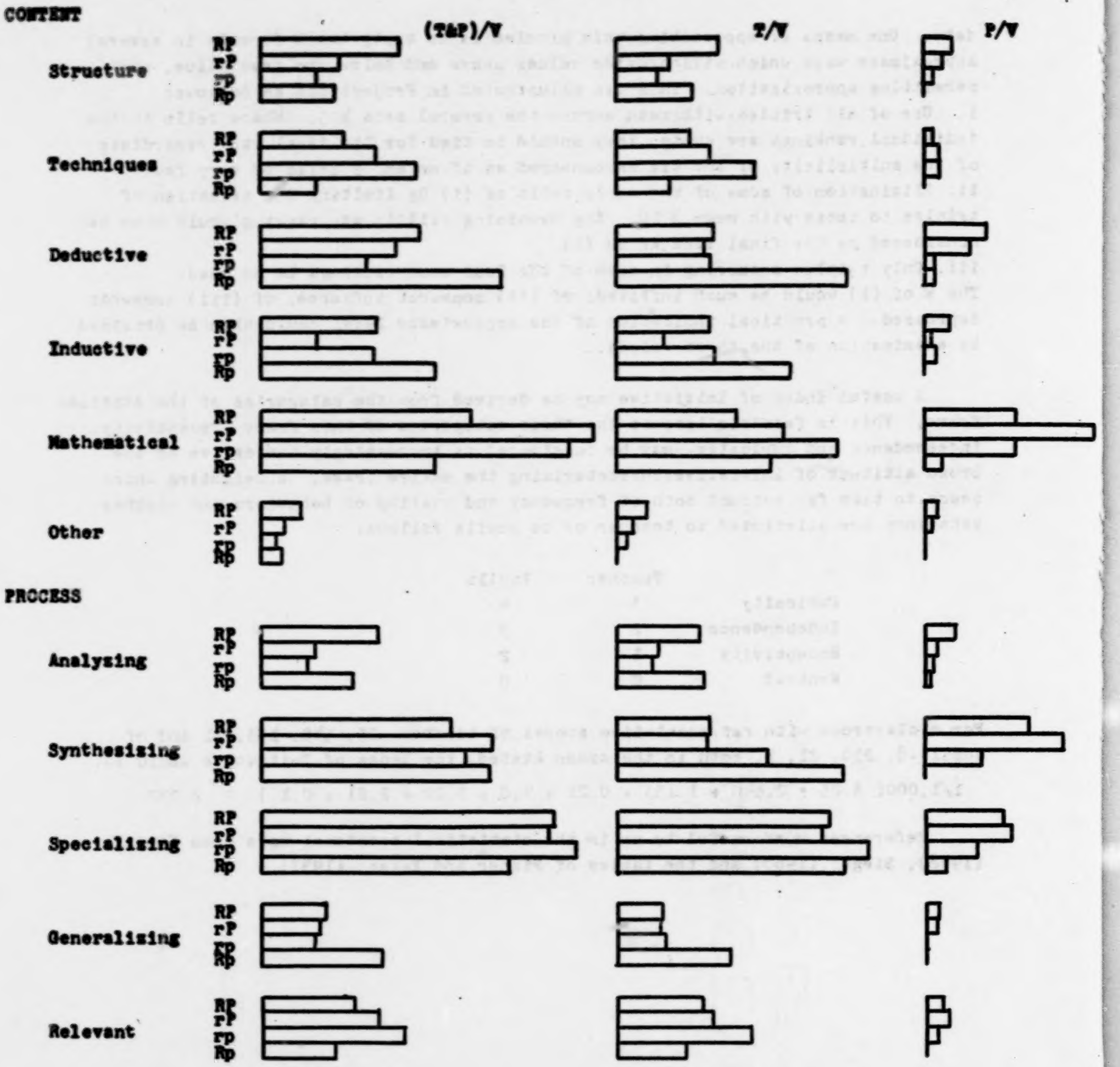


Figure 48 Comparison of single categories in individual frame scores of the four classroom types, RP, rP, rp, Rp. In the Content and Process frames only. Scale as before.

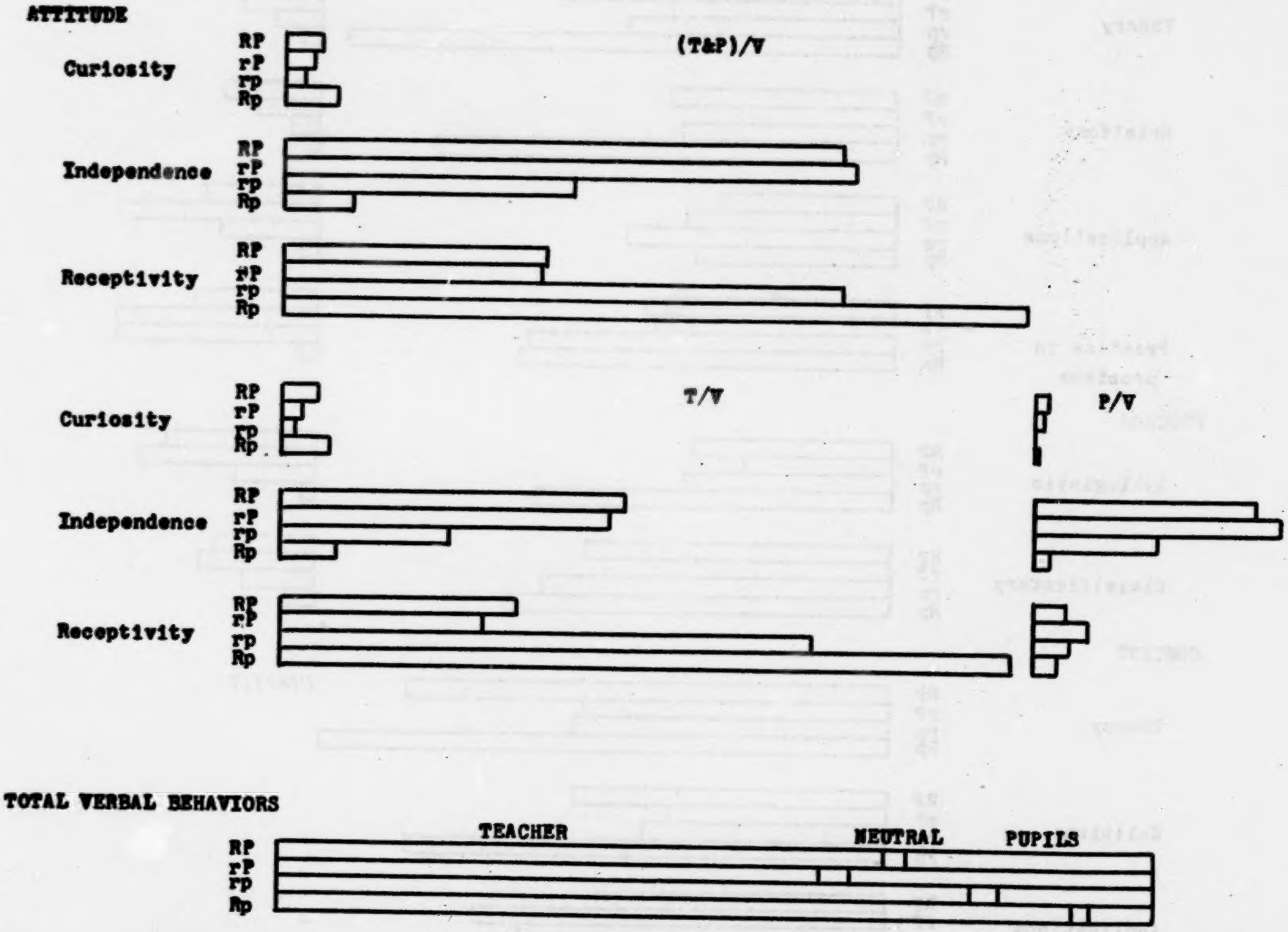


Figure 49 Comparison of single categories in individual frame scores of the four classroom types, Rp, rP, rp, Rp. In the Attitude frame only. Comparison of division of total verbal behaviors.

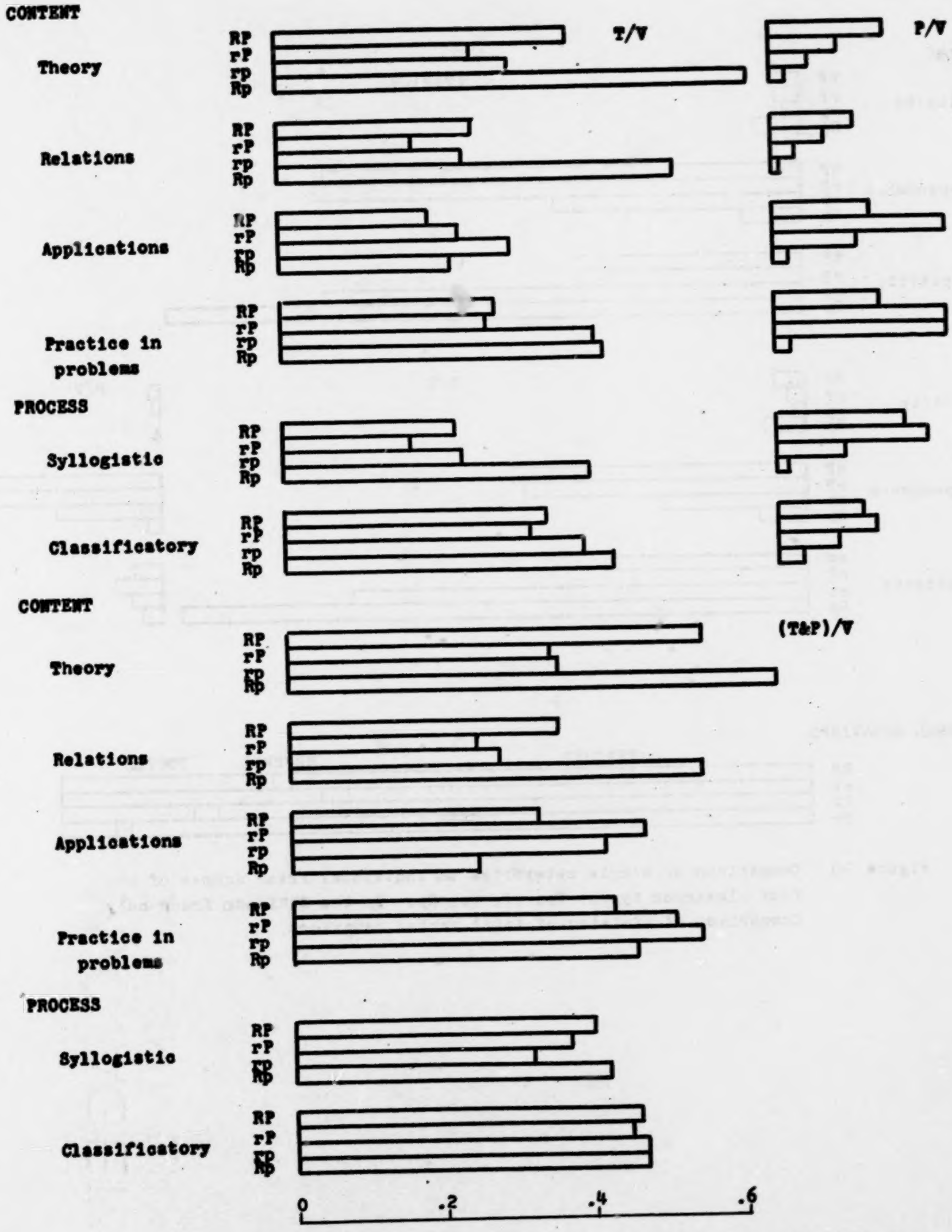


Figure 50 Comparison of combinations of categories in individual frame scores in Content and Process for the four classroom types, RP, rP, rp, Rp.

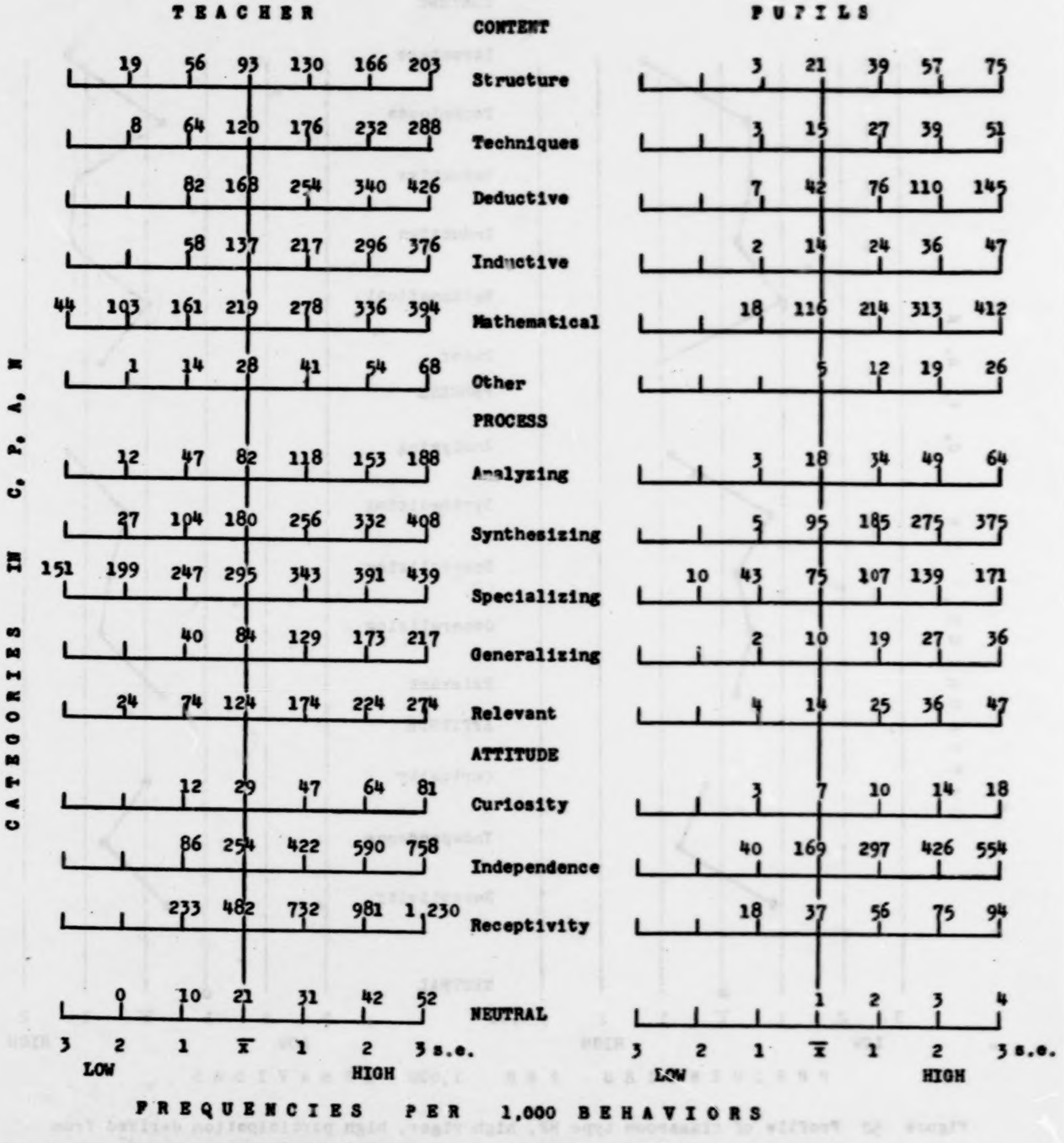


Figure 51 Scales for profile comparison of the frequencies in each category of Content, Process, Attitude and in Neutral of individual teachers and groups of pupils with mean values of twelve algebra classrooms and estimates of population variance.

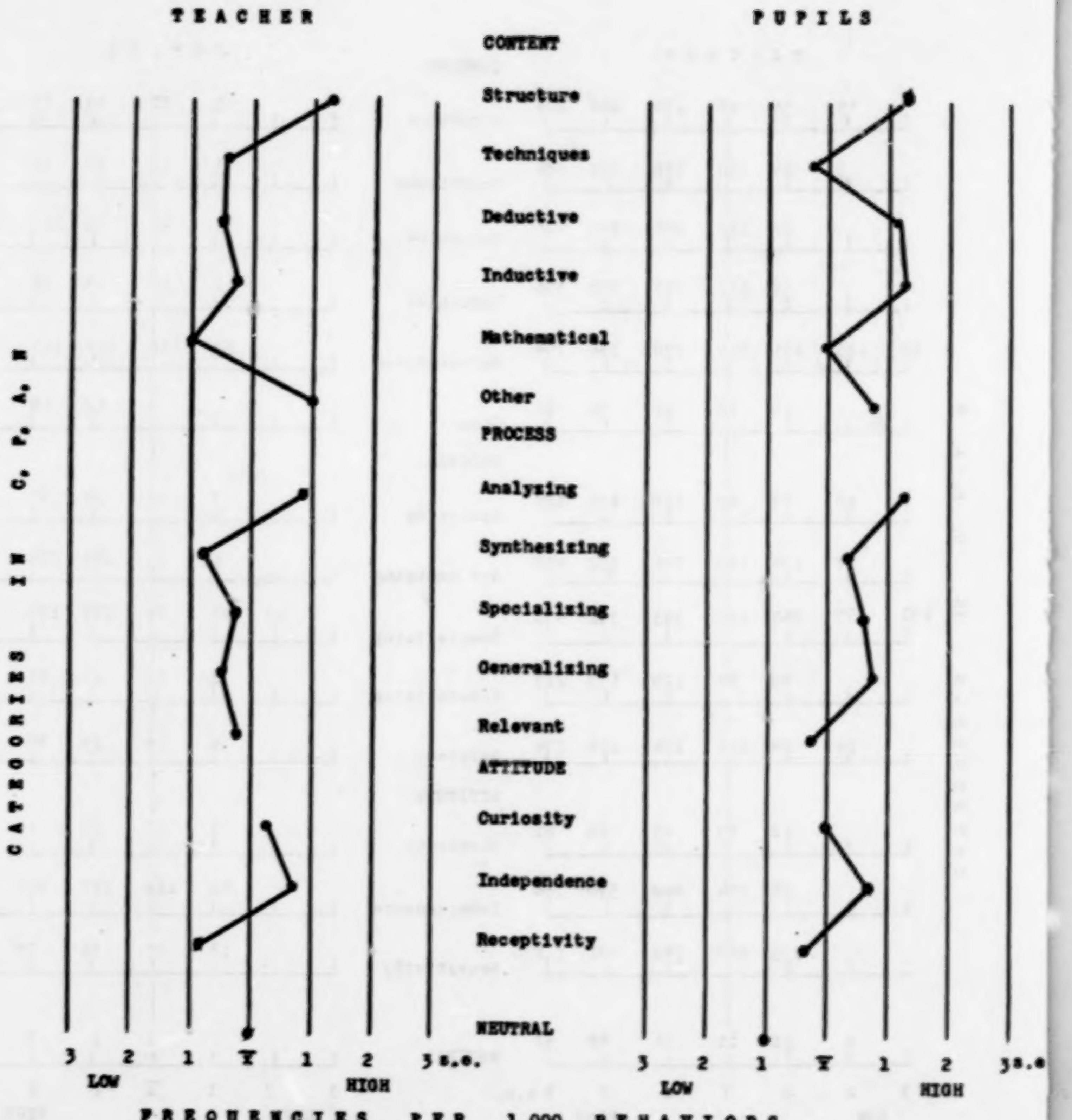


Figure 52 Profile of classroom type RP, high rigor, high participation derived from mean values of the frequencies in each category of Content, Process and Attitude, and in Neutral of three classes. Profile based on mean values of twelve algebra classrooms and estimates of population variance.

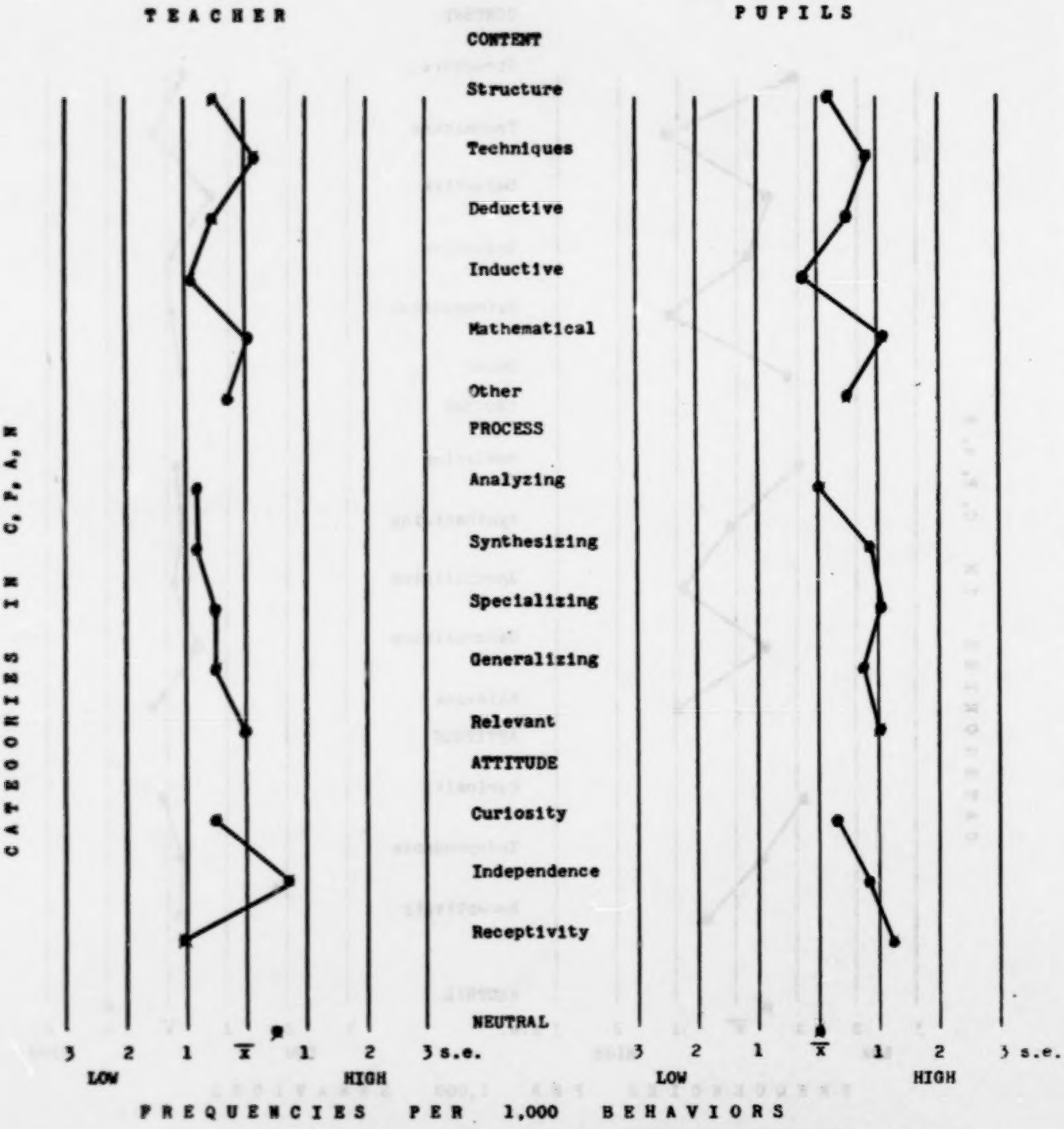


Figure 53 Profile of classroom type rP, low rigor, high participation, derived from mean values of the frequencies in each category of Content, Process and Attitude, and in Neutral of three classes. Profile based on mean values of twelve algebra classrooms and estimates of population variance.

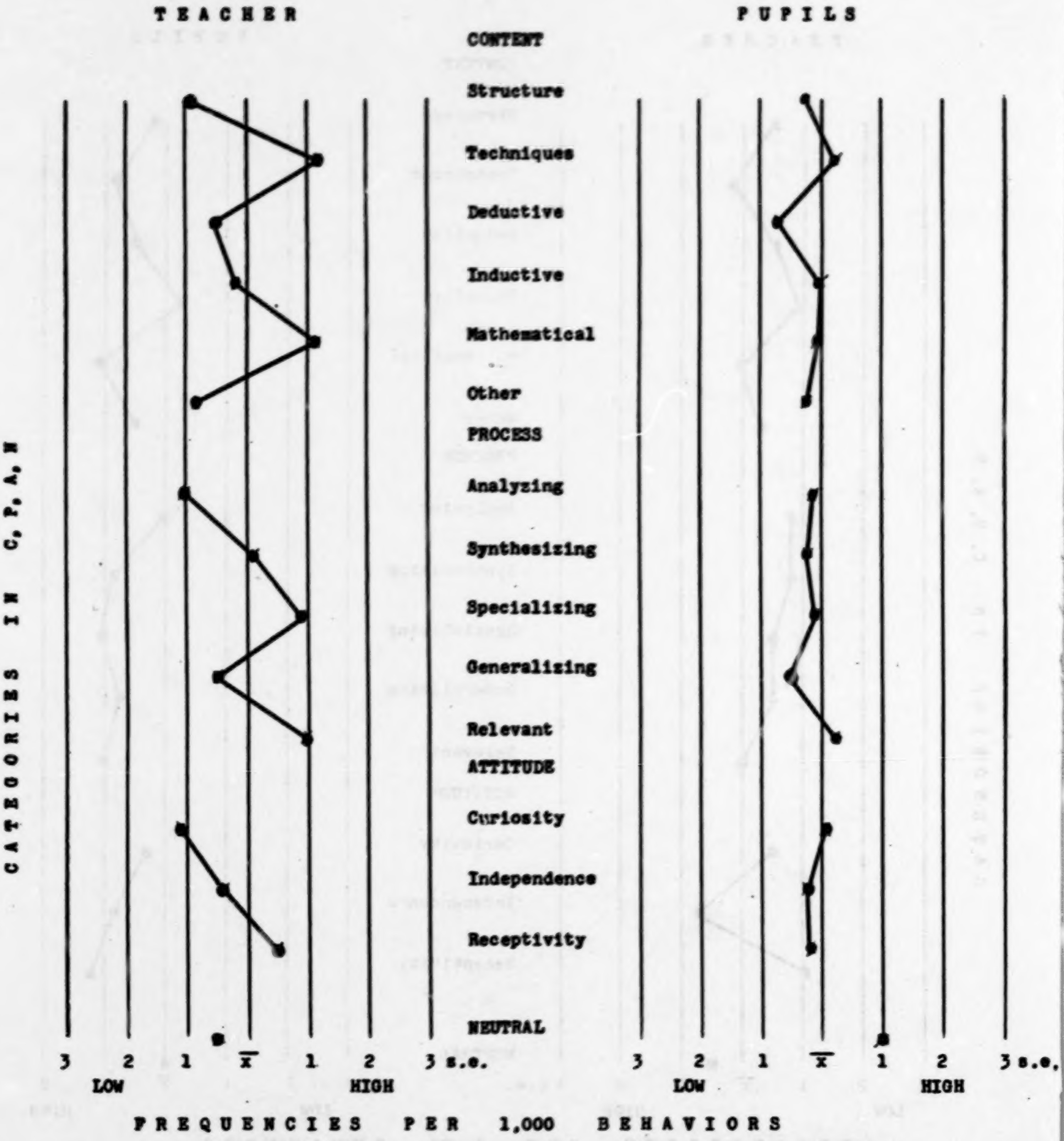


Figure 54 Profile of classroom type rp, low rigor, low participation derived from mean values of the frequencies in each category of Content, Process and Attitude, and in Neutral of three classes. Profile based on mean values of twelve algebra classrooms and estimates of population variance.

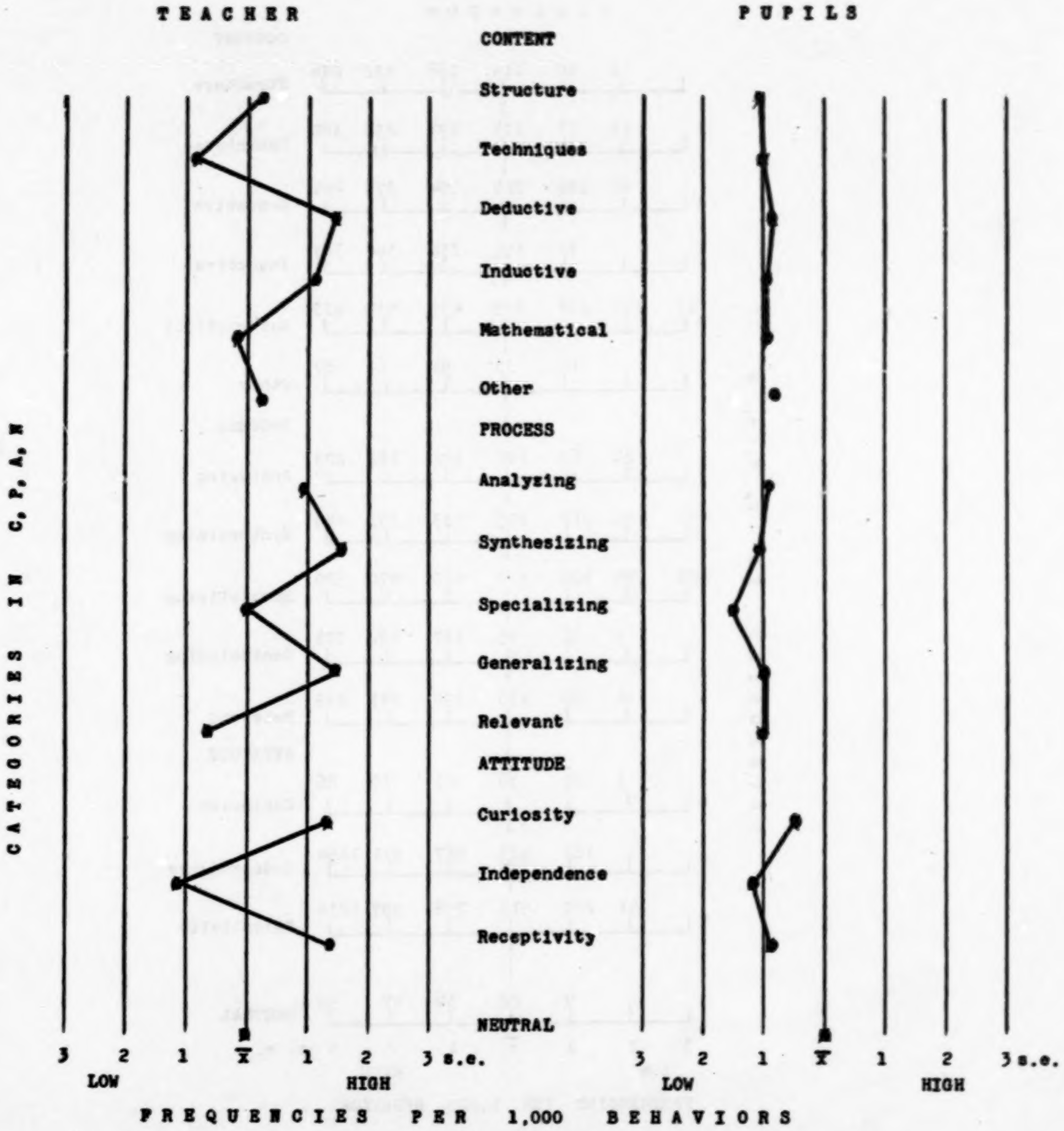


Figure 55 Profile of classroom type Rp, high rigor, low participation derived from mean values of the frequencies in each category of Content, Process and Attitude, and in Neutral of three classes. Profile based on mean values of twelve algebra classrooms and estimates of population variance.

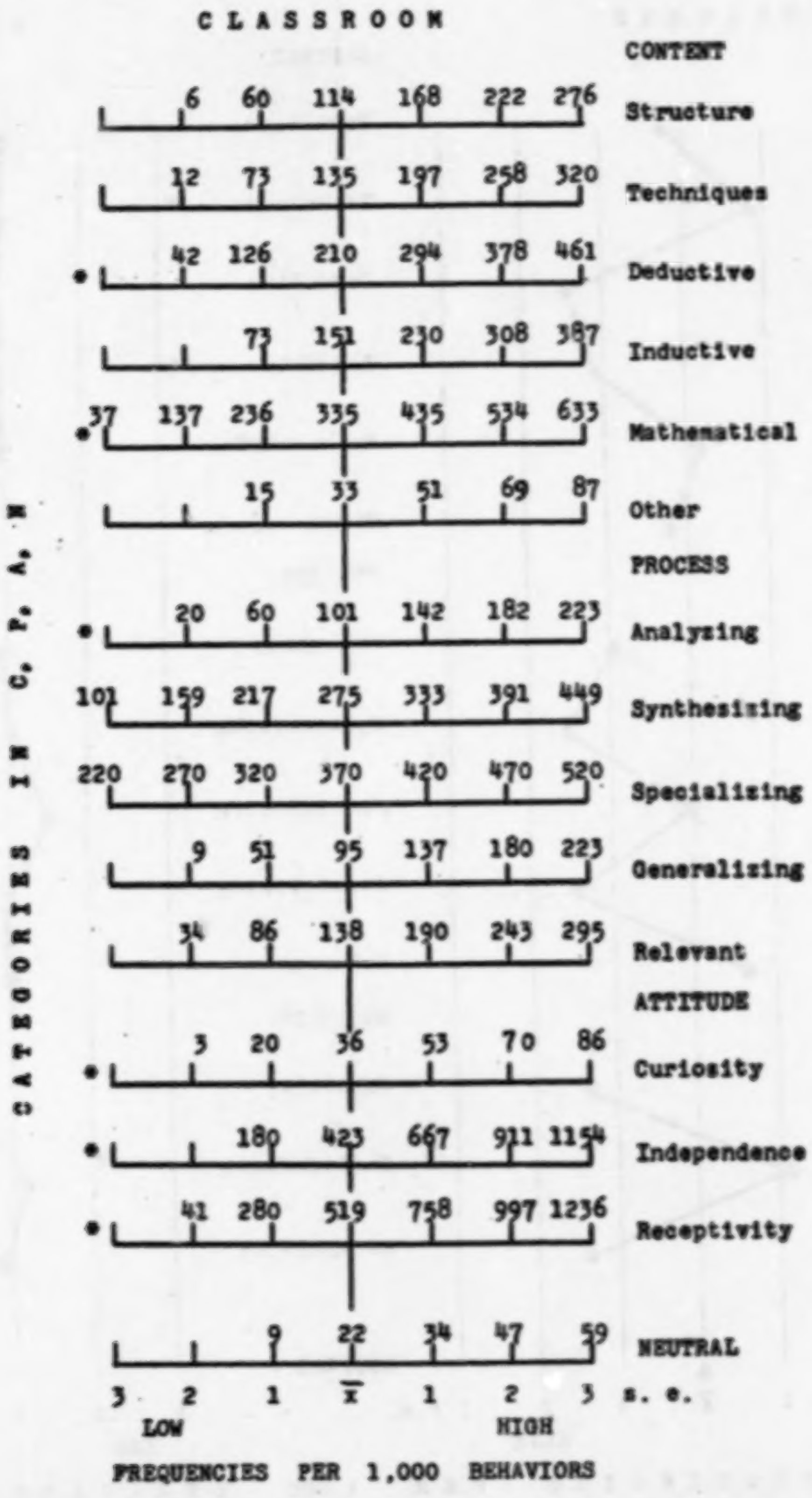


Figure 56 Scale for profile comparison of the frequencies in each category of Content, Process, Attitude and in Neutral of individual classrooms with mean values of twelve algebra classrooms and estimates of population variance.
* Starred categories particularly distinctive in classroom scores.

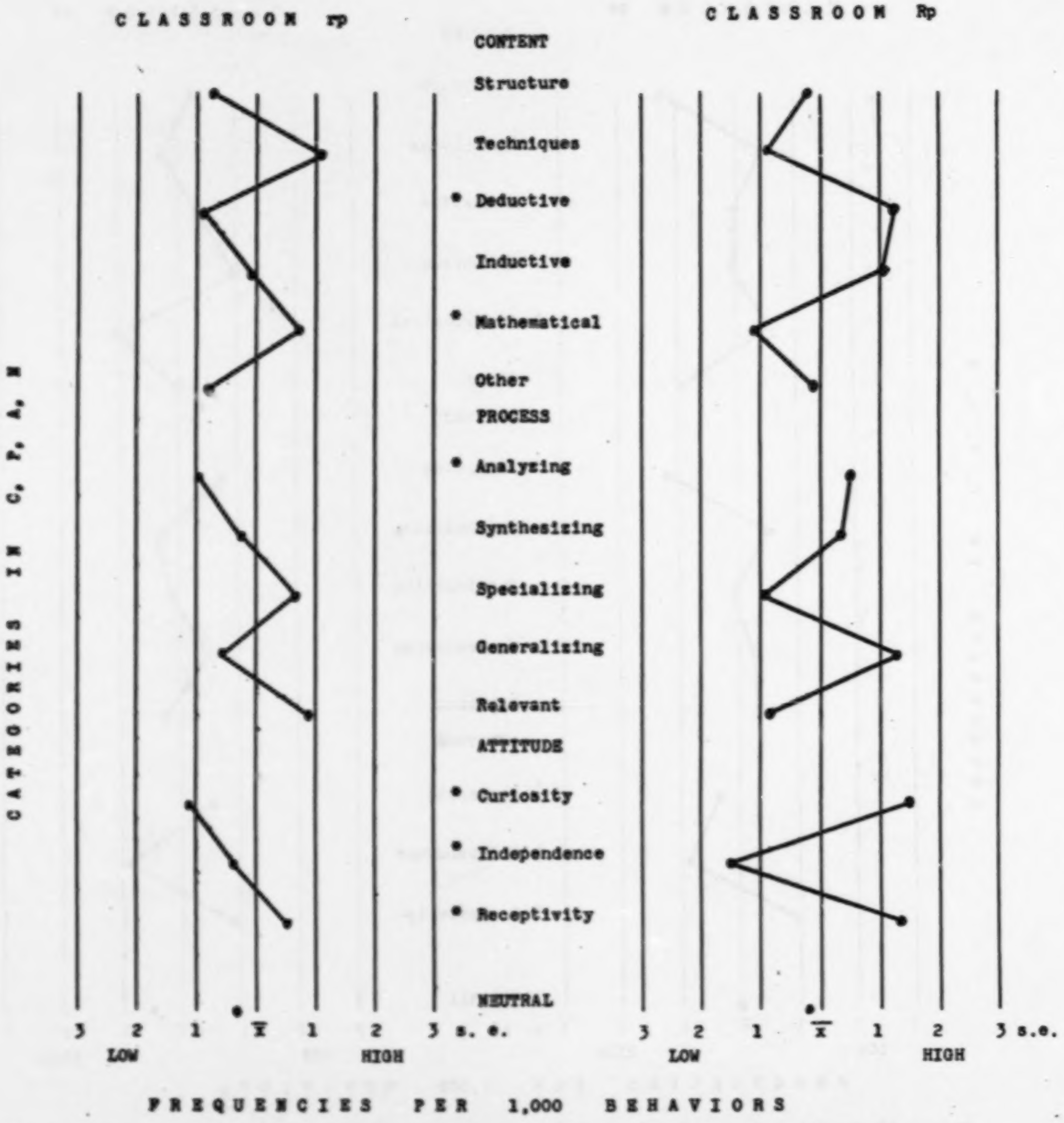


Figure 57 Profile of classroom types rp, low rigor, low participation and Rp, high rigor, low participation derived from mean values of the frequencies in each category of Content, Process and Attitude, and in Neutral of three classes. Profile based on mean classroom values of twelve algebra classrooms and estimates of population variance.
* Starred categories particularly distinctive in classroom types.

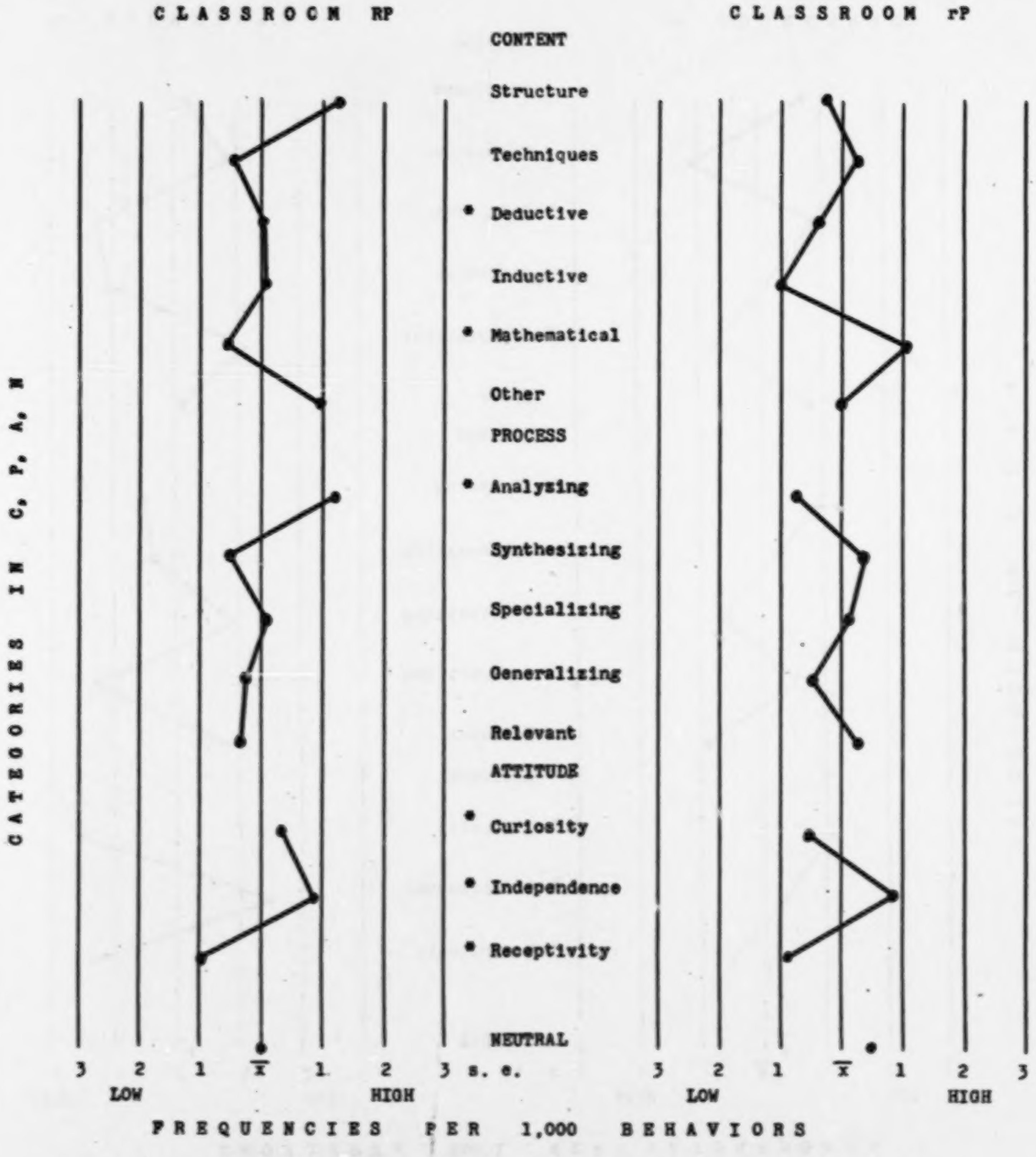


Figure 58 Profile of classroom types RP, high rigor, high participation and rP, low rigor, high participation derived from mean values of the frequencies in each category of Content, Process and Attitude, and in Neutral of three classes. Profile based on mean classroom values of twelve algebra classrooms and estimates of population variance.

* Starred categories particularly distinctive of classroom types.

Table 59 Calculation of Kendall Coefficient of Rank Correlation, W, among the rankings of triple frame scores for the three classrooms of type RP.

Numerical Symbols of Triple scores	Frequencies by Classrooms			Rankings by Classrooms			ΣR	$\lvert R_j - \bar{R}_j \rvert$	$(R - R_j)^2$
	I	XI	XIII	I	XI	XIII			
112	44	42	38	7	6	7½	20½	32	1024
212	31	21	27	10½	15	13	38½	14	196
321	9	13	38	22	21½	7½	51	1½	2¼
612	9	13	23	22	21½	16	59½	7	49
113	5	17	16	26½	17	20	63½	11	121
213	3	12	3	29	23	34	86	33½	1122¼
313	2	5	13	31	30½	21½	83	30½	930¼
122	10	-	7	19½	34	31½	85	32½	1056¼
222	90	77	29	3	3	12	18	34½	1190¼
322	128	116	71	1	1	2	4	48½	2352¼
622	16	30	19	16	11	18	45	7½	56¼
223	8	19	9	24½	16	27½	68	15½	240¼
323	12	27	26	18	13	14	45	7½	56¼
623	-	9	8	33½	26	29½	89	36½	1332¼
431	2	5	10	31	30½	26	87½	35	1225
631	-	8	12	33½	28	24	85½	33	1089
132	57	22	76	5	14	1	20	32½	1056¼
232	50	32	40	6	9½	6	21½	31	961
332	100	38	57	2	7	3	12	40½	1640¼
632	42	72	44	8	4	5	17	35½	1260¼
133	17	16	49	15	18½	4	37½	15	225
233	17	29	20	14	12	17	43	9½	90¼
333	20	43	37	12	5	9½	26½	26	676
433	4	1	12	28	33	24	85	32½	1056¼
533	2	16	6	31	18½	33	82½	30	900
633	5	35	37	26½	8	9½	44	8½	72¼
142	10	9	25	19½	26	15	60½	7	49
242	31	11	13	10½	24	21½	56	3½	12¼
642	15	3	7	17	32	31½	80½	28	784
143	9	9	17	22	26	19	67	14½	210¼
243	8	14	9	24½	20	27½	72	19½	361
451	17	6	8	14	29	29½	72½	20	400
552	63	32	12	4	9½	24	37½	15	225
553	38	92	31	9	2	11	22	30½	930¼
									22,951.75

$$W = \frac{s}{(1/12)K^2(N^3-N)-K\frac{\Sigma T}{T}}$$

$$W = \frac{12\ (22{,}952)}{3^2(34^3-34)-3(186)}$$

$$= 0.78$$

$$\chi^2 = K(N-1)W$$

$s = 22{,}952$

$\chi^2 = 77.2$ df $= 33$

$K = 3$

$\frac{\Sigma T}{T} = \frac{186}{12}$

$p = 0.001$

$N = 34$

TREATMENT OF DATA

SAMPLE STABILITY

If it is desired to confirm the choice of the ten day period of observation in providing a representative sample of classroom behaviors, the stability of the accumulating individual frame scores may be examined as follows. For a particular class, such as classroom I, the teacher and pupil raw scores are accumulated day to day to provide successive samples of behaviors collected throughout the observation period. Noting that each verbal behavior is classified once in each of the three frames or in neutral, the total behavior for each of the successive samples may be calculated by combining teacher and pupil category scores in any one frame and in neutral: $\sum \text{Category}_i + N$ with i from 1 to 7, content; 1 to 5, process; 1 to 3 attitude. Table 61. These ten sets of cumulative raw scores together with the ten successive values of total classroom verbal interaction are used to determine ten corresponding sets of proportions, Table 62. Thus:

$$\begin{array}{l}\text{Proportion 3-day sample}\\ \text{Teacher I--Structure}\end{array} = \frac{\text{Cumulative raw score-structure}}{\text{Cumulative raw verbal total}} = \frac{30.0}{271.5} = 0.110\ (p_3)$$

For each of these sample proportions the corresponding standard error is determined. Table 63. Thus:

$$\begin{array}{l}\text{Standard error-proportion}\\ \text{Teacher I--Structure}\end{array} = \sqrt{\frac{p(1-p)}{n}} = \sqrt{\frac{0.110 \times 0.990}{271.5}} = 0.019$$

These sets of proportions may then be tested for stability. The simple differences of successive pairs of proportions in a single category is obtained and this difference is compared with the standard error of the proportion of the smaller sample. Table 64. Thus:

$$\begin{array}{l}\text{Simple difference proportions}\\ \text{day 3-4, Teacher I--Structure}\end{array} = |p_3 - p_4| = |0.110 - 0.094| = 0.016$$

In this example it is noted that the difference between the two sample values of the category proportion is less than one standard error, an indication of stability. In the Table 64 of simple differences, differences significant at 34, 5 and 1% are noted and starred accordingly. It is noted that the numbers of simple differences significant at the 5% level is small, indicating stability of the sample for classroom I.

If it is desired to reduce the number of days necessary for completing a sample below the ten suggested, this may be considered if the study is limited to the frames process or attitude or if a sufficient number of aspects of a topic have been covered to permit stability of content. In Project 816 this is discussed at some length with the minimum suggested under these conditions as six days.

Triple frame scores may be assessed for stability by comparing rankings accumulated in a partial and a total sample by means of the Spearman coefficient of rank correlation which is illustrated in the examination of observer reliability.

Table 61 Cumulative frequencies in each category of Content, of Process and of Attitude, and in Neutral for Teacher I and Pupils I, derived from ten periods of classification.

DAYS CATEGORY	1	2	3	4	5	6	7	8	9	10
Content					TEACHER I					
Structure	15.0	26.0	30.0	34.0	45.5	45.5	55.5	77.5	92.5	109.5
Techniques	15.0	22.0	30.5	37.5	47.5	56.5	63.5	69.5	73.5	81.5
Deductive	14.0	17.0	36.0	53.5	63.0	73.0	86.0	112.0	150.0	162.5
Inductive	0.0	12.0	20.5	31.0	37.0	39.0	53.0	55.0	56.0	61.0
Mathematical	18.5	40.5	62.0	82.5	100.0	107.0	126.5	136.5	153.5	170.5
Other	0.0	0.0	0.0	3.0	11.0	22.0	28.5	30.5	30.5	30.5
Process										
Analyzing	7.5	15.5	21.0	26.5	36.0	37.0	49.0	61.0	76.0	80.0
Synthesizing	12.5	27.5	47.0	58.5	63.5	69.5	76.0	104.5	129.5	138.5
Specializing	16.5	31.5	52.0	78.5	109.0	115.0	149.5	167.5	186.5	220.5
Generalizing	8.5	11.5	18.5	27.0	30.0	35.0	45.5	47.0	56.0	59.0
Relevant	17.5	32.5	41.5	53.0	67.5	88.5	98.0	108.0	116.0	125.0
Attitude										
Curiosity	1.0	1.0	1.0	4.5	8.5	15.5	18.5	22.5	22.5	22.5
Independence	48.5	96.5	136.5	181.5	231.5	256.5	311.5	362.5	420.5	468.5
Receptivity	13.0	21.0	42.5	57.5	66.0	73.0	88.0	103.0	121.0	133.0
Neutral	2.0	11.0	11.0	12.5	12.5	12.5	13.5	15.0	15.0	18.5
Content					PUPILS I					
Structure	7.5	7.5	9.5	9.5	11.5	11.5	14.0	23.5	26.5	30.5
Techniques	0.5	1.5	2.5	4.0	4.0	5.0	6.5	6.5	8.5	12.5
Deductive	2.0	2.0	8.0	22.5	26.5	28.5	32.5	39.5	46.5	51.0
Inductive	0.0	7.0	7.0	7.5	9.0	11.0	16.0	16.0	16.0	17.0
Mathematical	13.5	28.5	46.5	54.5	66.5	70.5	73.5	75.0	76.0	89.5
Other	0.0	0.0	0.0	0.0	1.0	1.0	1.0	1.0	1.0	1.0
Process										
Analyzing	1.5	2.5	4.0	5.5	6.5	6.5	7.5	10.0	14.0	15.0
Synthesizing	16.0	30.0	47.0	57.5	66.0	71.0	75.0	83.5	88.5	95.5
Specializing	4.0	13.0	20.0	28.0	40.5	43.5	51.5	59.5	63.5	79.5
Generalizing	3.5	3.5	5.0	9.0	10.5	11.5	15.5	16.5	18.5	19.5
Relevant	1.0	3.0	5.0	7.5	7.5	9.5	10.5	11.0	12.0	16.0
Attitude										
Curiosity	1.0	1.0	1.0	4.5	6.5	6.5	7.0	7.0	7.0	7.0
Independence	22.0	46.0	71.5	92.0	112.5	122.5	140.0	159.5	175.5	200.0
Receptivity	3.0	5.0	8.5	11.0	12.0	13.0	13.0	14.0	14.0	18.5
Neutral	0.0	0.0	0.0	0.0	0.5	0.5	0.5	0.5	0.5	0.5
Total T & P	90.5	181.5	271.5	363.5	449.5	499.5	591.5	683.5	775.5	867.5

Fractional scores result from averaging joint classification by two observers.

Table 62 Cumulative proportions in each category of Content, of Process and of Attitude, and in Neutral for Teacher I and Pupils I, derived from Table 61. All decimals omitted in proportions.

DAYS CATEGORY	1	2	3	4	5	6	7	8	9	10
Content					TEACHER I					
Structure	166	143	110	094	101	091	094	113	119	126
Techniques	166	121	112	103	106	113	107	102	095	094
Deductive	155	094	133	147	140	146	145	164	193	187
Inductive	---	066	076	085	082	078	090	080	072	070
Mathematical	204	223	228	227	222	214	214	200	198	197
Other	---	---	---	008	.024	044	048	045	039	035
Process										
Analyzing	083	085	077	073	080	074	083	089	098	092
Synthesizing	138	152	173	161	141	139	128	153	167	160
Specializing	182	174	192	216	242	230	253	245	240	254
Generalizing	094	063	068	074	067	070	077	069	072	068
Relevant	193	179	153	146	150	177	166	158	150	144
Attitude										
Curiosity	011	006	004	012	019	031	031	033	029	026
Independence	536	532	503	499	515	514	527	530	542	540
Receptivity	144	116	157	158	147	146	149	151	156	153
Neutral	022	061	041	034	028	025	023	022	019	021
Content					PUPILS I					
Structure	083	041	035	026	026	023	024	034	034	035
Techniques	006	008	009	011	009	010	011	010	011	014
Deductive	022	011	029	062	059	057	055	058	060	059
Inductive	---	039	026	021	020	022	027	023	021	020
Mathematical	149	157	171	150	148	141	124	110	098	103
Other	---	---	---	---	002	002	002	001	001	001
Process										
Analyzing	017	014	015	015	014	013	013	015	018	017
Synthesizing	177	165	173	158	147	142	127	122	114	110
Specializing	044	072	074	077	090	087	087	087	082	092
Generalizing	039	019	018	025	023	023	026	024	024	022
Relevant	011	017	018	021	017	019	018	016	015	018
Attitude										
Curiosity	011	006	004	012	014	013	012	010	009	008
Independence	243	253	263	253	250	245	237	233	226	231
Receptivity	033	028	031	030	027	026	022	020	018	021
Neutral	---	---	---	001	001	001	001	001	001	001

Total T & P on which proportions based:

90.5	181.5	271.5	363.5	449.5	499.5	591.5	683.5	775.5	867.5

In each sample if teacher and pupil proportions combined in each category, then

$$\sum C_1 + N = \sum P_1 + N = \sum A_1 + N = 1{,}000$$

Table 63 Standard errors of cumulative proportions in each category of Content, Process and Attitude, and in Neutral for Teacher I and Pupils I, derived from Table 62. All decimals omitted.

DAYS CATEGORY	1	2	3	4	5	6	7	8	9	10
					TEACHER I					
Content										
Structure	039	026	019	015	014	013	012	012	012	011
Techniques	039	024	019	016	015	014	013	012	011	010
Deductive	038	022	021	019	016	016	014	014	014	013
Inductive	---	018	016	015	013	012	012	010	009	009
Mathematical	042	031	025	022	020	018	017	015	014	014
Other	000	000	000	005	007	009	009	008	007	006
Process										
Analyzing	029	021	016	014	013	012	011	011	011	010
Synthesizing	036	027	023	019	016	015	014	014	013	012
Specializing	041	028	024	022	020	019	018	016	015	015
Generalizing	031	018	015	014	012	011	011	010	009	009
Relevant	041	028	022	019	017	017	015	014	013	012
Attitude										
Curiosity	011	006	004	006	006	008	007	007	006	005
Independence	052	037	030	026	024	022	021	019	018	017
Receptivity	037	024	022	019	017	016	015	014	013	012
Neutral	015	018	012	010	008	007	006	006	005	005
					PUPILS I					
Content										
Structure	029	015	011	008	008	007	006	007	007	006
Techniques	008	007	006	005	004	004	004	004	004	004
Deductive	015	008	010	013	011	010	009	009	009	008
Inductive	---	014	010	008	007	007	007	006	005	005
Mathematical	037	027	023	019	017	016	014	012	011	010
Other	---	---	---	---	002	002	002	001	001	001
Process										
Analyzing	014	009	007	006	006	005	005	005	005	004
Synthesizing	040	028	023	019	017	016	014	013	011	011
Specializing	022	019	016	004	013	013	012	011	010	010
Generalizing	020	010	008	008	007	007	007	006	005	005
Relevant	011	010	008	008	006	006	005	005	004	005
Attitude										
Curiosity	011	006	004	006	006	005	004	004	003	003
Independence	045	032	027	023	020	019	017	016	015	014
Receptivity	019	012	011	009	008	007	006	005	005	005
Neutral	---	---	---	002	001	001	001	001	001	001

Table 64 Simple differences of successive cumulative proportions in each category of Content, of Process and of Attitude, and in Neutral for Teacher I and Pupils I. Absoluted values, decimals omitted.

DAYS CATEGORY	1-2	2-3	3-4	4-5	5-6	6-7	7-8	8-9	9-10
Content				TEACHER	I				
Structure	023	033*	016	007	010	003	019*	006	007
Techniques	045*	009	009	003	007	006	005	007	001
Deductive	061*	029*	014	007	006	001	019*	029**	006
Inductive	066	010	009	003	004	012	010	008	002
Mathematical	019	005	001	005	007	000	014	002	001
Other	---	---	008	016***	020**	004	003	006	004
Process									
Analyzing	002	008	004	007	006	009	006	009	006
Synthesizing	014	021	012	020*	002	011	025*	014	007
Specializing	008	019	024	026*	012	027*	008	005	014
Generalizing	031	005	006	007	003	007	008	003	004
Relevant	014	026	007	004	027*	011	008	008	006
Attitude									
Curiosity	005	002	008*	007	012**	000	002	004	003
Independence	004	029	004	016	001	013	003	012	002
Receptivity	028	041*	001	011	001	003	002	005	003
Neutral	039***	020*	007	006	003	002	001	003	002
Content				PUPILS	I				
Structure	042*	006	009	000	003	001	010	000	001
Techniques	002	001	002	002	001	001	001	001	003
Deductive	011	014*	033***	003	002	002	003	002	001
Inductive	039	013	005	001	002	005	004	002	001
Mathematical	008	014	021	002	007	017*	014	012	005
Other	---	---	---	002	000	000	001	000	000
Process									
Analyzing	003	001	000	001	001	000	002	003	001
Synthesizing	012	008	015	011	005	015	005	008	004
Specializing	028*	002	003	023*	003	000	000	005	010
Generalizing	020	001	007	002	000	003	002	000	002
Relevant	006	001	003	004	002	001	002	001	003
Attitude									
Curiosity	005	002	008**	002	001	001	002	001	001
Independence	010	010	010	003	005	008	004	003	005
Receptivity	006	003	001	003	001	004	002	002	003
Neutral	---	---	001	000	000	000	000	000	000

---

* Differences between 1 and 2 s. e.

** Differences between 2 and 3 s. e.

*** Differences greater than 3 s. e.

TREATMENT OF DATA

OBSERVER RELIABILITY

Observer reliability will be discussed in terms of the testing of level of agreement in the individual frame and triple frame scores, the two types used in this instrument. In addition, a method for detailed examination of cross-classification by two observers to test agreement on very large samples of behaviors and to permit examination of categories susceptible to misinterpretation or bias will be presented.

Individual frame scores

To facilitate this discussion, a matrix of cross classification by two observers of behaviors collected for the teacher component in classroom I is provided, Table 67a.

If a rapid check of agreement is desired--during a study or training period--the use of binomial probability paper in plotting total single category values (the individual frame scores) is suggested. An appropriate significance level, e. g. $p = 0.05$ may be chosen. This is illustrated in Figure 68. With two experienced observers it is assumed that the split will be 50-50 and in the total of 162 behaviors classified by the pair in the structure category that each will record 81 behaviors in this category; similarly in techniques, with a total of 116 behaviors classified by the pair, it would be expected that with perfect agreement 58 would be recorded by each observer. In these two cases the actual numbers recorded were: O_1-79, O_2-83 for structure; O_1-49, O_2-67 for techniques. The expected 50-50 split may be indicated on graph paper by a line drawn at an angle of 45 degrees. The binomial paper is designed not only to facilitate the drawing of a line at an appropriate expectancy angle, but in addition permits determination of the distance of each plotted point from this line by drawing lines parallel to the line of split 1, 2 and 3 s. e. away. The plotting of the agreement of the seven single categories of Content in Figure 68 indicates no differences significant at the 5% level; indeed 5 of the seven are within 1 s. e. of the line of perfect agreement. This excellent paper was designed by Mosteller and Tukey (1949).

A second method, also recommended in much of the literature, is that of calculating simple percentage agreement. In our case we find this of much less value than the plotting described above; and than the use of chi-square described below, because of the ready means by these methods of determining the level of significance of the differences. In particular, the total n's vary so much in single categories during successive observations that comparability is limited sharply. If percentage comparison is chosen, Flander's treatment (1960) is suggested. In Table 67b, percentage agreement of Observers 1 and 2 in classification of Content in classroom I is recorded. The percentages determined as follows:

Percentage agreement, Teacher I, Content--structure: $\frac{2 \times 58}{79 \quad 83} \times 100 = 72$

Percentage agreement, Teacher I, Content--entire frame: $\frac{313}{440} \times 100 = 71$

Bales (1950) suggested a minimum of 50% agreement even in categories involving considerable inference on the part of the observer. It is noted that in this frame a level of 70% was maintained for Teacher I scores.

A third method we recommend is the comparison of the totals of the set of single categories across an entire frame. This comparison may be made by chi-square because we have established the independence of the single categories from each other. This comparison is illustrated in Table 67c. The number of degrees of freedom for this analysis is $k(r - 1)$ where k represents the number of column (in this case 2); and r the number of rows (7). Because the observers necessarily view the same number of behaviors during joint classification, the column totals are fixed and equal. However, in moving successive random samples of behaviors for joint classification where the row totals for individual categories cannot be expected to remain the same, no further restriction is required. This differs from the more customary (k - 1)(r - 1). For Teacher I, Content, a X^2 of 5.35 with df of 12 is determined. The probability of occurrence of such a value is 0.99 (Fisher and Yates, 1957, p 45). Thus a null hypothesis would be retained and association of the category totals of the two observers is concluded.

During training periods, when disagreement may be very marked in one or two categories, assessment of agreement might usefully be made category by category using the chi-square--similar to single category comparison discussion earlier in the comparison section. This would provide numberical probability values corresponding to the graphic check with binomial probability paper.

Triple frame scores

An example of a test for agreement between classifications of triple frame scores by two observers is given in Table 69. Using triple frame scores where mean observer classification is $\geqslant 5$, these scores are ranked for each observer separately. The two sets of ranks are compared by means of the Spearman coefficient of rank correlation, r_S. For the case where ties occur:

$$r_S = 1 - \frac{S(d)^2}{1/6(n^3-n) - (T^1+U^1)}$$

$S(d)^2$--sum squared deviations
n --number of ranks
T^1 --$\sum_t 1/12(t^3-t)$ ties in rank one
U^1 -- as above, ties in rank two
Kendall, 1948, p 39: 3.10

In the case of Teacher I, with n equal to 21, the r_S correlation between the rankings was 0.86. From Siegel, 1960, p 284, it is noted that a value as large as this is significant at the $p < 0.01$ level. Thus the null hypothesis, that the rankings of the two observers are unrelated is rejected and an association between triple frame classification of the two observers for Teacher I is concluded, $p < 0.01$. On the other hand, for Pupils I, with n equal to 6, r_S to 0.66, the probability of occurrence of an r of this size under H_0 if 0.07 (Kendall, 1948, p 172). Here the null hypothesis is retained and no association between the triple frame classification of the two observers for Pupils I can be concluded, $p = 0.05$.

If the number of observers exceeds two, the test of level of agreement may be extended in each of the above examples.

Table 67a Matrix of cross classification by Observer 1 (column) and Observer 2 (row) of 440 individual behaviors from classroom I in the Content frame for Teacher I components only.

Content	S	T	D	I	M	O	N	Total
Structure	58	2	9	5	8	1	0	83
Techniques	1	39	9	0	12	5	1	67
Deductive	9	0	75	3	5	0	0	92
Inductive	1	1	10	30	3	0	1	46
Mathematical	8	6	9	5	88	1	1	118
Other	2	0	0	0	1	14	0	17
Neutral	0	1	0	1	2	0	11	15
Total	79	49	112	44	119	21	14	438

Table 67b Percentages of agreement, and corresponding frequencies of Observers 1 and 2 in the single categories of Content, and across the entire frame from the 440 behaviors from classroom I for Teacher I components only.

	S	T	D	I	M	O	N	Entire
Percentage	72	67	73	67	73	74	76	71
Frequency	82	58	102	45	119½	19	14½	438

Table 67c X^2 comparison of totals of behaviors classified by Observer 1 and 2 within the Content frame for Teacher I.

Content	O_1	O_2	$O_1 - O_2$	E	$\|O - E\|$	$\|O - E\|^2$	$\|O - E\|^2/E$
Structure	79	83	162	81	2	4	0.049
Techniques	49	67	116	58	9	81	1.397
Deductive	112	92	204	102	10	100	0.980
Inductive	44	46	90	45	1	1	0.022
Mathematical	119	118	237	118.5	0.5	0.25	0.002
Other	21	17	38	19	2	4	0.210
Neutral	14	15	29	14.5	0.5	0.25	0.017
							for each observer

$X^2 = 2(2.677) = 5.35$; $df = 2(7 - 1) = 12$

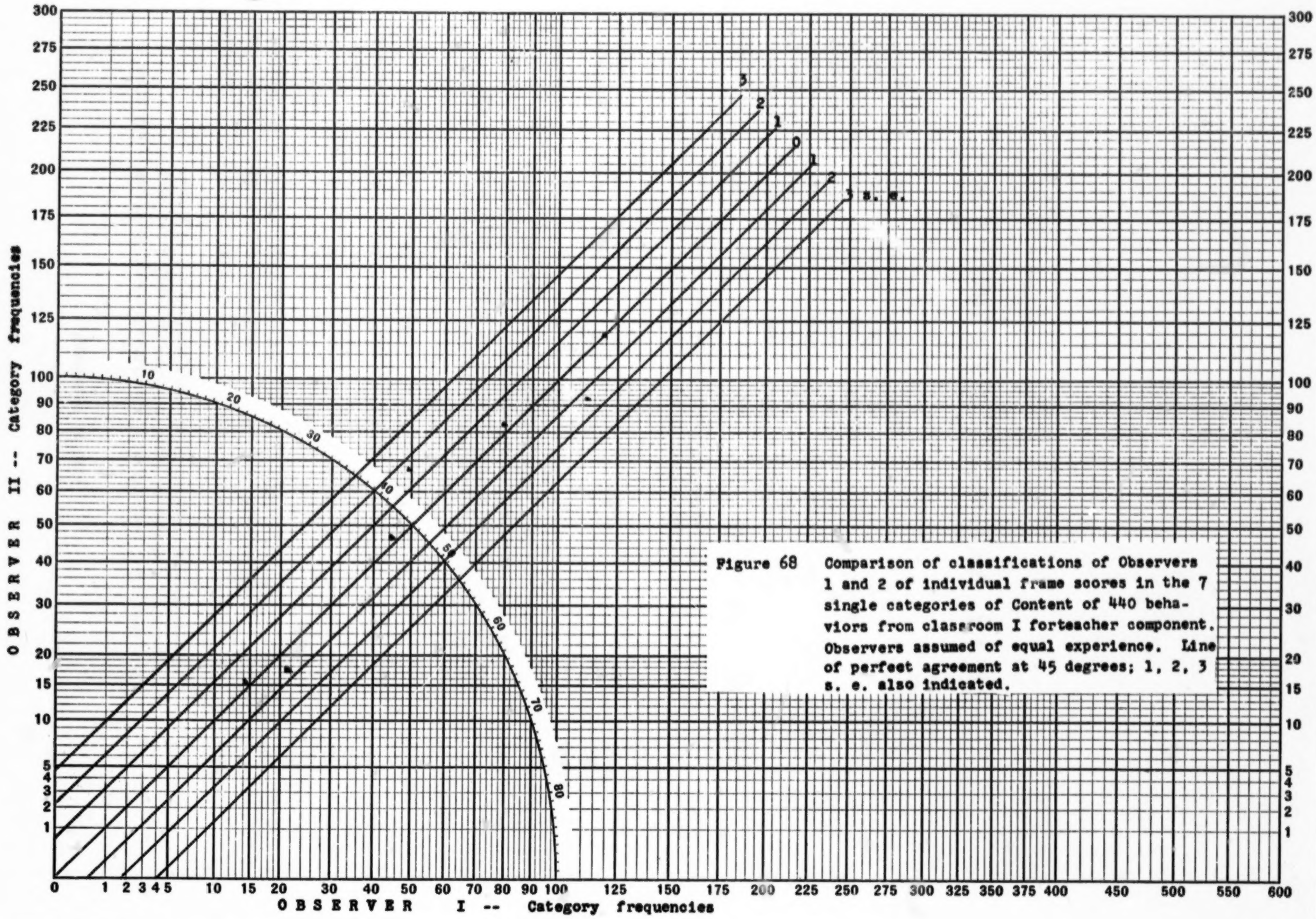


Figure 68 Comparison of classifications of Observers 1 and 2 of individual frame scores in the 7 single categories of Content of 440 behaviors from classroom I forteacher component. Observers assumed of equal experience. Line of perfect agreement at 45 degrees; 1, 2, 3 s. e. also indicated.

Table 69 Calculation of Spearman Coefficient of Correlation, r_s, between the rankings of triple frame scores for Observer 1 and 2, Teacher I and Pupils I.

Numerical Symbolism of triple frame scores in Content, Process and Attitude, respectively.	Observer I		Observer II		$\lvert d \rvert$	d^2
	frequency	rank	frequency	rank		
TEACHER I						
332	60	1	54	1	0	0.00
322	37	2	36	3	1	1.00
552	31	4	41	2	2	4.00
222	33	3	35	4	1	1.00
132	26	7	27	6	1	1.00
232	27	5.5	23	7	1.5	2.25
632	27	5.5	20	9.5	4	16.00
112	22	8	20	9.5	1.5	2.25
553	15	11	28	5	6	36.00
212	21	9	13	11.5	2.5	6.25
333	7	15.5	21	8	7.5	56.25
242	16	10	8	15	5	25.00
133	12	12	13	11.5	0.5	0.25
233	8	13.5	10	13	0.5	0.25
642	8	13.5	9	14	0.5	0.25
622	6	17	4	20	3	9.00
223	5	18.5	5	19	0.5	0.25
312	7	15.5	3	21	5.5	30.25
612	4	20.5	7	16	4.5	20.25
143	5	18.5	6	17.5	1.0	1.00
122	4	20.5	6	17.5	3.0	9.00
PUPILS I						
322	38	1	45	1	0	0.00
332	31	2	17	4	2	4.00
222	23	3	21	2.5	0.5	0.25
132	11	4.5	21	2.5	2	4.00
232	11	4.5	5	6	1.5	2.25
242	7	6	7	5	1	1.00

Teacher I:

$S(d^2)$--221.5

n--21

T^1--2.5

U^1--1.5

$$r_s = 1 - \frac{221.5}{1539.5} = 0.86$$

$p < 0.01$
significant

Pupils I:

$S(d^2)$--11.5

n--6

T^1--0.5

U^1--0.5

$$r_s = 1 - \frac{11.5}{34.0} = 0.66$$

$p = 0.07$

not significant, 0.05.

TREATMENT OF DATA

Cross-classification

When large aggregates of teacher and pupils behaviors, jointly classified, are available, a comprehensive test of observer agreement in the individual categories may be made by X^2 analysis applied to the entire matrix of cross classifications. If desired the relationship may be expressed as a coefficient of Contingency. The significance of the association may be inferred by comparison of the coefficient of contingency (actual) with the maximum possible, or may be determined by noting the probability of the occurrence of the X^2 value. An example of comprehensive matrixes examined in this way occurs in Table 71. Included are matrixes for teacher and pupils components in each of the three frames of reference. X^2, Coefficients of Contingency, actual and maximum are included. The necessary formulae for these calculations are:

$$X^2 = \Sigma(O - E)^2/E \quad \text{or a useful modification} \quad n\left(\Sigma\frac{n_{ij}^2}{n_i n_j} - 1\right)$$

O-observed frequency
E-expected frequency

n-overall total
n_{ij}--cell ij
n_i, n_j--total row or column

$$C = \sqrt{\frac{X^2}{n + X^2}} \quad \text{(actual)} \qquad C = \sqrt{\frac{t - 1}{t}} \quad \text{(maximum in square table, t rows)}$$

In all of the matrixes in Table 71, the probability of occurrence of X^2 of the given sizes was, in each case, <0.01.

In addition to the numerical indexes of agreement, close examination of the variation within the several cells of the tables pointed up categories where observer disagreement is concentrated. This may be random or may result from inadequate definition of categories, or may be caused by careless use of particular categories by an observer, or may indicate observer bias. This particular example is discussed in some length in Project 816. However, comment on the Attitude frame is included here. In the categories of Attitude, only one area of disagreement is noted: between receptivity and independence. This is different in the teacher and pupils matrixes. A bias of Observer 1 towards independence or Observer 2 towards receptivity in the teacher behaviors is apparent. A slight bias in the opposite direction is noted in the pupils behaviors.

Table 71 X^2 comparisons and C coefficients of Contingency from joint classification by Observer 1 (column) and Observer 2 (row) of 7735 behaviors from 14 classrooms in the Content, Process and Attitude frames. Teacher and Pupils scores.

TEACHER

Content	S	T	D	I	M	O	N	Total
Structure	582	19	75	52	83	6	0	817
Techniques	29	732	57	37	118	16	26	1015
Deductive	53	29	719	85	65	4	2	957
Inductive	26	11	49	503	37	10	1	637
Mathematical	96	90	82	59	1230	7	17	1581
Other	11	11	3	2	33	164	2	226
Neutral	3	16	2	5	11	24	891	930
Total	800	908	987	743	1577	209	939	6163

X^2 20.64 x 10^3

C_{Actual} 0.88

$C_{Maximum}$ 0.92

Process	A	Sy	Sp	G	R	N	Total
Analyzing	292	62	146	18	10	3	531
Synthesizing	31	713	256	33	20	12	1065
Specializing	127	279	1520	106	139	3	2174
Generalizing	35	35	100	296	22	1	479
Relevant	13	13	117	27	771	29	984
Neutral	1	1	16	1	17	892	930
Total	499	1109	2155	481	979	940	6163

X^2 11.42 x 10^3

C_{Actual} 0.81

$C_{Maximum}$ 0.91

Attitude	C	I	R	N	Total
Curiosity	117	31	27	0	175
Independence	19	1891	285	10	2205
Receptivity	25	454	2336	38	2853
Neutral	1	10	28	891	930
Total	162	2386	2676	939	6163

X^2 11.35 x 10^3

C_{Actual} 0.81

$C_{Maximum}$ 0.87

PUPILS

Content	S	T	D	I	M	O	N	Total
Structure	145	1	30	9	29	0	0	214
Techniques	3	91	7	2	18	0	1	122
Deductive	5	1	279	6	26	1	0	318
Inductive	2	11	14	48	7	1	0	73
Mathematical	15	9	20	26	751	5	0	802
Other	1	2	0	0	8	24	0	35
Neutral	0	0	2	0	0	0	6	8
Total	171	105	352	67	839	31	7	1572

X^2 4.50 x 10^3

C Actual 0.86

$C_{Maximum}$ 0.91

Process	A	Sy	Sp	G	R	N	Total
Analyzing	61	21	35	4	0	0	121
Synthesizing	11	610	76	1	2	0	700
Specializing	17	72	414	3	15	1	522
Generalizing	7	4	32	60	0	0	103
Relevant	1	5	22	0	90	1	119
Neutral	0	0	1	0	0	6	7
Total	97	712	580	68	107	8	1572

X^2 2.94 x 10^3

C_{Actual} 0.81

$C_{Maximum}$ 0.89

Attitude	C	I	R	N	Total
Curiosity	31	10	0	0	41
Independence	13	1198	75	1	1287
Receptivity	0	58	176	0	234
Neutral	0	4	0	6	10
Total	44	1270	251	7	1572

X^2 3.10 x 10^3

C_{Actual} 0.81

$C_{Maximum}$ 0.81

VALIDITY

CONTENT AND CONSTRUCT

In considering the validity of this observational instrument with the large measure of inference in its defined categories, the evidence accumulated for the four aspects of validity--content, construct, predictive and concurrent--and for the reliability of the instrument and of the observer should be cited. As yet, the tests of the four types have been limited to content and construct. The latter two certainly require attack by comparison of the results of this instrument with those of other direct instruments, or with indirect measurements such as paper and pencil tests of pupil change. Suggestions for this occur in project 816 and in suggested studies of this Manual.

First, content validity. Considerable support, both logical and empirical, may be noted. The place of verbal behaviors as appropriate representation of thinking has been discussed affirmatively in "Rationale". Also supported in the Rationale was the triple viewpoint of these verbal behaviors in providing a comprehensive assessment of language. That the categories of the three frames of content, process and attitude reflect important aims of mathematics teaching has been extensively considered by Wright (1957) and Proctor (1960). The behavioral definitions of the categories have been presented in successive forms of the instruments with increasing precision, and always within the broad requirements of category sets: exhaustiveness, exclusiveness and unity. While the nature of classification--that a behavior should be classified once and only once in each frame forced exclusiveness logically, evidence for the actual independence of the categories of each frame has been obtained. In project 816 with its 10-day sample of 16 classrooms, and in a semester-long study of one classroom, scattergrams plotted between pairs of categories within each frame showed no indication of correlation. Despite the empirical evidence of independence, and the definition of exclusiveness, we feel that it should be noted that the three categories of Attitude are related serially in increasing emphasis of initiative from receptivity to curiosity. Where independence of categories is a pre-requisite of a statistic, such as chi-square, this evidence supports the use of such statistics within individual frames. A second series of scattergrams plotted between pairs of categories across the three frames revealed a number of sharp correlations. These have yet to be investigated further.

Empirical assessment of sample size sufficient to provide representative behaviors, yet not needlessly large, received its most complete treatment in Project 816. Wright (1957) noted the importance of viewing a complete topic, and data at that time suggested the minimum number of days of observation for single category stability ranged from 10 to 25. From a semester-long study of a single algebra classroom with a more refined instrument (1959), ten days--selected either at random or in sequence--provided a sound sample in single categories of the entire 77 days of observation. In project 816, confirmation of the adequacy of a ten day sample was sought as described in "Treatment of Data--Stability". Detailed assessment both of individual frame and triple frame scores emphasized the soundness of the ten day sample with its provision of approximately 1,000 behaviors--verbal and non-verbal. In addition, where study is limited to the process or attitude frames, or if an adequate number of phases of a topic have been covered, this sample may be derived from as few as six days of observation of 50 minutes in length.

Finally, the number of subjects (i. e. classrooms) to be observed before normative data is provided has been considered only in project 816. As noted in more detail in Construct validity, groups of three classrooms were sufficient to differentiate classrooms selected at the extremes of rigor and participation.

Second, construct validity. Strongly supported. The initial forms of the categories showed clear differentiation of individual classrooms, yet greater differences between classrooms with different teachers and pupils than between classrooms where the same teacher was guiding different groups of pupils through the same subject matter. The differences noted were in keeping with broad descriptions of the several classrooms. (Wright, 1957, 1959)

In project 816, distinctive patterns in the areas of content, process and attitude defined by the Instrument were established for the four types of classrooms varying in degree of mathematical rigor and student participation. These distinctions were noted in terms of single categories within each area, by meaningful combinations of categories within content and process, and by triple combinations of categories across the three areas simultaneously.

Finally, it was sharply apparent that if our study in project 816 had been limited to one or two dimensions of the classroom, e. g. to a few categories within one area, or to one area, that wide variation in behaviors of classrooms of a single type would have suggested distinctions between teachers and classrooms quite unjustified by the independent variables, general reputation and by any one of the complete views of the dependent variables. It seems to us a marked advantage of this instrument that by whichever of the three means a complete view of the classroom variables is taken, certain apparent discrepancies in wide variation of single dimensions could be accepted as reasonable and indeed as quite desirable.

RELIABILITY OF THE INSTRUMENT

Early studies established the validity of the fifteen second division and the use of alternate intervals for classification and for judgment. Classification from a seven and a half second division, a fifteen second division and a thirty second division were compared with that of a natural division. The fifteen second division provided results readily classifiable in contrast to the shorter division which seemed to cut too many ideas in parts. In addition, more material could be classified than in the longer division, and plenty of time for judgment was available if alternate intervals were classified. Then comparison of the patterns from the two sets of alternate intervals was favorable. Finally, agreement of the patterns from the fifteen second division and from a natural division was high. The former was preferable to a direct use of a natural division, which might on the surface seem more desirable, because of the complexity of the instrument, requiring classification from three viewpoints simultaneously. A fixed interval removed the need for broad judgment of beginning and end of 'natural' intervals by selection of an optimum fixed-length interval.

A second check was that of repeated classification of selected interaction recorded in typescripts, made at three-weekly intervals. High agreement between repeated classifications was noted in general. Particular categories where agreement was low were then refined. All these comparisons are discussed in Wright (1957, 1959), and were based on individual frame scores of the initial rougher definition of the categories of each frame.

In project 816 two tests related to the reliability of the instrument were made: the test of sample stability, noted above in content validity, and examination of categories still susceptible to misinterpretation and observer bias, noted above in "Treatment of Data--observer reliability"). Both the chosen ten day sample, and the refinement of definition of categories in the Manual likely to increase the levels of agreement recorded for project 816, support the reliability of the current form of the instrument.

RELIABILITY OF THE OBSERVER

Only results with the form of the instrument used in project 816 are pertinent. Immediately before this project was begun five observers, all secondary school mathematics teachers were trained by the principal investigator in the use of the instrument while they observed a class of gifted students with the principal investigator as teacher studying elementary topics in probability and statistics. The assistant investigator acted as an experienced observer classifying all interaction for comparative purposes. Following a ten day training program modelled essentially along the lines suggested in this manual--but with the principal investigator for consultation throughout--the following indicators of agreement were obtained.

Trainee	Percentage Agreement			No. categories within		
	Content	Process	Attitude	1	2	3 s. e.
1	65	49	79	16	19	25
2	82	75	94	22	28	28
3	41	36	60	13	19	26
4	64	63	87	14	20	25
5	83	71	94	25	27	28
Mean	67	58	86	18	23	26

In the percentage agreement check, each trainee's total n was approximately 900. The overall total for mean determination of percentage agreement was 4334. In the binomial probability paper check, for each trainee, 26 categories were plotted: for each of teacher and pupil--5 Content (deductive and inductive were still viewed as one category); 5 Process; 3 Attitude;1 Neutral.

In project 816 the two experienced observers reached the following levels of % agreement in entire frame totals: content ranged from 70 to 89; process from 64 to 89; attitude from 80 to 96. Details of individual categories within each frame are available in the project, chapter III. However this percentage test is least valuable because pf widely varying n's and extremely low values in some categories.

The two main tests of reliability of the scores used in later interpretation were X^2 across the category totals within each individual frame and r_S between rankings of triple frame scores. These tests were applied in fourteen of the sixteen classrooms of the study. Of all comparisons of individual frame scores for teacher and pupils in content, process and in attitude only in one case, Teacher Attitude, class IX, was difference significant at the probability level, 0.05. The ranking tests on the highly complex triple frame scores also emphasized the high levels of agreement. All teacher rankings were associated at a probability level less than 0.01 and seven of the fourteen pupil rankings were associated at 0.05 or practically.

A much finer test of observer agreement than required by the scores used for interpretation of results was that of X^2 and C from the cross classification matrixes of some 8,000 behaviors jointly classified (detailed in Observer Reliability, this Manual). The high agreement shown behavior by behavior would permit assurance in use of results of these two observers for such study as development throughout a single lesson, variation from lesson to lesson, etc.

CARE IN USE OF THE INSTRUMENT

1. While this Manual is prepared to facilitate self training, it is still desirable where convenient to train with an experienced observer. Where this is not possible two or more novices will learn more readily than one working alone. In all cases, the importance of getting into the classroom and classifying from the live situation as outlined in the training procedures cannot be overemphasized. An intermediate step, of course, would be the use of a tape of classroom interaction. Indeed, such a tape has been prepared with this report, based on selected aspects of the typescripts included. However, an important part of classroom activity is still absent.

2. It is essential that the observer be well prepared in mathematics.

3. While this is designed for use in a mathematics classroom, with some modifications the instrument is likely suitable for observation in science. With more complete redefinition of specific categories--even adding or changing categories --a specialist in another subject matter field might find some useful comparisons. Probably the categories of Attitude would change most; those of Process least. Because the specialist in a subject area acts as observer, the actual effect of the subject is absorbed leaving the categories free for comparison across several areas where they occur in common.

4. This instrument is obviously adaptable to an evaluation program. If this is the intent, because of the detailed and revealing nature of results, this should be done only with the approval of the teacher concerned and with the written results available for constructive discussion both for the observer and for the teacher. In our own experience, although we were not using this instrument in making value judgments, the experience we have both had in the classroom with its consequent concern with the problem of teaching--not of the teacher--was particularly pleasing to the teachers we observed. In every case the administration and the teachers were most generous in their time, discussing broad and specific approaches to the problems of the classrooms we were studying.

5. While the possible total of ten days necessary for determining a representative sample is indeed time consuming, the richness of results is a major compensation. In addition, both administration and teachers observed appreciated the several days taken before judgment of a classroom was made.

6. The data supplied for comparative purposes should be interpreted in the light of the validity and reliability studies detailed above.

SUGGESTED STUDIES

This instrument may be directed to a variety of specific uses. Its value has been apparent throughout our several studies in the sharpening of the attention of the observers using it to details in the content, the process and the attitude areas during discussion in mathematics lessons. In particular a heightening of interest in essential considerations of subject matter and of specific behaviors of teacher that stimulate or of pupils that demonstrate a desired end, has been apparent in many student teachers using this in a methods course. It has also been considered that by redefinition of exact categories in the three frames by a specialist in another field to meet the peculiar needs of that subject area, comparison of categories common to two or more subject matter areas would be possible. It is likely that the definitions of the Process frame would be most stable permitting direct comparison of relative emphasis of these functional aspects of logical thinking.

In suggesting a few specific studies, we note several that would be fruitful in understanding of the mathematics classroom; in addition this instrument may be applied to interesting examples of mathematical discourse outside the classroom; and may be used in further validation of itself or to supplement information available from other instruments direct or indirect in their measurement.

I. Description and comparison

1. of classroom patterns in topics of mathematics markedly different, e. g. geometry, algebra, probability.
2. of classroom patterns in mathematics at varying grade levels.
3. of classroom patterns with the same teacher and subject matter but different groups of pupils
4. of actual classroom patterns within some theoretically interesting framework.
5. of classroom patterns in different phases of a lesson or topic; e. g. development of new work, solving specific problems.

II. This instrument may be applied to interesting examples of mathematical discourse outside the classroom

1. in the laboratory situation of classroom, small group, individual.
2. in analysis of textbooks.

III. This may be useful in methods courses in mathematics teacher training programs in sharpening attention to aspects of content, process and attitude significant in the classroom. Also by direct observation to select significant behaviors of teacher and pupils.

IV. Empirical validity and reliability

1. Comparison of observation with this instrument with that from other direct measures.
2. Comparison of results with this instrument and with indirect measures of related variables.
3. Study of observer agreement.
4. Correlation of pairs of categories across the three frames.

APPENDIX I

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APPENDIX II

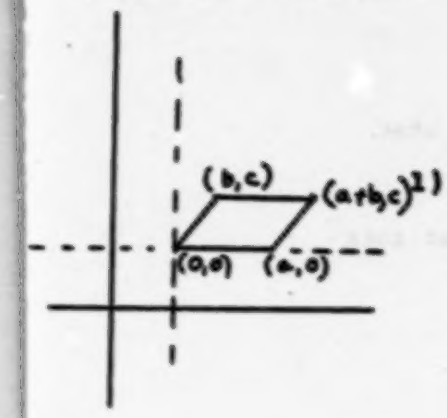
TRAINING TYPESCRIPTS

Number	Appendix Page	Classification Page
1	81	26
2	91	26
3	103	27
4	113	27
5	122	28

Typescript #1

Analytic geometry

Blackboard notes



1) **T:** I want to use this parallelogram to indicate two or three things, and I want to relate the materials of this chapter to

locus, the topics which some of you may not have all of the conclusions for right at your finger tips.

2) First, in order to show the relations between diagonals, parallel sides,

area, and other properties of the parallelogram, I'll suggest that we may move the coordinate axes

3) so that they intersect anywhere we choose, because if you had put the parallelogram on the coordinate axes, - I notice you haven't, - move the origin so that

it's placed at one vertex of the parallelogram. Now mark on the sketch these coordinates, that point

4) as 0, 0. Incidentally you may find as you continue your study of mathematics, you may find that coordinates are

marked in that way ⟨3,4⟩ often instead of with parentheses. I don't know why this particular form of printing

5) has been introduced in the last several years, but one sees it frequently, our text has used it you know. I'll mark this point (a,0).

I'll indicate the coordinates of another point (b,c), and then I'll leave it to you to write the coordinates of the remaining vertex.

6) How can they be expressed?

Gussie?

P: bd

T: 3 d did you say?

P: bd

7) **T:** bd, you multiplied them. I believe we can express them in terms of the coordinates we already have, I don't

believe it's necessary to introduce any new ordinate,
or abscissa.

Jane
P: a + b and c.
T: Very good; indeed, that'll do it. Now what
basis would we have for using

8) the same value of an ordinate at this vertex as at this
one, Ken?
P: The same distance
T: What is the same distance?
P: The distances are equal

T: Will you speak up so we
can hear you, Ken.
P: The two distances are equal.
T: I'll put a mark on there that will give some
Continue your sketch now with the diagonal,

9) so we have represented a line and the coordinates at each
end of the line, segments, that is;

draw, both diagonals.
Now in terms of the - that's a very small sketch.

10) you know what I do with that (T. tears boy's paper in two
and throws it away; amusement).

Write the coordinate
of intersection of the diagonals now,

11) and then read to us what you wrote.

Do you have the
abscissa

12) and the ordinate?
P: (b+a) over two, c over a, c over two.
T: (b+a) over two, c over two for values of abscissa and ordinate

respectively. Do those seem reasonable? Linda?
P: Wouldn't it just be a over two, it wouldn't be b?
T: a over two, yes

13) a over two, that would be half of a, so if I started here
at the original and go half the distance to a along the
line I might stop right in there somewhere. What does
inspection

of the illustration tell you about using half
a, half c as the coordinates of this point? What
does an inspection of the sketch suggest?

14) Martha?

P: That it's wrong.

T: Very good, we'll discard it then. I'll keep the coordinates here. Martha has indicated a highly important thing there

that one's conclusions are often correctly based on intuition. This also indicates the help one gets from having a good,

15) a big illustration, one that's useful (laughter). The coordinates

of the diagonal having a plus slope and the coordinates of the diagonal having a minus slope, since it's one point, the

16) coordinates are the same. What conclusion then, if you had studied no plane geometry at all but algebra only, what conclusion could you reach about the diagonals of a parallelogram? Valerie?

P: Bisect each other.

T: Yes. And many of the conclusions can be verified in coordinate

17) geometry, analytic geometry. Let's take another idea. Locus of an equation,

The equation, well let's see. I won't use a linear equation, which you could possibly by intuition and a sketch reach

18) some conclusions about. Let's take $y^2 + x^2 = 15$. The locus of an equation

contains all points and only those points which

19) satisfy the equation. Can you give us coordinates of one point

that satisfied the equation $x^2 + y^2 = 15$, one pair of coordinates, one set of values for x and y?

20) Bonnie

P: Square root of 5, the square root of 10. Bob

T: Square root of 5, square root of 10. Are those satisfactory.

P: Yes

T: How can you show that those are satisfactory, what

21) computation can you make to support that opinion.

P: You'd have to substitute

T: Well let's see if they do. Square

root of 5 for x^2+Square root of 10 for y, squared. Can you suggest another pair of coordinates where satisfy it equally well

x	y
$\sqrt{5}$	$\sqrt{10}$
$\sqrt{7}$	$\sqrt{8}$
0	$\sqrt{15}$
$\sqrt{15}$	0
$\sqrt{6}$	$\sqrt{9}$
$\sqrt{11}$	$\sqrt{4}$

22) Bonnie? Burst? John ?

P: Square root of 7, the square root of 8.

T: Let's give her half a dozen more quickly, one such pair, Dave?

23) P: Zero, square root of 15

T: I just wrote it. And another pair, Ben?

P: Now, one final pair

P: Square root of 11, square root of 4.

T: Take x y coordinates and roughly plot a few of these

24) until you can tell us what is the locus of all points represented by the equation, then you'll have the locus of the equation.

omitted (pupils plot points on paper)

25) omitted (pupils continue to plot points)

How many points have you plotted so far, Barbara?

P: Four

T: Four points, you've plotted four points already. Do those four points begin to outline any geometric figure? Barbara

26) P: No

T: If they did you might infer what the entire figure would be and so not plot any more. When you get to the part that you can make that reasonable inference then you won't need any more of the mechanics of just plotting points. What does intuition suggest about the few points you

27) have plotted, Ben.

P: That it's a straight line.

T: That it's a straight line. Will you verify that with the sketch you have so far, Gussie? Do the points that you have plotted so far seem to lie on a straight line. Gussie?

P: I don't have that many.

T: How many would she need?

P: Two.

T: How many would she need, Linda?

28) P: Well, she'd need at least three to be sure. P. one
T: How many would she need, Bob ?
P: Three
T: Three?
P: You could do it with one seeing that that's going to be a circle.
T: Oh yes.

One point, plus the center.
P: The center is the origin.
T: The center is the origin in this case?
P: Yes
T: Well then we could use the
P: The square root of 5

29) T: The square root of 5?
P: You could use the square root of 15, that would be the radius of the thing
T: of 15, very good, radius = square root of 15

The thing I'm trying to emphasize now is, one need not mechanically, and finally after plotting 12 or 15 points discover that

30) you might have much earlier recognized the equation as that of a circle with its center at the origin.

Take a reverse relationship. Locus of an equation, equation

31) of a locus, which may help to emphasize that operations in mathematics work forwards and backwards,

and besides the opposite direction, deserve some attention. On coordinate axes, locate what you believe to be the point (2,1).

32) What quadrant, Martha?
P: Quadrant 1.
T: Quadrant 1. Now in this last relationship

$x^2 + y^2 = 15$,
we had seen that there are several pairs, ordered pairs, several sets of values that satisfy the equation

33) Consequently all the points that have coordinates satisfying this equation lie on the circle, and every point on the circle has coordinates

satisfying this equation. Now let's go instead of from the equation to its locus, from the locus to an equation. So where here you might have

34) thought of a point moving so that its coordinates always satisfy this equation, now think of a point moving but you don't know what the equation is, the equation we shall write, eventually,

if we have enough time. A point moves so that it is always the same distance from the origin as it is from a point whose coordinates are 2,1.

35) Sketch it, move that point so that it remains the same distance from the origin as it is from the point (2 ,1) What is its path?

Is it a geometric figure? Valerie?
P: Yes.
T: What is its name?
P: Circle.
T: Speak up so we can hear you.
P: Circle.

36) T: Anyone disagfee with that? Along here, there's half of the people in this row. Richard?
P: Well I believe this is a line which is perpendicular to perpendicular bisector to the line joining the point (2,1) to

the origin.
T: Any comment to offer to that? No comments.
You don't have any ideas at all

37) about it. Do you still believe that it's a circle, Tom? This locus might be classified as the locus of all points equally distant from two given points,

and if we take two given points and represent the locus of all points equally distant from them, what kind of geometric figure do we have, Ken?
P: A perpendicular bisector, A straight line.

38) T: I'll represent the locus in white. This is you're right, Richard, the locus is a straight line.

not any straight line, it's the perpendicular bisector of the line segment joining these two points, we'll find the equation of this line and have the equation of the locus.

39) How can we find the equation of a line? We know it's perpendicular to another line segment, and we know the coordinates of the ends of that line segment.

How shall we go about finding the equation of this locus, ? Kenny?
P: You can find the slope of the first line, and since

40) the second line is perpendicular to the first line, since the lines are perpendicular, you know the slopes are negative reciprocals, and since you know both points of the second line, first line, rather, you can find its distance, and you know it's the perpendicular bisector, so you know it's only half way

so you can find a new point.

T: First the slope of the line segment joining (2,1) to

41) (0,0). Perhaps you can by inspection tell us what that is? Martha?

P: One half.

T: One half. Any disagreement there

Anyone who doesn't know how she has determined the slope of this line? How long is this ordinate, Jerry?

P: It's 2, - or 1,

42) 1.

T: And the abscissa, Jerry?

P: Two.

T: Now we ought to realize that the slope is one-half. The slope then of the locus,

probably you can tell us, Bob ?

P: Minus two.

T: Minus two; well, we now have m = 2, and how can we use that information

43) to find the equation of the line, Paul?

P: Well, first you have to know the midpoint which is one

T: Shall we try Linda on that. Will you

tell us the coordinates of this point, Linda?

P: One, and one half.

T: Right

44) (1, 1/2). Now we know it goes throught the point, 1, 1/2, we know its slope, write the equation in standard form, and then read it aloud.

Omitted (pupils work on problem)

45) Omitted (pupils work on problem)

Omitted (pupils work on problem)

46) Do you have it Barbara? What is it?

P: $4x + 2y - 5 = 0$

T: How can we test and know whether or not that is correct?

47) Only one line satisfies the conditions, and only one equation for one line is acceptable. How can we test this equation,

and know that it is the equation of the locus

David?

P: Well we can put it in standard form -

T: Speak up so we can hear you.

P: We can put the equation in standard form and see if its slope is the negative reciprocal.

48) T: Why is that not a suitable test? Bonnie ?

P:

T: Speak up so we can hear you

P: You know that its slope

will be the negative reciprocal because you used that formula originally.

T: Yes, we can devise a better test than just to use the data in about the same way as we used it before. What do you suggest, Ben?

49) P: Well, the coordinates, 1, 1/2 should satisfy the equation.

T: Exactly, certainly. Do they ? Of course we used that information also to get the equation, to derive

the equation.

Well, I've taken another point here, I'll call it point P, and when I join it to

50) the origin, and when I join it to the point whose coordinates are (2,1) then, those distances should be equal. If they aren't,

and if the coordinate of the point satisfy this equation then you see we'll be using a relationship not used in deriving the

51) equation earlier. Now how can we find the coordinates of the point P. How can we show that P is the same distance from the origin as it is from the point (2,1).

or maybe I should say, can we? show this point P is equally distant from the origin and from (2,1). I'll call this distance a,

52) and the distance from point P to (2,1) I'll represent by the letter b, if a b then

P lies on the locus. How can we represent the length of the segment a?

53) Jerry?

P: Well, you could, it's the hypotenuse of one of those triangles, you could do it that way if you want.

T: Yes, and so we can represent it in terms of ?

P: The, let's see, the distances,

one of them would be the square root of.

T: Let's call the coordinates of P, (x,y).

P: Oh, O.K.

T: That, you see, makes it general, so that point P represents any

54) and every point including the midpoint on that oblique line which is marked locus. Now how can we represent the length of segment a.

P: Well, this seems sort of

if you want to prove that a b you can do it much easier than that I think though, can't you, you've got an angle and a side, and you've got 2 sides, and an angle right there.

T: Well what shall I write?

P: $a = b$, you could write it just like that.

55) Well, you've got 1, you've got two right triangles so you've got one angle, you've already got that that line bisects the line with coordinates (0,0) and (2,1)

alright that's one side, and you've got one side which is a common side, and you've one angle between those two sides so you know that they're equal.

T: We've assumed that they're equal. You see all that depends on whether or not

56) P lies on the locus. Cranston, what idea have you?

P: The distance is equal to the square root of $x^2 + y^2$.

T: The distance or length of segment a

equals the sum of the squares of the abscissa and ordinate. How can I fill in these blank

57) spaces correctly, then? John.

P: x - 0 gives x, and y-0

or just y.

T: Then the length of segment a is equal to the square root of what quantity, Bonnie Burst?

P: $x^2 + y^2$

T: Now let's take

58) the segment b, yes, the segment b, using the distance formula now,

the length of segment b. Can you tell us how to represent the length of segment b using the distance formula, Gussie? We want your voice on this record

P: x - 2 plus

59) + y - 1.

T: Anyone suggest any changes in the way we represented

segment a and segment b? If those are correct, and if $a = b$.

60) then the square root of $x^2 + y^2$ would equal the square root of x - 2 quantity squared plus y - 1 quantity squared. Do you believe we can show that

those two expressions are equal? What do you suggest is

Typescript #2

lackboard notes:

Commission - grade 7.

1) T: Now open your books to p. 101 please. Now the topic is working on commission

orking on Commission

working on commission. Does that word have any meaning to you? Al?
P: It means a supplement for an extra pay

2) after a job is done.
T: Do some people work entirely on commission, is their entire income on a commission basis, would you say, Elizabeth?

P: Some people do
T: Would you think that's a pretty good way to earn a living? Do you have any thoughts on that matter? Jack
P: Well maybe a person can keep working and welling for a company if they are a salesman.

3) T: Do you think they can earn a pretty good living that way if they are an adequate salesman?
P: Yes
T: There are many people that do work on commission and this is applying some of our understanding

of percent to condition say in the business world; we are not going to do a lot of applied work in this field of mathematics, the idea is for us to learn the relationships involved in this subject

4) and when it comes time to apply them and you understand what you are doing you'll be able to apply themm alright. For many businesses today when a person is needed in the field of mathematics that they want to train

the person themselves but in order to train them they have to have an adequate understanding of the principles involved in the subject so I feel if you understand the principles then you will be able to apply the principles but occasionally we do use a few simple applications and

Rate of Commission

5) this assignment indicates a few. Now, what do you think is meant by the rate of commission? The rate of commission. What idea do you have about

that, Bob?
P: Well, I think it's probably how much money a person gets, for how much he sells, like if he sold some $2.00 worth of something he might get a dollar

6) commission on it.
P: Then you say the dollar would be his commission?
P: His rate. Well, the rate of commission would be $1.00 and to every $2.00
T: Oh I see, it's sort of a ratio, a comparison of two numbers.

You would say then if he sold two dollars worth of goods and he got to keep a dollar then you would say his rate of commission is a ratio of one to two, is that right?
P: Yes

$\frac{1}{2} = \frac{50}{100} = 50\%$

7) T: I wonder if we could establish that one to two then, as perhaps in another notation, Charles.
P: You could put it 50/100 or 50%
T: We could establish that as an equality

of 50 divided by 100 and we know that tha's 50%. So then we could say that this person who sells $2.00 worth of goods and gets to keep a dollar out of each two dollars

8) he takes in is working on what, Pete?
P: 50% commission
T: A 50% rate of commission. His commission is the $1.00 isn't it or his commission - what did you want to say.

P: 50% of the two dollars.
T: Of the $2.00 - that's not a bad deal if you can get it. Well now you are going to set up problems of that type and what I want you to do

9) is not necessarily set up a ratio the way we did today but make a mathematical sentence out of your problem. Now let's take one and see what we can

get out of that. Let's look at No. 5 on P. 101. Will you read if for us, Jack?
P: If watermelons were hard to sell and rate of commission was 12½%, Mary sold a large one for $1.50, two small ones for 85 cents a piece

10) find her commission.
T: Now I want you to make a mathematical sentence out of

that, using some symbol to represent her commission and

then we want to find the replacement set for that variable, and we understand that the set can be only the numbers with which we are familiar

11) and what are those numbers? For about the 150th time - what are they Nancy?
P: The non negative rational numbers?
T: Alright so who would like to try their hand at making a mathematical statement out of this number?

1.50+.85+.85

Judy?

P: Could it be $1.50 plus 85 cents divided by 12½ hundreds?

12) I mean two plus and then plus 85 cents again, because he wold two.
T: What

do you want to do with that?
P: Divide it by 12½ hundreds
T: How do you want me to write that?
P: As a fraction.

$\frac{1.50+.85+.85}{\frac{12\frac{1}{2}}{100}} = N$

13) T: Like that - that equals n. Do you agree with that, Jim?
P: No, I think it should be, you multiply it by $\frac{12\frac{1}{2}}{100}$ or divide it by N and get $\frac{12\frac{1}{2}}{100}$

$\frac{12\frac{1}{2}}{100}(1.50+.85+.85) = N$

T: O.K. So Jim wants to change this Judy, and he says he wants to do this.

14) and now he wants to multiply this times all this and so in order to do this how would you indicate that? Just say 12½ hundreds times
P: No, you would put a parenthesis around the $1.50 and .85¢ and I guess you could around the $1.50 but you don't really need it.

T: Why don't we really need it around
P: Cause it's the only one left, you wouldn't need to put any mark anyway, you just leave it.

15) T: O.K. I wonder if, that, in our problems that we had today, let's pick out one of those, let's say we pick out

15% of 200, let's make a mathematical statement out of that. We want to find

15% of 200

$\frac{15}{100} = \frac{n}{200}$

16) 15% of 200, Holly.
P: You could say 15/100th equals n over 200 ? and then
T: That's a mathematical statement and that's surely right. I wonder if there's another one that you could set up.

Perhaps a more (cough) Since we have set this up

17) today as 15 over 100 this is the way you work the problem today and that is surely a mathematical sentence that can be obtained

$\frac{15}{100} \times 200 = n$

from this statement. I wonder if there's another way that you can express it. Elizabeth.

P: 15/100 times 200. 200 equals n.

T: Yes we could establish it like this. Now lets see if we can derive

18) this from starting with what you have here. I imagine most of you did this using the rule,

$\frac{15}{100} = \frac{n}{200}$

didn't you? Well for one this simple probably you didn't, but suppose you did use the slide rule and if you recall we have a number of different ways to find the replacement set for n.

19) What would be a way that you could set this up so that it might and up looking like that one over there? Charles?

P: 15 times 200 divided by 100.

$\frac{15 \times 200}{100} = n$

T: From where, from what principle that we have had previously this year has that been derived?

20) P: Well, if a over b equals c over d that is true, this is true if and only if a times d equals b times c.

if $\frac{a}{b} = \frac{c}{d}$

then $ad = bc$

T: All right. What is the a times d in that expression, Larry?

P: 15 times 200

T: All right. What is the b times c in that expression, Terry?

21) Terry?

P: The 100 x n.

T: But we don't have that so how did we acquire this result when it should be 100 x n? Holly?

P: We divided it by the quotient of 15 x 200 to get n.

T: In other words what you are saying is that if 15 times 200 equals 100 times n then n must be equal to

22) what we have here. Is that what you mean? We have obtained that intuitively from the fact that what operations are inverse operations, Dee?

P: Multiplication and division

T: Well does this look

like this? John? Is this the same as this?

P: Yes

T: Is it all right if we would write this.

$\frac{15 \times 200}{100} = n$

23) Is that all right. Because actually we could think of 15 hundredths and 200 as what of n? They would be what of n. This is 15/100 x 200 so 15 over 100 and 200 could be thought of a what? Bob.

P: Factors.

T: Factors of n, where n

24) would be thougt of as what? Nancy.

P: That would be the product.

T: Yes the product. Now what I want you to do for assignment twelve,lis that right, is to work the ten problems on p 101

25) and set up mathematical sentences like this and then of course find the replacement for n, and the procedure that you use will be up to you after you set up the statement. Now the next thing I want to do is discuss any

26) questions or arguments you want to give me concerning the test that you took. Now Pete and Terry you haven't taken this but I'll make up another for you. I guess we'll let you stay here. Any questions on the test? Alice?

P: On Test B could you please give me the questions on 4

27) and 5.

T: In B? 120% expressed as a ratio reduced to lowest terms is? How do we do that? We're going to express 120% as a ratio in reduced form, what is the first thing you would have to do?

28) P: Find its decimal equivalent.

T: So what is that?

P: 120/100

T: What's that equal to?

$120\% = \frac{120}{100} = \frac{6}{5}$

P: 6/5's or 1 1/5.
T: Number 5 finding the replacement for n in the equation 7/3 equals N/100.

$\frac{7}{3} = \frac{n}{100}$

29) we know that 7 times 100 is another name for what? Dee?

P: 3 times n.
T: 7 times 100 and 3 times n are merely names for

30) the same thing. All right. Pete.
P: What was the problem on B. What was the instruction.
T: Find the replacement for n that makes the statement true.

You took a double take on that, didn't you?
P: Yes.
T: Pete wrote for the replacement for n--he wrote all his answers as per cent.

$\frac{350}{15} = \frac{n}{100}$

31) In other words he did the first one and he said 350/15 equals N/100--he found the proper replacement for n but he thought

well I don't know what he thought really. But what is the replacement for n, Pete?
P: Well on that one it's 2.333.
T: Well if

32) he worked it out it would be that. If he used a slide rule it would be 2.33 but then on the end of that he stuck this which made it incorrect

2333%

doesn't it because he is replacing n with what, Judy?
P: per cent and he can't do that because you already have 100 as denominator.
T: So this is merely a number. It's a first member

33) of the ordered pair isn't it? Now what he could do is express 2.333 over 100 as what, Dee?
P: 2.333

per cent.
T: Yes. O. K. Any others, Bob?
B: On test A, on Section D where you had that chart

34) the first step was to change common fractions into lowest terms. Well could you do those? Those three problems of the first set.

T: Well here were the four problems on test A. The first thing was to get a fraction in lowest terms, the second thing was to write it as a decimal fraction, the third as an ordered pair in which 100 was a second member and the fourth as a per cent.

	[illegible]	decimal fraction	ordered pair	%
.625 18% 825/100 4/11	625/1000			

Now in the first case which of the columns has already been filled, Larry?

P: The decimal fraction column.

T: Now Bob wants to know how to express .625 as a fraction, a common fraction, reduced to lowest terms. Now how do you do that Peter?

P: Well you could find the old way of finding the fraction if you want to reduce the fraction.

T: We have a decimal fraction, how are you going to reduce it? That's what we want to know. This is what's given and we want to write it as a common fraction reduced to lowest terms.

P: First you find the common fraction.

T: Well that's what he wants us to do.

P: Well that would be the decimal fraction .625 thousandths

T: All right, now this is the first thing you have to know and this is something you should know, we have had it previously. And then Pete said we use the procedure of reducing a fraction to lowest terms which is to do what, Craig?

P: Well you would find a factor that both of the numbers contain, you divide that into them

T: Fine. The thing that we actually want to do is to find that common factor if we want to reduce it to lowest terms, Craig.

P: Well the largest common factor.

T: The largest common factor. Do you want that done, Bob, or is that clear now?

P: Well I see what I did on that, I made a mistake. I had 625/100 instead of 1,000.

T: You see what was wrong with that?

P: Yes, and then I can see on the third one I had the problem, instead of over 10, over 100.

T: It should be over 100--this column says the second member must be 100--so what would that be?

P: Which column?

T: This column 100 do you want the fraction to thus so that 100 is the second member of the ordered pair appearing.

P: So then you would write it 62.5/100.

41) T: That's right. And if you're able bo do this then that becomes what?

P: 62½ %

T: Now in some cases I have circled part of that chart. Actually those answers are wrong, but I didn't count those as being wrong because of the

42) fact that you made a mistake previous to that and if you used that previous error and used that number to obtain the others then made no mistake I gave you credit for it although it was still the wrong answer.

Carol?

P: What's the question on no. 5, test A, section A?

T: If you worked 75 % of the problems on this test correctly, you could say you have

43) ____ correct problems out of each _____ problems on the test.

P: If you put after my question--what if there were twelve problems on the test--what did you mean by

that?

T: I just asked you a question--if you worked 75 % of the problems on this test and there are twelve and you work 75 % of them correctly then you could say you have how many correct

44) problems out of how many problems on the test--if there were twelve problems on the test?

P: Well 75 % is 3/4 and would be 8, wouldn't it?

T: Well not on Wednesdays. (laughter)

P: Wait let me see now, I mean nine.

T: Nine out of twelve in other words. Another way to express it, you might say if you worked 75 % of the problems on any test it means that for how

45) many problems you worked you have how many correct?

P: For every 100 you get 75; oh, for every 12 you get 9.

T: Yes or for every what, Barbee?

P: Every 4 you get 3, right.

T: Every 4 you get 3, right

so I gave you credit for it so say 75 correct problemr out of each 100 on the test.

You could also establish that you get three out of every four.

46) What if there were only two problems on the test?
P: You would get 1½ right.
P: This is on the chart. I had one right. I had 4/11 equals 36.4 and I

put it as one with--oh I see what you did instead I didn't make it as a 100 when you said before you know if you got a

47) previous error and needed the same thing.
T: Yes, but you did another error. Anything else on the test? You understand your errors?
P: On test B

on the first problem what was the question?
T: 350 divided by 15 equals n over 100. Is that what you mean? Problem 1 in part B?
P: Yes.
$\frac{350}{15} = \frac{n}{100}$ T: 350 divided by 15

48) equals n divided by 100. You do not see your error?

P: I had 150 divided by 35 instead of 350/15.
T: You copied it wrong?
P: Yes.
T: Jim?
P: Will you read the answers for test B and the C part and the B all the ones.

49) T: For each statement below write a ratio to show the comparison--write an equal ratio with 100 as a second term and express

the ratio as a per cent--now which of the problems do you want under that?
P: Only the second one.
T: All the second ones for all of them. Two wringers out of five throws--write an equal ratio with 100 as the second term.
P: I want the answers.
T: You think I marked it wrong?

50) Two wringers out of five throws, now we want to write that so we have 100 as a second term. It says, write an equal ratio with 100 as the second term.

So what would it be?
P: 40.

T: 40 all right. The second one was 6 problems correct out of 9 worked, 6 problems correct out of 9 worked, and you have 100 here--so what will that be?

51) P: 66 and 2/3; 66.6.
T: 66 and 2/3 out of 100, is that right; with the slide rule it would be 66.6

or 66.7
P: That's what I had!
T: You sure?
P: Yes.

52) T: But that isn't the answer. The answer is 66.6 over 100. It's not 66.6.
P: I thought it was a replacement.

T: Yes, my question is to write an equal ratio with 100 as the second term. Now Jim, you understand it perfectly, I don't question that, but the fact is you did not do what the problem required did you

53) really?
P: No.
T: All right. That's why I marked it wrong. You want the other, Jim, or not?
P: No.
T: O. K. Alice?
P: Section B on test A, the first word in A and B I

had 87 and 1/2 hundredths.
T: Now wait a minute, what are we doing? .875?
We have here now .875 and we want

54) to reduce that to the common fraction in lowest terms. How do we do it? It's just like this one, now if you understand this one.
P: Make 87.5 over 100.
T: All right.
P: Or 875 over 1,000.

T: Well if you have the 875 over 1,000 all we are going to do now is get that as a fraction reduced to lowest terms so

55) why don't you just reduce this fraction, Alice? We want it as lowest terms--in lowest terms.

What would you suggest

doing here? To reduce 875/1,000 to lowest terms?
P: Find the prime factors.
T: All right, you don't necessarily have to find
the prime factors if you see a large

56) common factor there immediately you don't have to break it
down to primes first. What one do you see?
P: Well I can see five there.
T: See any larger one than five that's common?

P: I'm not sure about 75.
T: 75? Well if you are not sure of that you
might be sure of what?

57) One larger than 5 but not as large as 75--what might that
be, Bob?
P: 25.
T: 25, you might try that. Of course would I really have
required you to show anything here?

except the final result?
Barton?
P: Well you should know what that equals.
T: Yes, we were supposed to have memorized some
of these equivalents--weren't we--so that all you
had to put down here

58) is what? Pete?
P: 7/8.
T: Now what does that tell you about the greatest common
factor in both of 875 and 1,000?

What is the greatest
common factor, Dave?
P: 25.
T: No, you should know, people--everyone of you
in this class should know immediately what it is.

59) By one slight division you should be able to tell me. Jack?
P: 125.
T: How do you know, Jack?
P: Well, I've done it so often that each time I did it I
knew it had to be well it could

be over 25 because you di-
vide once and you get over 10, besides the fact
that I memorized it, I memorized the fraction.
T: I'm saying though now that you have this you

60) surely should know what you divide 1,000 by to get 8--how
could you tell?

P: You know it had to be more than 25.
T: Oh, but how could you tell without even estimating?
P: Divide 875 by 7.

T: Or divide 1,000 by 8, wouldn't that tell you? And that's how you get 125. So we have divided 875 over 1,000 , divided by 125 over 125 and that equals 7/8.

All sketches were on black board before class began

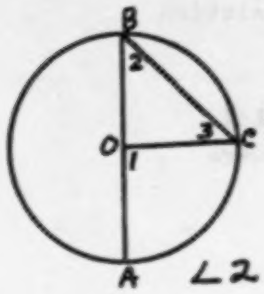
Typescript #3

Geometry, Grade 10

1) T: I have a memorandum which states, no work for Sherry, no work for Tom, we are happy. As a reminder

be last Friday again. Make use of scratch paper now and let's get an organization

Case 1



2) of theorems that are related and can be combined

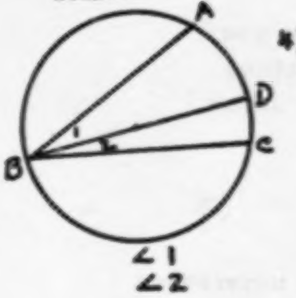
I separated these into case 1, case 2, and case 3.

3) In case 1, how would you distinguish case 1 from case 2, from case 3? What differences, if any, are there, what similarities are there?

Jim

P: Well, in case 1 there is uh they have only 1 chord

Case 2



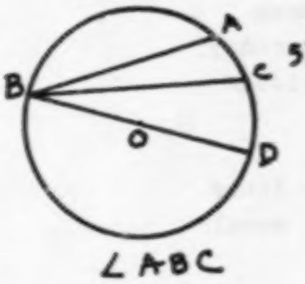
4) and there is also a radius that doesn't appear in the others.

T: Only have one chord. Let's look at line A B. What kind of line is this, Beasley?

P: uh a chord

T: It's a chord, so we have two chords. Over here we have 3, here we have 3, so one difference would be the number of chords we have. Would you regard that as an

Case 3



5) important difference? Jim

P: Well, I don't know what we're gonna do.

T: Very good, indeed. Well I do.

We're going to have some expression here for angle 2, some expression here for angle 1A and angle 2 and here for angle A B C and then to summarize all

6) of these in one statement. In case 1, and in case 2, and in case 3 what do you notice about the vertex of the angles I have written? Ed.

P: They're

on the circle.

T: The sides of the angle that I have written in case 1 are chords and the same is true in each of the other two cases.

7) but there is a difference in that angle A B C differs from angle 1 from angle 2 from angle 2

in this step

in what way? Sally

P: uh well angle A B C is made up of two chords neither of which are a diameter,

8) and

T: neither are a diameter is that right neither is a diameter would be better (laughter) What is the relation between angle 2, angle 3,

angle 1 in the first illustration, if any? I should of course say but there may not be any, let me look

9) Dick

P: uh angle 1 equals angle 2 plus angle 3

T: Now angle 1 is

if I had enough chalk I'd have you do some writing here, Robby, you need the practice

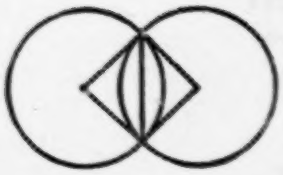
T: Angle 1 is related to an arc of the circle, which arc, and what is the relationship?

10) Doug.

P: It is equal in degree measure to arc AC

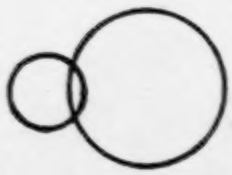
≐ T: Written that way (g) Now make sure that you understand what Doug has said about angle 1 being equal in degree measure

because we shall have soon and possibly have had in the last day or so, last Friday last Thursday, I mean, some arcs that were equal but not in linear units but in degrees only. For example,



11) in such a relationship as uh this, we may have the lines drawn which yield equal angles and therefore yield equal arcs

in degrees, but if the symbols are unequal we still may have the equal angles



12) and the arcs equal in degrees but not equal in linear units. What do I mean by linear units, Sue?

P: Well for example inches is the length to be applied

T: in.

P: inches or whatever you want to measure with

T: centimeters

P: sure

T: and to what degrees you are saying, Doug, is equal to

P: arc AC
T: so I shall write here

13) angle 1 may have substituted for it its equal arc AC. May I make the same kind of substitution for angle 3, Bill

$\angle 1 \doteq \overset{\frown}{AC}$

P: No

T: Well why may I sometimes may I make that substitution, other times not make it? You didn't object to my making that substitution here,

14) Bill
P: No
T: But you wouldn't agree to it being made here, and rightly so. What is the reason for that difference?

Yes here, no there. The reason, Joe
P: I can't see -
T: stand up so you could see over Herb's shoulder.

15) The exercise might be beneficial too, Joe. Stand up (laughter)

$\angle 3 + \angle 2 = \angle 1$

P: Angle 3 plus angle 2 equals angle 1.
T: Very good, that's reason enough. Now let that stand

then now what do we do about angle 2? Well angle 2 is to be the subject of the sentence. Angle 2 is the angle I had written first. So I write it that way.

$\angle 2 + \angle 3 = \overset{\frown}{AC}$

16) A central angle such as angle 1 hum has as many degrees written this way as the arc cut off by the sides of the angle. What is the relationship between angles 2 and 3 hum in this sketch, if any? Stetson

$\angle 2 = \angle 3$

17) P: Well they-re equal
T: They are equal, the reason why they're equal, Sally
P: Subtraction axiom
T: The reason why they are equal, Helen?
P: In the triangle

the two sides are equal so opposite angles are equal.
T: Yes so you see I don't need to hunt for any hum arc that Bill didn't agree to being used here for angle 3, I'll just put in another angle

$\angle 2 + \angle 2 = \widehat{AC}$ 18) 2 since 2 and 3 are equal, and now I can make angle 2 the subject of the sentence, by what authority, Robert?

$\frac{2\angle 2}{2} = \frac{\widehat{AC}}{2}$

P: Division axiom
T: Speak up so we can hear you Robert.
P: Division axiom
T: Very good. Let's move over here. We may note that vertex of angle 2 being on the circle is repeated here, one side of angle 2

(case 2)

$\angle 1 \overset{\circ}{=} \frac{1}{2}\widehat{AD}$ 19) is a chord, the other side of angle 2 is a chord; even BC, the lettering is the same. How then may we express angle 1

in this sketch in terms of degrees? Doug
P: uh it's ½ arc A D (teacher writes)

$\angle 2 \overset{\circ}{=} \frac{1}{2}\widehat{BC}$ 20) and why may I write angle 2 in degrees is equal to ½ arc BC, Johnny?
P: ??
T: Using the addition axiom

Jim will you write the conclusion

$\angle 1 + \angle 2 \overset{\circ}{=} (\widehat{AD} + \widehat{BC})$ 21) Thank you. You know why I asked you to write that?
P: ??

T: Case 3. Angle B

(case 3)

$\angle ABC \overset{\circ}{=} \frac{\widehat{AC}}{2}$ 22) can be expressed in terms of an arc, Margy
P: equals ½ arc AC

T: and uh angle ABD, in like manner, John
P: equals ½ arc AD

$\angle ABD \overset{\circ}{=} \frac{\widehat{AD}}{2}$

$\angle DBC \overset{\circ}{=} \frac{\widehat{CD}}{2}$ 23) T: By what authority then may I write angle D B C equals ½ C D, if any? I'll put

the degree sign in, hum, by what authority, if any, hum, Betsy?

24) P: Subtraction axiom
T: Subtraction axiom, very good. hum. Does it seem possible now, Miss ? without very much effort to use the subtraction axiom

where a few weeks ago we were always looking to see, were there three sets of equalities so that we had equals subtracted from equals to get equals. Almost no one has to look back now to see

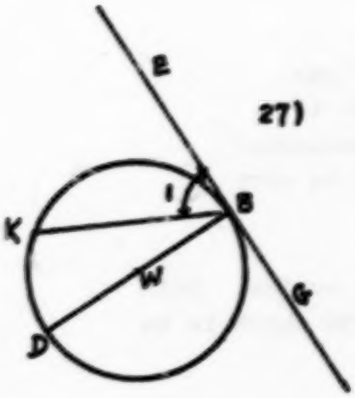
25) which has the application of subtraction axiom and addition axiom because you have only two of the three

equalities required or because you ignore the use of parenthesis and write something like that, where we have half of one

of the arcs but not half of the other. One sentence now which summarizes case 1 plus case 2 plus case 3. One sentence.

26) One statement in summary, Bill.

P: The inscribed angle equals one-half, equals in terms of degrees one-half the arc that the sides intercept
T: Very well



27) Very well. Your neighbor might wish to say it differently, but that's accurate and that's complete. In the next illustration

angle 1 is not an inscribed angle. What does it lack? Earlier somebody pointed out that angle B in each of the first three illustrations has its vertex on the circle, I believe Eddie pointed that out to us

28) Well it still has its vertex on the circle so that isn't a difference. The difference between the inscribed angles which Bill summarized and the

new angle one which has its vertex on the circle, the difference must lie with regard something other than the position of the vertex. What difference do you observe, Ann?

29) P: Well, an inscribed angle is made up of two chords, and
T: Very well. I'll write angle 1 plus angle KBD

and leave the right member of that sentence to you. Write the entire sentence,

$\angle 1 + \angle KBD \doteq 90^\circ$ 30) reread it and then if you still believe that what you wrote is a true statement, read it aloud,

Angle 1 plus angle KBD equals, - what is it equal to, Steve?
P: 90 degrees
T: That was a very short way

31) of finishing a statement. Can someone give us a reason why angle 1 plus angle KBD under these conditions yields a right

angle? Can you see it Steve?
P: If a diameter and a tangent meet at the same point on a circle they are perpendicular.

32) T: Has he jumped to some conclusion there that is not warranted by the data given, Steve?

P: I don't think so.

T: Someone might inquire why he for example assumed that to be a tangent. Now that he has chosen to use it as such, let's agree with him the EBD

33) is tangent to the circle W, and from the point of contact the diameter and chord are drawn in the circle. Then we have angle 1 plus angle KBD equals 90 degrees for the reason he gave, except it would be better I guess here to say perpendicular lines

form right angles than to say a perpendicular, no a diameter drawn to the point of contact of a tangent is perpendicular to the tangent. The reason you gave would be very fine for a reason why lines

34) are perpendicular but not why angles are right angles. Well with that established then, I'll write angle KBD which is an inscribed angle, and

$\angle KBD \doteq \frac{1}{2}\widehat{KD}$

therefore it falls in the classification that Bill has used for the first three cases, equals ½ arc KD, that's in degrees

35) so it seems I can immediately say what angle 1 is equal to in degrees. Will you try finishing that statement? Jessie

P: 90 degrees - ½ arc KD

$\angle 1 \doteq 90^\circ - \frac{1}{2}\widehat{KD}$

36) T: Very well. How can we rewrite this right member which has in it a whole number and a fraction multiplied by a whole number

I'll believe I'll try to rewrite the entire sentence and write angle 1 equals, and have a denominator of 2. What should I make the numerator then? Jim

37) P: Arc KD

T: By what authority may I rewrite the right member that way? Sydney

$= \frac{180^\circ - \widehat{KD}}{2}$

38) Steve?

P: Well you really didn't do anything, you just changed things around so they're still in the same order.

T: I changed two fractions to the same denominator, one had the denominator 1 - I made its denominator 2. I did not operate on both members of the equation; there was no axiom to

$\angle 1 \overset{\circ}{=} \overset{\frown}{DKB} - \overset{\frown}{KD}$

39) cover such. However, there is an axiom that covers the authority for the new and last statement I've written here, namely that angle 1 equals one-half the difference arc DKB

and arc KD. Perhaps you can tell us what that is, Bill?

40) Here I was only using the right member of this last line but now,

now the reason is entirely different, it's not changing two fractions to the same denominator, they have the same denominator now as they had in this next to last line. The reason, Laura?

P: (mumbling)

T: The reason, Margie?

41) P: Substitution axiom.

T: Yes, certainly. What was the quantity that was substituted for its equal, Betsy?

P: DKB here was substituted for 90 degrees

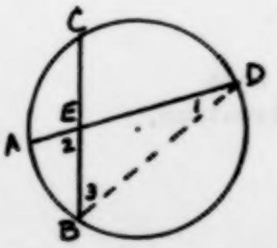
T: I believe that I shall not leave it this way for the final statement. Maybe you can suggest what would be a bit simpler

$\angle 1 = \frac{\overset{\frown}{KB}}{2}$

42) than this expression Johnnie?

P: Angle 1 equals $\frac{KB}{2}$

T: Now this next arrangement should require less time than the last one did. We have an arrangement angle 2



43) angle 3, angle 1. Tell us what it is, Richard?

P: Angle 2 equals angle 3 plus angle 1

T: And angle 3, an inscribed angle, half arc CD, angle 1, an inscribed angle, half arc AB, so angle 2

$\angle 2 = \angle 3 + \angle 1$

44) Margery, is equal to

P: Half arc CD plus AB,

$\angle 2 = \frac{\overset{\frown}{CD}}{2} - \frac{\overset{\frown}{AB}}{2}$

in parentheses, CD plus AB

T: Correction - no correction? Go on with the next one? No, not quite.

$\angle 2 = \frac{1}{2}(\overset{\frown}{CD} + \overset{\frown}{AB})$

45) What do you notice, Margie?

$\angle 2 = \frac{\overset{\frown}{CD}}{2} + \frac{\overset{\frown}{AB}}{2}$

P: On one you have a minus sign, and on the other you have an addition sign.

T: Yes, do you know why I have one minus and the other plus?

P:

T: That's more consistent, isn't it?

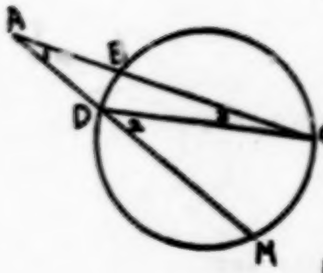
The vertex is inside, the vertex is out

46) any comment up there, Ed?

P: Well it should be plus.

T: Well that's what she said because Margie objected, so this should be plus,

possibly the preceding should be plus, and the preceding and is, now is everything alright? We'll let Ann reread that as often as she needs until, she is,



47) well, I wouldn't say in agreement with this but until she is quite sure it is incorrect or correct, whichever it is. Here I notice

the angle I have written is outside the circle rather than inside, rather than on as before, but there still is the angle 1, angle 2, angle 3.

$\angle 2 \; \angle 3 \; \angle 1$

$\angle 2 = \angle 3 + \angle 1$

48) How are they related in this new sketch, - the three angles?

As before is angle 2 equal to angle 3 plus angle 1, it was in the last sketch. Does angle 2 equal angle 3 plus angle 1 here and for the same

49) reason as before, Joe?

P: Yes sir

T: Now what substitution shall I make if I wish to make angle 1 finally the subject of the sentence?

What kind of angle is angle 2, because of its location, Doug?

P: It's an inscribed angle

T: Therefore

$\frac{\overset{\frown}{CM}}{2} = \frac{\overset{\frown}{BD}}{2} + \angle 1$

50) since angle 2 is inscribed in the circle how may I express it in terms of arcs?

P: It's equal to one-half arc CM.

T: And angle 3?

P: It's equal to one-half arc BD.

T: So we're ready for a statement about angle 1

51) in terms of arcs. This angle formed by two secants, secants ABC and ADM, that's the way they are read, and conversely

if you wish to write secant ADM, it must be in the position where the first letter you speak is outside the circle.

52) Angle 1 the subject of the sentence then, Rich?
P: Angle 1 equals B, no $\overset{\frown}{CM}/2$

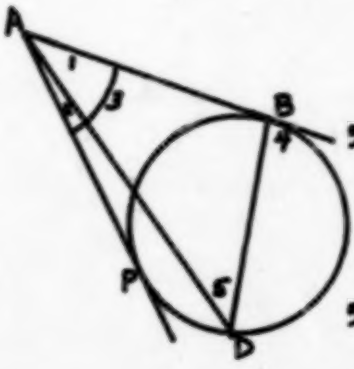
$\angle 1 = \frac{\overset{\frown}{CM}}{2} - \frac{\overset{\frown}{BD}}{2}$

minus $\overset{\frown}{BD}/2$

53) T: Write out

$= \frac{1}{2}(\overset{\frown}{CM} - \overset{\frown}{BD})$

the statements for the next step, Jim, where angle 2 becomes the subject of the sentence, the conclusion.



54) P: Write the equation?
T: We want finally angle 3 equals

P: (writes on board) $\angle 2 = \overset{\frown}{PD}/2$, $\angle 1 = \frac{\overset{\frown}{BD} - \overset{\frown}{BC}}{2}$

55) $\angle 3 = \frac{\overset{\frown}{BD} - \overset{\frown}{BC} + \overset{\frown}{PD}}{2}$

/

56) $\angle 2 = \frac{\overset{\frown}{PD} - \overset{\frown}{PC}}{2}$

$\angle 2 = \frac{\overset{\frown}{PD}}{2}$

$\angle 1 = \frac{\overset{\frown}{BD} - \overset{\frown}{BC}}{2}$

$\angle 3 = \frac{\overset{\frown}{BD} - \overset{\frown}{BC} + \overset{\frown}{PD}}{2}$

$\angle 2 = \frac{\overset{\frown}{PD} - \overset{\frown}{PC}}{2}$

T: Any comments, Sydney?

P: angle 2 $= \frac{\overset{\frown}{PD} - \overset{\frown}{PC}}{2}$

T: Sydney has noted

57) that if angle 1 is half the difference of 2 arcs angle 2 must be half the difference of two arcs.

(P. still writing) Now will

$\angle 3 = \frac{\overset{\frown}{BD} - \overset{\frown}{BC} + \overset{\frown}{PD} - \overset{\frown}{PC}}{2}$

58) that have any effect on your last statement, Jim?
P: (Writes correctly) angle 3 $= \frac{\overset{\frown}{BD} - \overset{\frown}{BC} + \overset{\frown}{PD} - \overset{\frown}{PC}}{2}$

T: Now shall

we accept that the way it's written, Barbie?

59) P: I think parentheses should be around BD and BC.
(she writes on board: $\angle 3 = \frac{(\overset{\frown}{BD} - \overset{\frown}{BC}) + (PD - PC)}{2}$

$\angle 3 = \frac{(\overset{\frown}{BD} - \overset{\frown}{BC}) + (\overset{\frown}{PD} - \overset{\frown}{PC})}{2}$

T: Will you speak up so we can catch your voice here

P: Parentheses should be around $\overset{\frown}{BD}$ —

60) BC and around arc PD - PC,
T: Can we

write that last statement with fewer terms? Try it, Jim.

Typescript #4

Algebra, Grade 11

Blackboard notes

1) T: How would you prove

$\forall n \quad 1+n=n+1$

for each counting number, n, 1 plus n equals n plus 1 without using

2) the commutative principle for addition? Gene.
P: Prove that four is hereditary and one has the property...
T: Wait a minute--prove what is hereditary?

P: Prove that the statement, the expression 1 plus n equals n plus 1, prove that that property is hereditary.
T: What is the property here? Hank.

$P: 1+\ldots=\ldots+1$

3) P! The property P asserts that 1 plus dot dot dot equals dot dot dot plu one.
T! 1 plus ... equals ... plus 1--oh, did I get this backwards.

P: No you got it right.
T: All right, this

4) generalization is--we're trying to prove that this generalization

has a property which is hereditary, the property being that 1 plus ... equals ... plus 1. Now let's show how this is hereditary.

5) Gene?
P: For each k, k plus 1 equals 1 plus k, then k plus 1 plus 1 equals 1 plus k plus 1.
T: You are following the pattern that

they have used in the book here and this has since been revised a little bit and I think it makes it a little clearer so that we see that we aren't using the thing we're trying

Suppose for some k
that $1+k=k+1$

6) to prove in the proof, so we say, suppose for some counting number k

that k plus 1, that k has this property and this is, that 1 plus k equals k plus 1, all right; then 1

Then
$1+(k+1)=(k+1)+1$

7) P:plus the quantity k plus 1 equals the quantity k plus 1 plus 1.

T: 1 plus quantity k plus 1 equals quantity k plus 1. How dc you know it does?

8) If we knew this, immediately then we'd have this. We could show that it is hereditary, but there is a gap

in here to show that this is equivalent to this. We somehow need to fill that gap.

$1+(k+1)=(1+k)+1$, (apa) 9) Without of course using the commutative principle so we can't just do this. Bob?
P: Can't you just associate

and have the first association 1 and the k in the first part, the left hand quantity, 1 plus k equals 1.
T: This is useing the associative principle for addition, all right?
P: And then

10) you can reassociate again to get it, well you can just put that thing down where that other blank is
T: Why?
P: Because you've

T: The commutative principle for addition, no, you can't use that.
P: Isn't it something, the second part of the recursive definition....

11) T: There's no recursive definition used here. John?
P: Oh, the hypothesis.
T: Yes. In every problem that we run into that we use induction we have a hypothesis.

For proving that it is hereditary we have to use that hypothesis so we suppose that some 1 plus k equals k plus 1, and here we

12) get equal 1 plus k, I beg your pardon, here we go right.. here, 1 plus k equals k plus 1, the recursive definition.

Hence......I beg your pardon.....Hence for each k

$\forall k$ if $1+k=k+1$ then $1+(k+1)=(k+1)+1$

13) if 1 plus k equals k plus 1, then 1 plus k plus one

equals k plus 1 plus 1. This shows that this property we have in mind is hereditary.

14) P: No--equals k plus 1 plus 1.
T: Thank you.

P: In your first step when you say then, when you have equals, is it 1 plus k?
T: Yes.

15) P: Well why do you have that instead of k plus 1?
T: How would you get k plus 1?
P: Well because you had you had k plus 1 before.

T: No you missed the boat on this. See this
property here?
P: Yes.

16) T: We've got written down, we put k plus 1 in the blank,
k plus 1 in the blank, that's what we'd like to be able
to do, but we don't, we put k plus 1 here and show that
we can come up with k plus 1 here,

in the other blank.
See 1 plus the quantity k plus 1 we know by the
associative principle for addition is 1 plus k
plus 1, and that in

17) turn, through our hypothesis, is k plus 1 plus 1. Bill?
P: Well what about doing something like, like taking this
quantity and then subtracting 1 from both sides of it and

then show that one quantity equals another
quantity, and then working from there.
T: What quantity are you going to take to begin
with?
P: Well, you could take something like k or,
well I used a instead,

18) I don't know why.
T: Time out....

he thinks this is candid camera.....(laughter)

19) T: What was the question? Oh you had some equation, you
were going to transform this equation, now what equation
did you have?

P: If you took a, it't not an equation just a
property you got 1 plus this, something like a

20) and equals a plus 1, and then you took and subtracted 1
from both sides of that because you know you can do that
because we'e always done it, I don't know what it's called
and then you, well all right

and then you go from there,
this property equals itself and then work from there.
T: All right, there are other ways of showing this,
true,

$1+n=n+1$

21) but I...no, now wait a minute, you know what, you want to
follow this through step by step and you've got something
like 1 plus n equals n plus 1, you want to write

$1+n-1=n+1-1$

1 plus n minus 1 equals n plus 1 minus 1,

22) how are you going to do this without using the commutative principle, somehow? You see to associate this one with this negative one we have to use the commutative principle.

P: Isn't that the addition transformation?
T: Yes
P: I added one to each side.
T: No, wait a minute. We know that n equals n for each n,

$n=n$

23) right? Now we know from our addition transformation that n plus 1 equals n plus 1. All right. Now what?

$n+1=n+1$

P: Can I, can I show what I did.
T: Umhum.

$1+k=k+1$

24) P: (at board) Well, I had this equation, 1 plus k equals k plus 1 and then by the addition transformation principle I added one to each side, and I got the quantity 1 plus k plus 1 equals quantity k plus 1 plus 1, and then I reasso- it and

$(1+k)+1=(k+1)+1$

$1+(k+1)=(k+1)+1$

25) got 1 plus quantity k plus 1 equals quantity k plus 1 plus 1, the same thing which is what you wanted to show.

T: Oh, all right, you are using here your hypothesis just the same, aren't you, supposing 1 plus k equals k plus 1, for each counting number k,

26) then 1 plus quantity k plus 1 equals quantity k plus 1 plus 1. All right this is an alternative which put in here is all right too. How about the first instance, is that easy? 1 plus 1 equals 1 plus 1, all right. Let's take another problem such as number 5.

#5.

$\forall n\ n^2+n$ is even

27) For each n, n^2 plus n is even. Louis?
P: First suppose for some counting number k, k^2 plus k is even.

28) Then the quantity k plus 1 squared plus quantity k plus 1 equals k^2 plus k plus 2 by the recursive definition.
T: k^2 plus ?
P: k plus 2
T: Now wait a minute.

29) P: She's confusing the next even number
T: Yes how do you know whether you have the next even number?

For instance, suppose we let n be some counting number,
pick
P: 3
T: All right, 3^2 plus 3 is what?

30) P: 12
T: 12 and that is even. Now suppose we pick the

next number, the next counting number 4, 4^2 plus 4,20--
that's not the next

31) even number.
P: It doesn't say it has to be next even number.
T: No it doesn't but that's the thing you're making use of by
adding 2 to this, you see.
P: Oh, I see.
T: So somehow we've got to go about this a

little differently.
Do we have anybody besides Bob and John? Bill?

32) P: Would there be a difference if you took all these different
examples, 1, 2, 3, and 4 wouldn't you find some sort of a pattern
which is the difference between the numbers?
T: But how could you prove this pattern holds for all of

these? Here again you would need mathematical induction.
P: That's not what you're trying to prove though.
Couldn't you just make use of the fact.
T: If you take instances and recognize

33) a pattern, this is nice, but how do you know that that pattern
always holds?
P: So you'd have to prove that first.
T: Right, right. John?

$(k+1)^2+k+1=k^2+2k+1+k+1$

P: I said, that same k, k plus 1, quantity squared
plus k plus 1 equals k^2 plus 2 plus 1, plus k plus 1.

34) by recursive definition.
T: No, by algebra
P: It could also be by recursive definition because you know
that $(k \text{ plus } 1)^2$.......
T: What recursive definition?

P: Well, couldn't you say
that you know the k plus 1 odd number or square number

35) is 2k plus 1 greater than the k^2 number so, that is $(k \text{ plus } 1)^2$.
Is 2k plus 1 greater than k^2?
T: Oh,

all right. I guess we could. We'd be using two recursive definitions, or using it twice, all right. Or just expanding we get this.

36) P: And then I commuted around and associated again and got the quantity (k^2 plus k) plus (2k plus 2)

$= k^2 + k + (2k+2)$

which is (k^2 plus k) plus 2(k plus 1), and then I said, now since

$= (k^2+k) + 2(k+1)$

37) (k^2 plus k) is even we know that there is some counting number a such that 2a equals k^2 plus k.

$2a = k^2 + k$

T: Say this once more.

P: Since k^2 plus k is even, we said that in the inductive hypothesis, that, then we know there exists

38) some counting number a such that 2a equals k^2 plus k.

Since we know k is even, then there is some a such that $2a = k^2 + k$

so $(k+1)^2 + (k+1) = 2a + 2(k+1)$

$= 2(a+k+1)$

T: Equals what?

P: k^2 plus k, and then I substituted into what I had put down there and I got....

39) Just put another equals underneath down there and say equals 2a plus 2(k plus 1),

which equals 2 times the whole quantity (a plus k plus 1),

40) and since a plus k plus 1 is a counting number, and 2(a plus k plus 1) equals (k plus 1)^2 plus (k plus 1), (k plus 1)^2 plus (k plus 1) is even.

T: All right, all right. I think we might...I'll tell you what we need, something back in Unit 4

41) where we did prove that addition of even integers is closed. Adding an even integer to an even integer you get an even integer.

P: Then you could have stopped right at that step.

T: That's right. Then by inductive hypothesis, k^2 plus k is even and since k plus 1 is a counting number, twice that is an even counting number, so we have the sum of an even number and an even number.

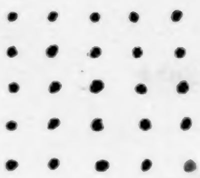
42) All right. Let' try

P: 6.

T: No, I want to hold off on that.

43) You remember yesterday we were investigating square numbers, figure numbers,

in particular, square numbers and trinagular numbers. We saw some patterns that existed here, and there's another pattern,

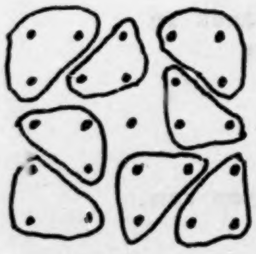


44) a rather interesting little thing. This is...which square number is this?
P: The fifth
T: The fifther square number, right, which means we've got, what is the fifth square number?
P: 25.
T: So we have indicated 25 points in square array. Now, the second triangle number has how

45) many points?
P: 3
T: 3, the second triangular number is 3 as we mentioned before. You recall, we had the triangular numbers, we have 1, 3, 6
P: 3, 6, 10, 15.
T: 15, 21, etc. The square numbers we have
P: 1, 4, 9, 16, 25, 36..
T: Very good,

46) Suppose I were to draw a loop around 3 of these dots that are indicated in the triangular array here, and we want to do that with lots of these. Can somebody come up and do this for us,

47) doing as many as we can
P: Without overlapping?
T: Without overlapping.
P: You mean you want groups of 3?
T: Right No three collinear but 3 in this triangular shape. It can be upside down, I don't care just so long as you get when you're finished you've got what looks like groups of triangular numbers.



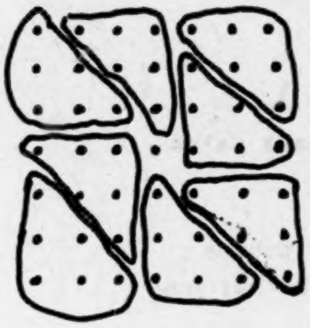
48) each with 3 dots in it. I'll do one more here, for instance we might take thi, all right, Joanie come on up and draw a loop around as many more of these as you can.

49) (Joanie draws loop)

50) How many did you get?

P: There are 8,

T: And one point left over. All right, I've got here a figurate number,



51) namely the seventh square number. Suppose we investigate doing the same thing,

and here we will take the third triangular number which has six dots. Somebody come up and do a similar process

52) to what she has done.

P: It's the same thing. It's the same thing.

T: All right, then draw your dots differently. Here I'll start off this way.

/

53) /

Like this, somebody come help me,

54) Bill.

Put as many of these in as you can.

55) P: Like a jig saw puzzle?

P: No. We won't get so many that way will we?

56) P: You get eight triangles.

T: How many of these do we have?

P: Eight.

T: Eight again. It is possible not to get this center dot as the dot that't left over each time.

57) For instance in this picture here I could put pink ones in here and leave this as the extra dot. Think we can?

P: Yes.

T: It had better be if this is the case.

P: Yeh you can.

58) T: Something's funny.

P: You included 4 in the top...you changed the top 2.

59) T: Well, I think you can, you try it. Let's see.

I did it right here. Well let's generalize this.

60) Here we had the fifth square number and we found it was equivalent to

let's see, how many triangular numbers did we have?
P: Eight.
T: And which triangular number was it?
P: Second, 6.
T: Square 5 equals 8 triangular plus 1.

Typescript #5

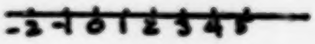
Algebra, Grade 9

Blackboard notes

1) T: Everybody see this microphone here? Everybody see that microphone there on the floor? The box over here, take one good look, then let's forget about them. O.K. Alright, now wel'll forget about them.

We are coming to the end of our unit and at the end of this unit we have some miscellaneous exercises which review some notions that we've picked up

2) throughout the unit, and kind of pull two or three ideas together. Let me introduce this notion this way



What's this a picture of, Sally?
P: Part of the number line
T: Right, alright. Do you recall

3) that we had some games in which we made some moves, say X moves to X plus 3,

$x \rightarrow x+3$

remember this?
P: (in unison) yes
T: you see that doesn't see your nodding heads (laughter) O.K. fine, suppose we start

4) at 2 where do we get in one jump? Sheryl
P: 5
T: Right, and two jumps. From the starting at 2, where do we go

2	5
17	20
0	3
-5	-2

in 2 jumps? Mary Beth
P: 8
T: Right, and so on and so forth. Now we might say that this is a kind of operations. What operation

5) would you say that this is? Well, we started at 2 and we got in one jump to 5. We started

at 17 in one jump we get what?
P: 20
T: 20 we start at 0 and in one jump we get to
P: 3
T: Start at negative 5
P: negative

6) 2
T: Alright. Does this suggest some operation? Go ahead
P: adding 3

T: Yes, adding 3. You say this is the operation of
adding 3.

7) P: Yes.
T: O.K. Can you find a sub set of the set of real
numbers

for which the operation adding 3 is closed
George?
P: Set of positive integers
T: Set of positive integers

8) Oh, then our picture would look something like this.
Which would this one be?
P: Zero

T: Alright, and this
P: Would be 1
T: and so on and so forth. Alright, so that if we
apply this rule we could start

9) with any one of those - just wait a minute, Dick
P: the zero isn't a positive integer
T: Alright, so what should we start with?
P: One.
T: O.K.

Alright. So we could start with any one of
these integers like 4 by this operation and we get
in one jump we get what
P: 7

10) T: 7 Alright. Can you make up an operation different from
this operation

uh such that this set of numbers that
we're working with for that operation is not closed ?

11) Scott
P: Well, you could change it to uh X to X minus 3 or you
could say
T: Alright, let's see, X goes

$x \to x-3$

to X minus 3 and uh we
pick any one of these like 4 and you apply this
operation you get 1 put
P: 1

12) anything less than 4 it comes out negative or zero
T: ? ? Alright, how about this operation

on this set
of real numbers, the set we had pictured here, Jimmy
P: It's gonna be closed
T: Yes it will be. Can you find a

13) subset different from this sub set for which this is closed? Some sub set of this?

Mary lou

P: uh non positive set

T: Alright are we including

14) zero

P: ? ?

T: Sally

P: I think maybe 1 minus 3 ???

T: Well how about zero?

Suppose we start at zero.

Zero's a new set of non-positive integers alright

P: um hum

T: Alright. Let's see if we can find a different

15) kind of operation other than adding 3. Well let's see. First of all let's find one different from this one for which this set

is not closed. Jimmy

P: Dividing by 3

$x \to \frac{x}{3}$

T: That's nice. X goes into X divided by 3.

16) Well can we find a different sub set in this set of positive intagers for which is is closed? This operation

Bill

P: The multiples, all multiples of 3

17) **T:** Let's see, for instance what would we have then? 3

P: 3

T: 3

P: 6 9 12

$-3 \quad 0 \quad 3 \quad 6 \quad 9 \quad 12$

uh

T: any down this way oh

P: uh zero

T: zero

18) **P:** negative 3

T: negative 3

P: negative 6

T: and so forth

P: yes

T: Alright, now let's see, it,this operation of dividing by 3 closed for

that set of intagers. Sally?

19) **P:** No it isn't

T: Pat

P: No

T: Why not

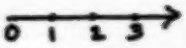
P: Well, if you'd use a multiple of 3 when you divide by 3 you don't get another multiple of 3, you get ???

T: you get an

integer but not a multiple of 3, that's right, so this would not be satisfactory. Do you have another one? Mary Lou

20) P: set of positive numbers maybe you better say integers
T: be hard way Jimmy
P: Well integers wouldn't work because sometime you're gonna get a fraction

it could be 2/3
T: So what would you like now?
P: A set of non negative numbers
T: Alright, in other words this would uh correspond to this picture, this point



21) would be the point zero, al~ight 1, 2, 3, and so forth. Now pick any member of this set operated on that number according to this

rule we get a number of this set is what she says. Now can you find a smaller set?

22)

Mary Dell
P: Could you use a set of positive integers
T: What

23) did Jimmy just say?
P: It wouldn't be closed
T: Why?
P: Because sometimes when you divide by 3 you get a fraction as if you divide by 4 by 3
T: O.K. Sally.

P: Would it be positive ??? rationals
T: Hum positive rationals Divide any positive rational by 3 you get a positive rational?

24) What.
P: Yes
T: Let's see, how do we make a picture of that?

P: (Laughter) You can't
T: Can't
P: Well unless you want to diagram
T: Well what is the matter with this picture?

25) P: ???
T: What's this?
P: Because if you draw a picture like that in between the 1 and the zero ? some repeating decimals
T: Oh, well well
P: Repeating decimals are rationals well there are some things like pi

T: Yes pi is in here somewhere

P: ???
T: Square root of two words, square of two
P: ?? 1.4
T: Somewhere in here, yes, so we have

26) to exclude some of those, wouldn't we? Alright, so we take some points out (laughter) How many points will we have to take out?
P: Quite a few
T: Quite a few,

in fact, what about this line, or this picture, be a picture of a ray full of holes wouldn't it? You know something we will learn a little bit about later

27) is that there would be more holes in it than there would be points in it. Alright, let's find another sub set

(laughter) But that was a good one, in fact you were correct with that set. Cathy

28) P: um couldn't you have a set of negative

nonpositive

numbers?
T: Non positive reals
P: Um huh similar set

29) T: Doubt that You divide any non positive real number by 3 and you would get a non positive real number No
P: What about zeros

T: Zeros is zero nonpositive?
P: (all talking)
T: Alright, let me suggest

$\{x : 1 \le x \le 2\}$

30) a set for you. How about this set? Set of x say 1 is less than or equal to X is less than or equal to 2

How

many would think that this set

31) is closed on this operation? How many think it's not closed? Oh, you're still thinking. Alright

/

32) P: Well I think it would be closed because if you take a number which is less than then 2 or uh more than 1 and

you put that over the 3 your gonna get a number such as 1/3 or 2/3
T: But what
P: Is that number ?? not in that set
T: Is for instance, pick some number in this set.

33) P: 1
T: Divide by 3
P: 1/3
T: is 1/3 in the set
P: No

T: No. So what would you say now? the set is
closed under this operation of dividing by 3 or is
not closed?
P: No.
T: Not closed. Alright. Could you find a set kind
of like this one

34) that is closed? Dick
P: You show a set that shows that zero was less than
or equal to is less than or equal to 3

T: You've
some ideas on this? Pat

35) P: I think it is closed
T: Pat says it is closed under the operation dividing
by 3 - you said - how many agree with her - how many
disagree.

Alright - you disagree you have to counter
example.
P: Well, if you took 1/2 one and 1/2 that would be
3 halves

36) and then times 3 which would be divided by 3 4/3
(laughter)
T: 3 then divide this by 3
P: that would work
T: O.K. good. Can you find a smaller sub set of a

37) set of real numbers for which this operation is closed?
P: Smaller than the one that was just given?
T: um huh

by smaller I mean that the picture here
would be would have a measure shorter than the
measures in the one we just tried.
P: set of

38) such as zero is less than or equal to X less than or
equal to more (?)
T: yes, alright, that would certainly satisfy the
conditions wouldn't it, Mary Beth
P: ???

T: Set of X equals 9 That wouldn't work
(laughter)

39) P: set of X such as X is zero

T: How about that, Sarah?
Very good. Can you find another set like this?

40) Such as when we performed this operation on any number
of that set we get a number of that set

Shaking the
head means no.

41) Let's go back to this operation just a minute then
we'll come back to this one, can you find a sub set
of this set of real numbers here which has what

we
might call an upper bound, you see the set that
we have described here a moment ago, a set of X
1 is less than or equal to X

42) is less than or equal to 3 or 0 is less than or equal
to X is less than or equal to 3 has an upper bound
namely what, namely what?

$\{x : 0 \leq x \leq 3\}$

P: 3
T: Can you find an upper

bound for a set here
for which this operation is closed? Jimmy
P: I don't believe so because we can always
add 3

43) to whatever we get.
T: Right. Can you find a set that has a lower bound
Dick?
P: I don't think so because you keep right on going

T: lower bound?
P: Oh, a lower bound, no no you can't because no
matter no still can't

44) T: Carol
P: is you still can go one below ?? Sir - you can 5
trillion or any number higher but take 3 away from it

you get a number that's lower than that you just get
T: Oh take 3 away from it - what operation is that?
What operation are we dealing with?
P: I'm thinking of it in terms of negative numbers.

45) T: Well the 3 trillion didn't sound very negative
P: Negative alright negative 3 trillion or something
like that.

T: That's pretty negative
P: Then add 3 and you get a number smaller than that

You couldn't ever found the beginning, the end
whatever it is
T: This one that you had over here a minute ago,
Dick, does it have, this set, does it have a
lower bound?
P: Yes
T: What

46) is it?
P: Zero
T: Does it have an upper bound?
P: 3
T: 3. We just finished saying that we couldn't uh find
an upper bound here, how about a lower bound? George
P: Well,

take negative 1.
T: Alright, what do we do with negative 1?
P: Add 3 to it you won't uh get a negative answer
of course 3 isn't negative.

47) T: um hum. Mary Lou.
P: Well any number that you take that's a bigger number
than 3 and take the opposite of it to add to 3, well it's
hard to explain it but take negative 5, it's gonna be
negative 2, it's gonna keep getting smaller

and bigger,
a little bit larger (several talking) it'll get
larger so maybe you can't
T: Let's go back to one of these sets that you had
picked - you had picked

48) the set of non negative reals I believe - at one time
you did, now does that particular set pictured here

have a lower bound? Sally.
P: Yes.
T: And yes so therefore can you find one that
has an upper bound?

49) P: No
T: No. Alright
P: You can't find something with an upper bound?
T: For this operation adding 3, alright, can you
find an operation

that does have an upper bound,
an operation involving addition? Sally.
P: No cause you always

50) can add on there's always a line
T: Pat
P: Well couldn't your lower bound be whatever it is in the set up there?
T: Well, this
means that we're picking any number from our set you see and if we're including our lower bounds such as we did here you said less than or equal to, then or course

51) X would be that number which we're considering as a lower bound there. George?
P: Well it doesn't say what you're adding, you could add the opposite of the number
T: What do you mean. Opposite of what
P: Add the opposite of a positive
number.
T: Allright, Fred? Which is sometimes said as adding what kind of a number?
P: A negative number
T: A negative number. Then that would have an upper bound

52) you say? It could have, - I mean we could find a set, a sub set of this that has an upper bound and that set is closed under the operation adding negative 3
Can you find one that has a lower bound? - under that operation - adding negative 3? Jimmy

53) P: No, I don't believe so because it's..no matter how low your number is you can always add negative three to it and get a lower number.
T: Good.
P: Well we've already proved that that operation is closed under the set of real numbers, and in the set of real numbers there's no limit

54) to the real numbers so there couldn't be an upper bound and a lower bound.
T: Oh, you mean there can't be both.
P: Yes
T: For the operation of adding some real number, she says. Do you agree with that? Mary Lou?

55) P: That's what I was going to say but --
T: How about - Let's see, I'm thinking of one number perhaps that this doesn't work for - your statement

why?

P: Zero

T: Yes, the operation of adding zero. Now can we find a

56) set which is a subset of the real numbers for which this operation is closed. Evelyn?

George?

$\{0\}$

P: The set of zero?

T: What do you mean, the set consisting of zero? Let's see, pick a

57) member of this set (amusement) got one? - which one are you going to pick, zero, add zero to that, do we stay within our set?

P: Yes

T: Yes

Alright, can you find another set?

P: You can use any set, I mean just as long as it is one number like four, like the

58) set of four

T: The set consisting of four - that's closed under this operation

$\{4\}$

P: Well the set of adding negative integers, or of adding negative, non positive numbers - any set you'd pick,

it's going to work, as long as you don't mix the negative and the positive, or the non negative and the non positive.

T: Alright, now does this set have an upper bound?

P: Four

T: Does it have a lower bound?

P: Four

59) T: Can you find a set that has fewer members than that, that this operation, adding zero is closed for? Why?

P: No

T: No?

P: the empty set

T: The empty set, add zero to it, I wonder

60) what its upper bound is. Maybe we could use the operation of adding three again on this set, what would happen then?

P: Zero

T: Allright, let's expand our notion to two dimensions. Suppose we take the operation, I beg your pardon, these are